https://doi.org/10.31489/2024M1/140-148 *Research article*

A boundary value problem for the fourth-order degenerate equation of the mixed type

J.A. Otarova

Karakalpak State University named after Berdakh, Nukus, Uzbekistan
(E-mail: j.otarova@mail.ru)

Many problems in mechanics, physics, and geophysics lead to solving partial differential equations that are not included in the known classes of elliptic, parabolic or hyperbolic equations. Such equations, as a rule, began to be called non-classical equations of mathematical physics. The theory of degenerate equations is one of the central branches of the modern theory of partial differential equations. This is primarily due to the identification of a variety of applied problems, the mathematical modeling of which serves the study of various types of degenerate equations. The study of boundary value problems for mixed type's equations of the fourth-order with power-law degeneration remains relevant. In this work, a boundary value problem in a rectangular domain for a degenerate equation of the fourth-order mixed-type is posed and investigated. Well-posedness of the boundary value problem for a fourth-order partial differential equation is established by proving the existence and uniqueness of the solution. Under sufficient conditions, a solution to the problem under consideration was explicitly found by the variable separation method.

Keywords: fourth-order mixed type equation, Bessel functions, Fourier series, completeness, regular solution.

2020 Mathematics Subject Classification: 35M12.

Introduction

In the modern theory of partial differential equations, the studies of degenerate equations and equations of a mixed type occupy an important place, which is explained both by the theoretical significance of the results obtained and the presence of their practical applications in the gas dynamics of transonic flows, magnetic hydrodynamics, in the theory of infinitesimal bending of surfaces, in various sections of continuum mechanics and other branches of knowledge.

The fundamental results for second-order degenerate equations of elliptic type were obtained by academician M.V. Keldysh (1951), where first the cases were indicated in which the characteristic part of the domain boundary can be freed from boundary conditions, which are then replaced by the condition of boundedness of solutions. The work of M.V. Keldysh spurred numerous further studies in the direction he indicated. Later, A.V. Bitsadze noted in his work that the boundedness condition can be replaced by a boundary condition with a certain weight function. The results he obtained were then developed and generalized by O.A. Oleinik. It is also worth noting the works of V.P. Glushko, Yu.B. Savchenko [1], S.A. Iskhokov [2], S.N. Sidorov [3]. In particular, the work [2] was focused on the study of the unique solvability of a variational problem for an elliptic equation with a "non-power" degeneration. It is also noteworthy that a "power" degeneration was initiated in the research of M.I. Vishik and V.V. Grushin. Degenerate elliptic equations of high order, including their connection with pseudodifferential operators, were studied in the works of A.D. Baev [4–6].

K.B. Sabitov [7] investigated the Dirichlet problem for the second-order degenerate equation of the mixed type of first kind in a rectangular domain. By the methods of spectral analysis, the criteria of uniqueness of a solution that is constructed in the form of the sum of a Fourier series was established. The question of the correctness of the formulation of the Dirichlet problem depending on

Received: 10 July 2023; *Accepted:* 15 December 2023.

the degree of degeneracy was investigated for a mixed type equation of second kind by K.B. Sabitov and A.Kh. Tregubova (Suleimanova) [8, 9]. A boundary-value problem with nonlocal boundary conditions for a mixed type equation was studied by M.E. Lerner and O.A. Repin in work [10]. The uniqueness of solutions of the problem was proved by using the principle of extremum, the existence of solutions of the problem was proved by methods of integral transformations and equations. Nonlocal boundary value problems of Bitsadze-Samarsky type for a fourth-order mixed type equation were studied by L.R. Rustamova in [11]. Many authors also studied boundary value problems for degenerate equations [12–14]. In the present paper, the boundary value problem is studied for the fourth-order degenerate equation in a rectangular domain.

1 Problem formulation

In the domain

$$\Omega = \{(x,t) :\ 0 < x < b,\ -T < t < T\,,\ \ T > 0\}$$

we consider the equation

$$Lu \equiv \operatorname{sgn} t \cdot |t|^m u_{xxxx} - u_{tt} + a^2 \operatorname{sgn} t \cdot |t|^m u = 0, \tag{1}$$

where $m = const > 0,\ a = const \geq 0$. The equation $Lu = 0$ for $t > 0$ has the form

$$|t|^m \left(u_{xxxx} + a^2 u\right) - u_{tt} = 0, \tag{2}$$

for $t < 0$

$$|t|^m \left(u_{xxxx} + a^2 u\right) + u_{tt} = 0. \tag{3}$$

We denote $\Omega^+ = \Omega \cap (t > 0)\,, \Omega^- = \Omega \cap (t < 0)\,.$

Problem A. Find in the domain Ω a bounded function $u\,(x,t)$ satisfying the conditions

$$u\,(x,t) \in C\left(\bar{\Omega}\right) \cap C^{2,1}_{x,t}\,(\Omega) \cap C^{4,2}_{x,t}\left(\Omega^+ \cup \Omega^-\right), \tag{4}$$

$$Lu = 0, \quad (x,t) \in \Omega, \tag{5}$$

$$\frac{\partial^k u}{\partial t^k}\,(x,+0) = \frac{\partial^k u}{\partial t^k}\,(x,-0)\,, \quad k = 0,1 \tag{6}$$

$$\left.\begin{array}{r} u\,(0,t) = u\,(b,t) = 0, -T \leq t \leq T, \\ u_{xx}\,(0,t) = u_{xx}\,(b,t) = 0, -T \leq t \leq T, \end{array}\right\} \tag{7}$$

$$u\,(x,T) = \varphi\,(x)\,, \quad 0 \leq x \leq b, \tag{8}$$

$$u\,(x,-T) = \psi\,(x)\,, \quad 0 \leq x \leq b, \tag{9}$$

$\varphi\,(x)$ and $\psi\,(x)$ are given sufficiently smooth functions, moreover $\varphi\,(0) = \varphi\,(b) = 0$, $\psi\,(0) = \psi\,(b) = 0$.

2 The existence of a solution

To prove the existence of a solution to the problem, we use the method of separation of variables, i.e. particular solutions of equation (1) that are not equal to zero in the domain Ω, will be sought in the form of a product $u\,(x,t) = X\,(x) \cdot T\,(t)\,,$ satisfying zero boundary conditions (7). The following theorem holds:

Theorem 1.

$$\left\{\begin{array}{l} \gamma\,(k) = K_{1/(2q)}\,\left(p_k T^q\right) \neq 0, \\ \delta\,(k) = \tilde{Y}_{1/(2q)}\,\left(p_k T^q\right) \neq 0. \end{array}\right. \tag{10}$$

Proof. By substituting this product into equation (1), we obtain

$$X^{IV}(X) - \lambda^4 X(X) = 0, \quad 0 < x < b, \tag{11}$$

we solve equation (11) with conditions (7), which change to the following

$$X(0) = X(b) = X''(0) = X''(b) = 0, \tag{12}$$

$$T''(t) - \left(\lambda^4 + a^2\right) \operatorname{sgn} \cdot |t|^m T(t) = 0, \quad -T < t < T, \tag{13}$$

where λ is the separation constant.

The solution to problem (11), (12) has the form

$$X_k(x) = \sqrt{\frac{2}{b}} \sin \lambda_k x, \quad \lambda_k = \frac{k\pi}{b}, \quad k = 1, 2, \ldots \tag{14}$$

From equation (13), following [15], (with $\lambda = \lambda_k$) for $t > 0$ we obtain

$$T(t) = W(p_k t^q)\sqrt{t} = \sqrt{t}\, W(z), \tag{15}$$

in which $q = (m+2)/2$, $p_k^2 = \left(a^2 + \lambda_k\right)/q^2$. Then we obtain the modified Bessel equation [16]

$$W''(z) + \frac{1}{z} W'(z) - \left(1 + \frac{\nu^2}{z^2}\right) W(z) = 0,$$

where $z = p_k t^q$, $\nu = 1/(2q) = 1/(m+2) \in (0, 1/2)$, the general solution of which is determined by the formula as follows

$$W(z) = C_1 I_{1/(2q)}(z) + C_2 K_{1/(2q)}(z), \quad z > 0, \tag{16}$$

where $I_{1/(2q)}(z)$ and $K_{1/(2q)}(z)$ are the modified Bessel functions and C_1, C_2 are arbitrary constants. Taking into account (15), (16), the general solution (13) for $t > 0$ can be written as

$$T_k^+(t) = A_k \sqrt{t} I_{1/(2q)}(p_k t^q) + B_k \sqrt{t} K_{1/(2q)}(p_k t^q), \quad t > 0, \tag{17}$$

where A_k, B_k are arbitrary constants.

In the same way, from equation (13) for t<0 we get

$$T(t) = Z(p_k (-t)^q)\sqrt{-t} = \sqrt{-t} Z(z), \tag{18}$$

and the Bessel equation

$$Z''(z) + \frac{1}{z} Z'(z) + \left(1 - \frac{\nu^2}{z^2}\right) Z(z) = 0.$$

The general solution is written as

$$Z(z) = C_1 J_{1/(2q)}(z) + C_2 Y_{1/(2q)}(z), \quad z > 0, \tag{19}$$

where $J_{1/(2q)}(z)$, $Y_{1/(2q)}(z)$ are the Bessel functions. Taking into account (18), (19), the general solution of equation (13) for $t < 0$ can be written as

$$T_k^-(t) = C_k \sqrt{-t} J_{1/(2q)}(p_k (-t)^q) + D_k Y_{1/(2q)}(p_k (-t)^q), \quad t < 0,$$

where C_k, D_k are arbitrary constants.

Therefore, the solutions of equation (13) for $t > 0$ have the form of (17), and for $t < 0$ have the form of (21). To find the unknown constants A_k, B_k, C_k, D_k, we use the gluing conditions (6), that respectively change to the following conditions

$$T_k\left(0+0\right) = T_k\left(0-0\right), \tag{20}$$

$$T_k{}'\left(0+0\right) = T_k{}'\left(0-0\right). \tag{21}$$

Condition (20) is satisfied for any A_k, B_k if $D_k = -\pi B_k/2$, and condition (21) is satisfied for $C_k = \pi B_k ctg\left(\pi/(4q)\right)/2 \ - A_k$ and when $D_k = -\pi B_k/2$. Considering all of these, the solution of equation (13) can be written as

$$T_k\left(t\right) = \left\{ \begin{array}{c} T_k^+\left(t\right) = A_k\sqrt{t}I_{1/(2q)}\ \left(p_k t^q\right) + B_k\sqrt{t}K_{1/(2q)}\ \left(p_k t^q\right), \quad t>0, \\ T_k^-\left(t\right) = -A_k\sqrt{-t}J_{1/(2q)}\ \left(p_k(-t)^q\right) - \frac{1}{2}\pi B_k\tilde{Y}_{1/(2q)}\ \left(p_k(-t)^q\right), \quad t<0, \end{array} \right. \tag{22}$$

where

$$\tilde{Y}_{1/(2q)}\ \left(p_k(-t)^q\right) = \frac{\pi}{2\sin\left(\pi/2q\ \right)}\left[J_{1/(2q)}\ \left(p_k(-t)^q\right) + J_{-1/(2q)}\ \left(p_k(-t)^q\right)\right].$$

For function (22), the equality, $T_k{}''\left(0+0\right) = T_k{}''\left(0-0\right)$, holds i.e. functions (22) belong to the class $C^2\left[-T;T\right]$ and satisfy equation (13). Functions (22) are not limited, because $\sqrt{t}I_{1/(2q)}\ \left(p_k t^q\right) \to \infty$, therefore we assume $A_k = 0,\ \ \forall\, k \in N$, then

$$T_k\left(t\right) = \left\{ \begin{array}{l} T_k^+\left(t\right) = B_k\sqrt{t}K_{1/(2q)}\ \left(p_k t^q\right), \quad t>0, \\ T_k^-\left(t\right) = B_k\tilde{Y}_{1/(2q)}\ \left(p_k(-t)^q\right), \quad t<0. \end{array} \right. \tag{23}$$

3 The uniqueness of the solution

Theorem 2. If there is a solution to problem A, then it is unique when

$$\lim_{x\to 0+0} u_{xx}\left(x,t\right)\sin\frac{\pi k}{p}x = \lim_{x\to p-0} u_{xx}\left(x,t\right)\sin\frac{\pi k}{p}x = 0, \quad T \le t \le -T \tag{24}$$

and if condition (10) is satisfied for all $k \in N$.

Proof. Let $u\left(x,t\right)$ be the solution of problem (4)–(9). Consider the functions (14) which form into $L_2\left(0,b\right)$ a complete orthonormal system.

We denote

$$u\left(x,t\right) = \left\{ \begin{array}{l} u^+\left(x,t\right),\left(x,t\right)\in\Omega^+, \\ u^-\left(x,t\right),\left(x,t\right)\in\Omega^-. \end{array} \right. \tag{25}$$

We consider the integral

$$\int\limits_0^b u^+\left(x,t\right)X_k\left(x\right)dx = \alpha_k\left(t\right), \ \ k=1,2,\ldots \tag{26}$$

Suppose that the partial derivative $u_{xx}\left(x,t\right)$ satisfies conditions (24).

Differentiating (26) with respect to t twice, taking into account equation (2) and conditions (7) we have

$$\alpha''_k\left(t\right) - |t|^m\left(a^2 + \left(\frac{\pi k}{b}\right)^4\right)\alpha_k\left(t\right) = 0, \quad k=1,2,... \tag{27}$$

For negative values of t we denote the integral

$$\int\limits_0^b u^-(x,t)\,X_k(x)\,dx = \beta_k(t), \qquad k = 1,2,\ldots \tag{28}$$

By a similar transformation from (28) and (3) we obtain

$$\int\limits_0^b u^-(x,t)\,X_k(x)\,dx = \beta_k(t), \qquad k = 1,2,\ldots \tag{29}$$

Equations (27) and (29) for $\lambda = \lambda_k$ coincide with equation (13), i.e. for $t > 0,\ \alpha_k(t) = T_k^+(t)$, and for $t < 0$, there will be $\beta_k(t) = T_k^-(t)$, which means that functions $\alpha_k(t)$ and $\beta_k(t)$ are determined by functions (25). To find the coefficients B_k, we use the boundary conditions (8), (9), i.e. $\alpha_k(T) = \varphi_k,\ \ \beta_k(-T) = \psi_k$ (8), (9) and formulas (26), (28), then:

$$\alpha_k(T) = \int\limits_0^b u(x,T)\sin\frac{\pi k}{b}x dx = \int\limits_0^b \varphi(x)\sin\frac{\pi k}{b}x dx = \varphi_k, \tag{30}$$

$$\beta_k(-T) = \int\limits_0^b u(x,-T)\sin\frac{\pi k}{b}x dx = \int\limits_0^b \psi(x)\sin\frac{\pi k}{b}x dx = \psi_k, \tag{31}$$

then from (23), (30) and (31), taking into account condition (10), we have

$$\begin{cases} B_k = \frac{\varphi_k}{\sqrt{T}\gamma(k)}, & t > 0, \\ B_k = \frac{\psi_k}{\sqrt{T}\delta(k)}, & t < 0. \end{cases} \tag{32}$$

Substituting (32) into (23), we find the functions $T_k(t)$:

$$T_k(t) = \begin{cases} \varphi_k\sqrt{\frac{t}{T}}\frac{K_{1/(2q)}\,(p_k t^q)}{\gamma(k)}, & t > 0, \\ \psi_k\sqrt{\frac{-t}{T}}_k\frac{\tilde{Y}_{1/(2q)}\,(p_k(-t)^q)}{\delta(k)}, & t < 0. \end{cases} \tag{33}$$

Let now $\varphi(x) \equiv 0$ and $\psi(x) \equiv 0$. Then, from equalities (30), (31) and solution (33), it follows that $T_k(t) = 0, \forall\, k \in N$. Therefore, by virtue of (26) and (28)

$$\int\limits_0^b u^+(x,t)\,X_k(x)\,dx = 0, \qquad k = 1,2,\ldots$$

$$\int\limits_0^b u^-(x,t)\,X_k(x)\,dx = 0, \qquad k = 1,2,\ldots$$

Hence follows that $u(x,t) \equiv 0$ for all $x \in [0,b]$ and $t \in [-T;T]$, due to the completeness of system (24) in $L_2(0,b)$.

Based on the Bessel asymptotic formula [16], for large k, we have estimate

$$\begin{cases} \left|\sqrt{k}\delta(k)\right| \geq C > 0, \\ \left|\sqrt{k}\gamma(k)\right| \geq C_0 > 0. \end{cases} \tag{34}$$

Under conditions (10) and (34), taking into account (25) and (33), the solution to problem (1), (4)–(9) can be written as

$$u(x,t) = \sum_{k=1}^{\infty} T_k(t) \cdot \sin\frac{\pi k}{b}x. \tag{35}$$

Given (34) and if the functions $\varphi(x) \in C^{3+\gamma}$, $0 < \gamma < T$; $\varphi''(0) = \varphi''(b) = 0$, $\varphi(0) = \varphi(b) = 0$ and $\psi(x) \in C^{3+\delta}$, $-T < \delta < 0$; $\psi(0) = \psi(b) = \psi''(0) = \psi''(b) = 0$, then for φ_k and ψ_k the estimates $|\varphi_k| \leq C_3/k^{3+\gamma}$; $|\psi_k| \leq C_4/k^{3+\delta}$; C_3, $C_4 = const > 0$ are valid. Then the series (35) converges uniformly in the domain $\bar{\Omega}$ and it can be differentiated term-by-term twice with respect to, t and 4 times with respect to, x. Therefore, for the solution of problem A we have $u(x,t) \in C^{4,2}_{x,t}\left(\bar{\Omega}\right)$.

Since the constructed solution is $u(x,t) \in C^{4,2}_{x,t}\left(\bar{\Omega}\right)$, then the condition (24) of Theorem 2 is always satisfied.

Conflict of Interest

The author declares no conflict of interest.

References

1 Glushko V.P. Higher-order degenerate elliptic equations: Spaces, operators, boundary-value problems / V.P. Glushko, Yu.B. Savchenko // Journal of Soviet Mathematics. — 1987. — Vol. 39. — P. 3088–3148. https://doi.org/10.1007/BF01087547

2 Исхоков С.А. О гладкости решения эллиптического уравнения с нестепенным вырождением / С.А. Исхоков // Докл. АН — 2001. — Т. 378, № 3. — С. 306–309.

3 Sidorov S.N. Nonlocal problems for an equation of mixed parabolic-hyperbolic type with power degeneration / S.N. Sidorov // Russian Mathematics. — 2015. — Vol. 59. — P. 46–55. https://doi.org/10.3103/S1066369X15120051

4 Баев А.Д. Об общих краевых задачах в полупространстве для вырождающихся эллиптических уравнений высокого порядка / А.Д. Баев // Докл. АН — 2008. — Т. 422, №6. — С. 727–728.

5 Баев А.Д. Теорема о существовании и единственности решения одной краевой задачи в полосе для вырождающегося эллиптического уравнения высокого порядка / А.Д. Баев, С.С. Бунеев // Изв. Саратов. ун-та. Новая сер. Сер. Механика. Математика. Информатика. — 2012. — Т. 12, Вып. 3. — С. 9–17. https://doi.org/10.18500/1816-9791-2012-12-3-8-17

6 Баев А.Д. О вырождающихся эллиптических уравнениях высокого порядка и псевдодифференциальных операторах с вырождением / А.Д. Баев, Р.А. Ковалевский, П.А. Кобылинский // Докл. АН. — 2016. — Т. 471, № 4. — С. 387–390.

7 Сабитов К.Б. Задача Дирихле для уравнения смешанного типа в прямоугольной области / К.Б. Сабитов // Докл. АН. — 2007. — Т. 413, № 1. — С. 23–26.

8 Sabitov K.B. The dirichlet problem for a mixed-type equation of the second kind in a rectangular domain / K.B. Sabitov, A.Kh. Suleimanova // Russian Mathematics. — 2007. — Vol. 51. — P. 42–50. https://doi.org/10.3103/S1066369X07040068

9 Трегубова (Сулейманова) А.Х. Задача Дирихле и видоизмененные задачи для уравнений смешанного типа с характеристическим вырождением: автореф. дис. ... канд. физ. мат. наук / А.Х. Трегубова (Сулейманова). — Казань, 2009. — 18 с.

10 Lerner M.E. A problem with two nonlocal boundary conditions for a mixed-type equation / M.E. Lerner, O.A. Repin // Siberian Mathematical Journal. — 1999. — Vol. 40. — P. 1064–1078. https://doi.org/10.1007/BF02677530

11 Рустамова Л.Р. Нелокальные краевые задачи типа Бицадзе-Самарского для уравнения смешанного типа четвёртого порядка / Р.Л. Рустамова // Изв. вузов. Сев.-Кавказ. регион. Естественные науки. — 2007. — № 4. — С. 14–16.

12 Асхатов Р.М. Решение основных краевых задач вырождающихся эллиптических уравнений методом потенциалов / Р.М. Асхатов, Р.Н. Абайдуллин // Физико-математические науки. Учен. зап. Казан. ун-та. Сер. Физ.-мат. науки. — 2015. — Т. 157, № 1. — С. 5–14.

13 Энбом Е.А. Задача Коши для вырождающегося уравнения третьего порядка / Е.А. Энбом // Науч. докл. Ежегод. межвуз. 55-ой науч. конф. Самар. гос. пед. ун-та. — Самара, 2001. — С. 80–87.

14 Апаков Ю.П. Краевая задача для вырождающегося уравнения высокого нечетного порядка / Ю.П. Апаков, Б.Ю. Иргашев // Украин. мат. журн. — 2014. — Т. 66, № 10. — С. 1318–1331.

15 Моисеев Е.И. О решении спектральным методом одной нелокальной краевой задачи / Е.И. Моисеев // Дифференциальные уравнения. — 1999. — Т. 35, № 8. — С. 1094–1100.

16 Дунаев А.С. Специальные функции: учеб. пос. / А.С. Дунаев, В.И. Шлычков. — Екатеринбург: УрФУ, 2015. — 938 с.

Төртінші ретті аралас типті өзгешеленетін теңдеу үшін шекаралық есеп

Ж.А. Отарова

Бердах атындағы Қарақалпақ мемлекеттік университеті, Нүкіс, Өзбекстан

Механика, физика және геофизикадағы көптеген есептер эллиптикалық, параболалық немесе гиперболалық теңдеулердің белгілі кластарына кірмейтін дербес туындылы дифференциалдық теңдеулерді шешуге әкеледі. Мұндай теңдеулер әдетте, математикалық физиканың классикалық емес теңдеулері деп аталады. Өзгешеленетін теңдеулер теориясы қазіргі дербес дифференциалдық теңдеулер теориясының негізгі бөлімдерінің бірі. Бұл, ең алдымен, әртүрлі қолданбалы есептерді анықтаумен түсіндіріледі, олардың математикалық модельдеуі өзгешеленетін теңдеулердің әртүрлі түрлерін зерттеуге қолданылады. Осы уақытқа дейін өзгешеленетін теңдеулерге арналған есептер негізінен модельдік теңдеулер мен төменгі мүшелерінің жеткілікті тегіс коэффициенттері бар төменгі мүшелері бар теңдеулер үшін зерттелді. Төртінші ретті аралас типті дәрежелері өзгешеленетін теңдеулер үшін шекаралық есептерді зерттеу өзекті болып қала береді. Мақалада төртінші ретті аралас типті бір өзгешеленетін теңдеу үшін тікбұрышты облыста шекаралық есеп қойылып, зерттелді. Төртінші ретті дербес туындылы дифференциалдық теңдеу үшін шекаралық есептің корректілігі шешімнің бар болуы мен жалғыздығын дәлелдеу арқылы белгіленеді. Жеткілікті шарттар қойылған жағдайда қарастырылған есептің шешімі айнымалыларды бөлу әдісімен табылды.

Кілт сөздер: төртінші ретті аралас теңдеу, Бессель функциялары, Фурье қатары, толықтық, регулярлы шешім.

Краевая задача для вырождающегося уравнения смешанного типа четвёртого порядка

Ж.А. Отарова

Каракалпакский государственный университет имени Бердаха, Нукус, Узбекистан

Многие задачи механики, физики, геофизики приводят к решению уравнений в частных производных, которые не входят в известные классы эллиптических, параболических или гиперболических уравнений. Такие уравнения, как правило, стали называть неклассическими уравнениями математической физики. Теория вырождающихся уравнений является одним из центральных разделов современной теории уравнений с частными производными. Это объясняется, прежде всего, выявлением множества прикладных задач, математическое моделирование которых обслуживает изучение различных типов вырождающихся уравнений. До настоящего времени задачи для вырождающихся уравнений, в основном, исследованы для модельных уравнений и уравнений с младшими членами с достаточно гладкими коэффициентами при младших членах. Исследование краевых задач для уравнений смешанного типа четвертого порядка со степенным вырождением остаётся актуальным. В этой работе поставлена и исследована краевая задача в прямоугольной области для одного вырождающегося уравнения смешанного типа четвертого порядка. Корректность краевой задачи для уравнения с частными производными четвертого порядка устанавливается доказательством существования и единственности решения. При достаточных условиях найдено решение рассматриваемой задачи в явном виде методом разделения переменных.

Ключевые слова: смешанное уравнение четвёртого порядка, функции Бесселя, ряд Фурье, полнота, регулярное решение.

References

1 Glushko, V.P., & Savchenko, Yu.B. (1987). Higher-order degenerate elliptic equations: Spaces, operators, boundary-value problems. *Journal of Soviet Mathematics, 39,* 3088–3148. https://doi.org/10.1007/BF01087547

2 Iskhokov, S.A. (2001). O gladkosti resheniia ellipticheskogo uravneniia s nestepennym vyrozhdeniem [On the smoothness of the solution of an elliptic equation with non-power degeneracy]. *Doklady Akademii nauk — Reports of the USSR Academy of Sciences, 378*(3), 306–309 [in Russian].

3 Sidorov, S.N. (2015). Nonlocal problems for an equation of mixed parabolic-hyperbolic type with power degeneration. *Russian Mathematics, 59,* 46–55 https://doi.org/10.3103/S1066369X15120051

4 Baev, A.D. (2008). Ob obshchikh kraevykh zadachakh v poluprostranstve dlia vyrozhdaiushchikhsia ellipticheskikh uravnenii vysokogo poriadka [On general boundary value problems in a half-space for degenerate elliptic equations of high order]. *Doklady Akademii nauk — Reports of the Academy of Sciences, 422*(6), 727–728 [in Russian].

5 Baev, A.D., & Buneev, S.S. (2012). Teorema o sushchestvovanii i edinstvennosti resheniia odnoi kraevoi zadachi v polose dlia vyrozhdaiushchegosia ellipticheskogo uravneniia vysokogo poriadka [The theorem on the existence and uniqueness of the solution of a single boundary value problem in a strip for a degenerate elliptic equation of high order]. *Izvestiia Saratovskogo universiteta. Novaia seriia. Seriia Mekhanika. Matematika. Informatika — News of Saratov University. New Series. Series: Mechanics. Mathematics. Computer science, 12*(3), 9–17 [in Russian]. https://doi.org/10.18500/1816-9791-2012-12-3-8-17

6 Baev, A.D., Kovalevskii, R.A., & Kobylinskii, P.A. (2016). O vyrozhdaiushchikhsia ellipticheskikh uravneniiakh vysokogo poriadka i psevdodifferentsialnykh operatorakh s vyrozhdeniem [On degenerate high order elliptic equations and pseudodifferential operators with degeneration]. *Doklady Akademii nauk — Reports of the Academy of Sciences, 471*(4), 387– 390 [in Russian].

7 Sabitov, K.B. (2007). Zadacha Dirikhle dlia uravneniia smeshannogo tipa v priamougolnoi oblasti [Dirichle problem for a mixed-type equation in a rectangular domain]. *Doklady Akademii nauk — Reports of the Academy of Sciences, 413*(1), 23–26 [in Russian].

8 Sabitov, K.B., & Suleimanova, A.Kh. (2007). The dirichlet problem for a mixed-type equation of the second kind in a rectangular domain. *Russian Mathematics, 51,* 42–50. https://doi.org/10.3103/S1066369X07040068

9 Tregubova (Suleimanova), A.Kh. (2009). Zadacha Dirikhle i vidoizmenennye zadachi dlia uravnenii smeshannogo tipa s kharakteristicheskim vyrozhdeniem [The Dirichlet problem and modified problems for equations of mixed type with characteristic degeneration]. *Extended abstract of Candidate's thesis.* Kazan [in Russian].

10 Lerner, M.E., & Repin, O.A. (1999). A problem with two nonlocal boundary conditions for a mixed-type equation. *Siberian Mathematical Journal, 40,* 1064–1078. https://doi.org/10.1007/BF02677530

11 Rustamova, L.R. (2007). Nelokalnye kraevye zadachi tipa Bitsadze-Samarskogo dlia uravneniia smeshannogo tipa chetvertogo poriadka [Nonlocal boundary-value problems of Bitsadze - Samarsky type for a fourth-order mixed-type equation]. *Izvestiia vuzov. Severo-Kavkazskii region. Estestvennye nauki — News of Universities. North Caucasus region. Natural Sciences*, (4), 14–16 [in Russian].

12 Askhatov, R.M., & Abaidullin R.N. (2015). Reshenie osnovnykh kraevykh zadach vyrozhdaiushchikhsia ellipticheskikh uravnenii metodom potentsialov [The solution of the main boundary value problems of degenerate elliptic equations by the method of potentials]. *Fiziko-matematicheskie nauki. Uchenye zapiski Kazanskogo universiteta. Seriia Fizikomatematiches-kie nauki — Physics and mathematics, Scientific notes of Kazan University. Series physical and mathematical sciences, 157*(1), 5–14 [in Russian].

13 Enbom, E.A. (2001). Zadacha Koshi dlia vyrozhdaiushchegosia uravneniia tretego poriadka [Cauchy problem for a third order degenerate equation]. *Nauchnye doklady Ezhegodnoi mezhvuzovskoi 55-oi nauchnoi konferentsii Samarskogo gosudarstvennogo pedagogicheskogo universiteta — Scientific reports of the annual interuniversity 55th scientific conference of Samara State Pedagogical University,* 80–87 [in Russian].

14 Apakov, Yu.P., & Irgashev, B.Yu. (2014). Kraevaia zadacha dlia vyrozhdaiushchegosia uravneniia vysokogo nechetnogo poriadka [A boundary value problem for a degenerate equation of high odd order]. *Ukrainskii matematicheskii zhurnal — Ukrainian mathematical journal, 66*(10), 1318–1331 [in Russian].

15 Moiseev, E.I. (1999). O reshenii spektralnym metodom odnoi nelokalnoi kraevoi zadachi [On the solution by a spectral method of a single non-local boundary value problem]. *Differentsialnye uravneniia — Differential equations, 35*(8), 1094–1100 [in Russian].

16 Dunaev, A. S., & Shlychkov, A.S. (2015). *Spetsialnye funktsii [Special functions].* Ekaterinburg: Uralskii federalnyi universitet [in Russian].

*Author Information**

Jamila Amanbayevna Otarova — Candidate of physical and mathematical sciences, Associate Professor, Head of the Department of Differential equations, Karakalpak State University named after Berdakh, 1 Chardzhou Abdirov Street, Nukus, 230112, Uzbekistan / Karakalpakstan; e-mail: *j.otarova@mail.ru*; https://orcid.org/0000-0003-3661-7937.

*The author's name is presented in the order: First, Middle and Last Names.