C.R. Parvathy, A. Sofia*

PSGR Krishnammal College for Women, Tamil Nadu, India
(E-mail: parvathytopo@gmail.com, sofiaarjunan11@gmail.com)

A study on new classes of binary soft sets in topological rough approximation space

Soft binary relation is used to define new classes of soft sets, namely BR-soft simply open set and BR-soft simply* alpha open set, in topological rough approximation space over two different universes. The defined set provides information on the missing elements of a BR-soft set and can help in simplifying decision making. Approximation operators are defined and the characteristics of the proposed sets are studied with examples. The relationship between the defined sets and other soft sets is brought out. An accuracy check was done to compare the proposed method with other methods. It is identified that the proposed method is more accurate.

Keywords: Soft set, rough set, simply open, approximation space, topological space.

Introduction

Decision making becomes complicated while handling problems with inappropriate or uncertain data. To deal with complex problems with uncertainty, many researchers developed various mathematical tools and theories. Soft set theory, one of the prominent theories of uncertainty, is highly helpful in decision making due to the presence of its parameters. This theory was developed by Molodtsov [1] in 1999. Further developments in soft set theory and its application were done by many researchers [2–6]. Though soft and rough set theories are different for handling problems with uncertainty, efforts have been made to combine both for solving complex problems [7, 8]. The relationship between soft sets, soft rough sets and topologies were investigated by Li [9]. Covering soft rough sets and their topologies were also studied by many other researchers [10,11]. While dealing with soft rough sets, Feng [12] used parametrized subsets to find upper and lower approximations of a subset. These soft rough sets, soft β rough sets, soft rough approximations, soft pre-rough approximations etc., are further studied by many researchers in decision making [13–17].

Soft set theory was also extended over rough approximation space, nearness approximation spaces and ideal rough topological spaces in [18–20]. In recent years, theories of uncertainty have been extended over two universal sets. However, approximation operators between two different universal sets are less explored. Zhang and Wu [21] were the first to study approximation operators between two different universal sets by the constructive approach of a random approximation space. Following them, a few other authors started working over two different universes using fuzzy rough set, intuitionistic fuzzy rough set, neutrosophic set, etc. [22–25]. In [26], the author constructed a topological space, using the fuzzy b-q neighbourhood of one fuzzy topology and fuzzy b-closure of another fuzzy topology.

The concepts of simply-open and its irresoluteness were studied by Dontchev et al. [27]. Continuous functions, separation axioms of the e-$\mathcal{I}$ set and many other concepts like a-local function are studied in ideal topological spaces by Al-Omeri et al. [28–31]. El Sayed et al. [32] extended simply-open to soft set theory. In addition, El Safty et al. [33] defined the concept of Simply* alpha open sets in rough set theory which is a union of an alpha open set and nowhere dense set. This set is useful in the field of

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*Corresponding author.
E-mail: sofiaarjunan11@gmail.com

decision making as it contributes to attribute reduction. Though it is studied in the rough set, since the soft set contains a parametrization tool, it is appropriate to study simply* alpha open set over soft set. The choice of soft sets in decision making problems varies among different researchers. It is also seen from the literature that every soft set may not include all elements of the universal set. In such cases, information regarding the left-out elements of the universal sets is not emphasized. A decision-making process is highly reliable only by considering every option (element) related to the problem. For this, new classes of soft sets will have to be defined.

In this paper, BR soft simply* alpha open set and BR-soft simply open set are defined in BR-soft topological rough approximation space. The complement of soft sets is taken as in [34]. Apart from this, other classes of BR-soft near sets, namely BR-soft delta, BR-soft nowhere dense, BR-soft alpha open, BR-soft beta open, etc., are also defined over different universes to obtain their relationship between the defined sets. The topological properties of defined sets are studied. In addition, the accuracy of the proposed method is demonstrated using example problems and compared with the methods of Feng [12] and Yao [35].

In the following section (section 2) of the paper, the required preliminary definitions are given. In section 3, the sets are defined and their properties are studied. Example problems illustrating the application of the sets and their accuracy measures are given in section 4. This is followed by a conclusion and scope for future work.

1   Preliminaries

Definition 1.1. A soft set m_k is a mapping from a subset of a parameter set ($A \subseteq E$) to the power set of a universal set U. The collection of soft sets m_{k_i} over U forms soft topology τ, if the following conditions are satisfied.

   i $\emptyset, U \in \tau$.
   ii The arbitrary union of soft sets in τ is in τ.
   iii The finite intersection of soft sets in τ is in τ.
Then, (U, E, τ) is said to be a soft topological space.

Proposition 1.2. The following conditions hold in the soft topological space (U, E, τ).
   i $\emptyset, U$ are soft closed sets over U.
   ii The arbitrary intersection of a soft closed set is soft closed.
   iii A finite union of soft closed set is soft closed.

Proposition 1.3. [34] The following conditions hold in the soft topological space (U, E, τ).
   i $\emptyset^C, m_k^C$ are soft closed sets over U.
   ii The arbitrary intersection of soft closed set is soft closed.
   iii A finite union of soft closed set is soft closed.

Definition 1.4. $(U, \mathcal{R})$ denotes Pawlak's approximation space, $\mathcal{R}$ is an equivalence relation and $X \subseteq S$. Using R following operators were defined.

$$\underline{\mathcal{R}(X)} = \{x \in S : [x]\mathcal{R} \subseteq X\},$$
$$\overline{\mathcal{R}(X)} = \{x \in S : [x]\mathcal{R} \cap X \neq \emptyset\}.$$

If $\underline{\mathcal{R}(X)} \neq \overline{\mathcal{R}(X)}$, X is a rough set. Otherwise, X is definable.

2   Soft set over "n" different nonempty finite sets

Definition 2.1. $S_1, S_2, ...S_n$ be nonempty finite sets. K be the subset of a parameter set E. A pair (m, K) or m_K is called a soft binary relation over $S_1, ...S_n$, if (m, K) is a soft set (BR-soft set) over $S_1 \times S_2 \times ... \times S_n$.

Throughout this paper, we consider n=2, i.e., two non-empty finite sets say, S and T.

Definition 2.2. Let $(S, T, R_{m(s,t)})$ be a rough approximation space and τ_{BR} be a soft topology obtained from a soft binary relation over S, T. Thus, $(S, T, R_{m(s,t)}, \tau_{BR})$ is said to be BR-topological rough approximation space where the elements of τ_{BR} are BR-soft open and its complements are closed.

Example 2.3. Let $S = \{2, 3, 5\}, T = \{4, 6\}, E = \{e_1, e_2\} = K$. Let $S \times T = \{(2, 4), (2, 6), (3, 4), (3, 6), (5, 4), (5, 6)\}$. Thus, the soft binary relation over $S \times T$ is $m_k = \{(e_1, \{(3, 4), (5, 4)\}), (e_2, \{(2, 4), (3, 6)\})\}$. The soft relations induced from soft binary relation are as follows:

$$R(3, 4) = \{(e_1, \{(5, 4)\})\},$$
$$R(5, 4) = \{(e_1, \{(3, 4)\})\},$$
$$R(2, 4) = \{(e_2, \{(3, 6)\})\},$$
$$R(3, 6) = \{(e_2, \{(2, 4)\})\}.$$

Subbasis $S_B = \{\{(e_1, \{(5, 4)\})\}, \{(e_1, \{(3, 4)\})\}, \{(e_2, \{(3, 6)\})\}, \{(e_2, \{(2, 4)\})\}\}$.

The topology obtained by taking the finite intersection of an arbitrary union of elements of a subbasis is as follows:

$$\tau_{BR} = \{\emptyset, m_k, \{(e_1, \{(5, 4)\})\}, \{(e_1, \{(5, 4)\})\}, \{(e_1, \{(5, 4)\})\}, \{(e_1, \{(5, 4)\})\}, \{e_1, \{(5, 4), (3, 4)\}\},$$
$$\{(e_1, \{(5, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1, \{(5, 4)\}), (e_2, \{(2, 4)\})\}, \{(e_1, \{(3, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1,$$
$$\{(3, 4)\}), (e_2, \{(2, 4)\})\}, \{(e_2, \{(2, 4), (3, 6)\})\}, \{(e_1, \{(5, 4), (3, 4)\}), (e_2, \{(2, 4)\})\},$$
$$\{(e_1, \{(5, 4), (3, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1, \{(5, 4)\}), (e_2, \{(2, 4), (3, 6)\})\}, \{(e_1, \{(3, 4)\}), (e_2,$$
$$\{(2, 4), (3, 6)\})\}\}.$$

Then, $(S, T, R_{m(s,t)}, \tau_{BR})$ is BR-topological rough approximation space.

Definition 2.4. Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space. For each $m_{ki} \subseteq m_k$, the BR-topological approximation operators are defined as follows:

$$\underline{\tau}_{BR}(m_{ki}) = \cup \{m_{kj} \in \tau_{BR}; m_{kj} \subseteq m_{ki}\},$$
$$\overline{\tau}_{BR}(m_{ki}) = \cap \{m_{kj} \in \tau_{BR}{}^C; m_{ki} \subseteq m_{kj}\}.$$

In other words, $\underline{\tau}_{BR}, \overline{\tau}_{BR}$ is considered as interior and closure of the BR-topological approximation space respectively.

3 BR-Soft simply open, BR-Soft simply* alpha open sets

Definition 3.1. In a BR-topological rough approximation space, a BR-soft subset is called BR-soft nowhere dense if $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) = \emptyset$.

Definition 3.2. In a BR-topological rough approximation space a BR-soft subset is called a BR-soft simply* alpha open set if $m_{ki} \in \{\emptyset, m_k, (m_{kj} \cup m_{kl}) : m_{kj}$ is BR-soft α open, m_{kl} is BR-soft nowhere dense and m_k is BR-soft set$\}$.

The collection of BR-soft simply* alpha open set is denoted by $BR_S S^* \alpha O(m_{ki})$, the complement is BR-soft simply* alpha closed.

Definition 3.3. In a BR-topological rough approximation space, a BR-soft subset is called

  i BR-soft simply open set if $m_k = (m_{ki}) \cup (m_{kj})$, where (m_{ki}) is BR-soft open and (m_{kj}) is BR-soft nowhere dense.

   ii BR-soft delta set if $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki}))$.

   iii BR-soft semi locally closed set if m_k equals the intersection of BR-soft semi open set and BR-soft semi closed set.

   iv BR-soft b open set if $m_{ki} \subseteq (\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \cup \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})))$.

   v BR-soft alpha open if $m_{ki} \subseteq \underline{\tau}_{BR}(\overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})))$.

   vi BR-soft beta open if $m_{ki} \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})))$.

BR-soft simply open set, BR-soft delta set, BR-soft nowhere dense, and BR-soft b open are denoted as $BR_sSO(m_{ki}), BR_s\delta O(m_{ki}), BR_sNO(m_{ki})$, and $BR_sbO(m_{ki})$ respectively.

Proposition 3.4. Every BR-soft open set is BR-soft alpha open.

Example 3.5. Considering the topology taken in Example 2.3, where
$m_k = \{(e_1, \{(3,4),(5,4)\}), (e_2, \{(2,4),(3,6)\})\}$. Here, $BR_sNO(m_k) = \{\emptyset, \{(e_1, \{(5,6)\}, (e_2, \{(2,6)\})\}\}$. It is clear that the BR-soft set $m_{k_1} = \{(e_1, \{(3,4),(5,6)\}), (e_2, \{(2,4),(2,6)\})\}$ is BR-soft simply open, $m_{k_2} = \{(e_1, \{(3,4),(5,6)\}), (e_2, \{(2,4),(2,6),(3,6)\})\}$ is a BR-soft δ set.

Theorem 3.6. Every BR-soft simply open is BR-soft simply* alpha open.

Proof. Let m_k be a BR-soft simply open set. That is, m_k is a union of BR-soft open set and BR-soft nowhere dense set. Since every BR-soft open set is BR-soft alpha open, we denote m_k as a union of BR-soft alpha open set and BR-soft nowhere dense set. This proves the theorem.

The converse of Theorem 3.6 need not be true and is explained in the following example.

Example 3.7. Consider the topology taken in Example 2.3. Let the BR-soft subset be
$m_k = \{(e_1, \{(3,4)\}), (e_2, \{(2,4)\})\}$ which is a BR-soft alpha set.
Thus, $\{(e_1, \{(3,4),(5,6)\}), (e_2, \{(2,4),(2,6),(3,6)\})\}$, $m_{k_1} = \{(e_1, \{(3,4),(5,6)\}), (e_2, \{(2,4),(2,6)\})\}$ are both BR-soft simply* alpha open and BR-soft simply open. The BR-soft nowhere dense set $m_{k_2} = \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$ is BR-soft simply open but not BR-soft simply* alpha open.

Theorem 3.8. Every BR-soft open set is BR-soft simply open and BR-soft simply* alpha open set.

Proof. The proof is obvious for BR-soft simply open and by Theorem 3.6, the BR-soft open set is BR-soft simply* alpha open.

Remark 3.9. Though the union of the BR-soft alpha open set and the BR-soft nowhere dense set is BR-soft simply* alpha open, a BR-soft simply* alpha open set need not be BR-soft alpha open.

The following example explains Remark 3.9.

Example 3.10. The BR-soft set $m_{k_1} = \{(e_1, \{(3,4),(5,6)\}), (e_2, \{(2,4),(2,6)\})\}$ taken in example 3.7 is BR-soft simply* alpha open but not BR-soft alpha open. Since τ_{BR} is obtained by considering m_k, τ_{BR}^C is also taken with respect to m_k. Let us consider $m_{k2} = \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$.
Then, we have $\overline{BR}_S(\underline{BR}_S(m_{k2})) = \emptyset$, and $\overline{BR}_S(\underline{BR}_S(\overline{BR}_S(m_{k2}))) = \emptyset$ which implies $\overline{BR}_S(\underline{BR}_S(m_{k2})) \subset m_{k2}$, $\overline{BR}_S(\underline{BR}_S(\overline{BR}_S(m_{k2}))) \subset m_{k2}$.

Theorem 3.11. For a BR-soft subset m_{ki} in a BR-topological rough approximation space, the following conditions are equivalent:

   i m_{k_i} is BR-soft simply open.

   ii m_{k_i} is BR-soft semi locally closed.

   iii m_{k_i} is BR-soft delta.

   iv m_{k_i} is BR-soft nowhere dense.

Proof. (i) $\iff$ (ii) is obvious.

(ii) $\iff$ (iii) Let m_{ki} be BR-soft semi locally closed.
Then,

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq (\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \cap \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})))$$

and

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})).$$

Thus, m_{ki} is a BR-soft delta set.

(iii) $\iff$ (iv) Let m_{ki} be a BR-soft delta set. Then,

$$
\begin{aligned}
\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) &= \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \cap \underline{\tau}_{BR}(\overline{\tau}_{BR}(S \times T \setminus m_{ki})) \\
&= \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \cap (S \times T \setminus \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki}))) \\
&= \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \setminus \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})) \\
&= \emptyset.
\end{aligned}
$$

Theorem 3.12. Consider the BR-topological rough approximation space, then
i The arbitrary union of a BR-soft simply open set is BR-soft simply open.
ii The finite intersection of BR-soft simply open set is BR-soft simply open.
Proof.
i Let m_{k1}, m_{k2} be two BR-soft simply open sets. Then

$$
\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k1})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k1}))
$$

and

$$
\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k2})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k2})).
$$

By taking union we get,

$$
\begin{aligned}
\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k1})) \cup \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k2})) &\subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k1})) \cup \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k2})) \\
\implies \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k1} \cup m_{k2})) &\subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k1} \cup m_{k2})).
\end{aligned}
$$

Let $m_{k1} \cup m_{k2}$ be m_{k3}. Then, $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{k3})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{k3}))$.
ii m_{k_i} be the collection of BR-soft simply open sets where $i = 1, 2, \ldots$
Then,

$$
\underline{\tau}_{BR}(\overline{\tau}_{BR}(\cap_{i=1}^{n}(m_{k_i}))) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(\cap_{i=1}^{n}(m_{k_i}))).
$$

Hence, $\cap_{i=1}^{n}(m_{k_i})$ is BR-soft simply open.
Remark 3.13. In the BR-topological rough approximation space,
i BR-soft simply open, BR-soft simply* alpha open and BR-soft beta open are not comparable.
ii BR-soft simply open, BR-soft simply* alpha open and BR-soft b open are not comparable.
iii BR-soft simply open, BR-soft simply* alpha open and BR-soft preopen are not comparable.
Proposition 3.14. In the BR-topological rough approximation space,
i The union of the BR-soft simply* alpha open set is BR-soft simply* alpha open.
ii The finite intersection of the BR-soft simply* alpha open set is BR-soft simply* alpha open.
Remark 3.15. Every BR-soft delta set is BR-soft nowhere dense.
Theorem 3.16. In a BR-topological rough approximation space, every BR-soft simply* alpha open set is BR-soft alpha closed.
Proof. The proof is obvious from Definition 3.2.
Theorem 3.17. In a BR-topological rough approximation space, every BR-soft simply* alpha open set is BR-soft pre closed (resp. BR-soft beta closed).
Proof. Let m_{ki} be BR-soft simply* alpha open. From Theorem 3.16,

$$
\underline{\tau}_{BR}(\overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})) \subseteq m_{ki}.
$$

Since,

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \underline{\tau}_{BR}(\overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})) \subseteq m_{ki}.$$

Thus,

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq m_{ki}.$$

The proof is similar for BR-soft beta closed.

Theorem 3.18. For a BR-topological rough approximation space, the following conditions are equivalent:

  i Every BR-soft simply* alpha open set is a BR-soft δ set.

 ii Every BR-soft simply* alpha open set is BR-soft beta closed.

iii Every BR-soft simply* alpha open set is BR-soft pre closed.

iv Every BR-soft simply* alpha open set is BR-soft b closed.

Proof. (i) $\Longleftrightarrow$ (ii) m_{ki} be BR-soft simply* alpha open, BR-soft delta set. Thus, we have

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})).$$
$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})))$$
$$\subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki}))$$
$$\subseteq m_{ki}.$$

Therefore, $\overline{\tau}_{BR}(\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki}))) \subseteq m_{ki}$.

  (ii) $\Longleftrightarrow$ (iii) It is obvious from the above proof that $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq m_{ki}$.

  (iii) $\Longleftrightarrow$ (iv) Since m_{ki} is preclosed and $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \subseteq \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki}))$.

We have,

$$\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) \cap \overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})) = \underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki}))$$
$$\subseteq m_{ki}.$$

A diagrammatic representation of the above-mentioned concepts is given below (Fig)

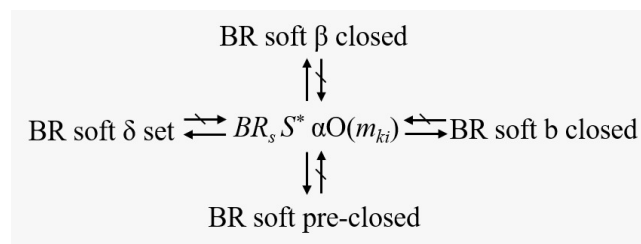Figure. Diagrammatic representation

Proposition 3.19. Every BR-soft simply* alpha open is BR-soft semi open (resp. BR-soft beta open) based on *Proposition 1.3*.

Proof. For every BR-soft simply* alpha open m_{ki},

$$\underline{BR}_S(m_{ki}) \subseteq m_{ki} \subseteq \overline{BR}_S(m_{ki})$$
$$\Longrightarrow \overline{BR}_S(\underline{BR}_S(m_{ki})) \subseteq \overline{BR}_S(m_{ki})$$
$$\Longrightarrow m_{ki} \subseteq \overline{BR}_S(\underline{BR}_S(m_{ki})).$$

Hence, BR-soft simply* alpha open is BR-soft semi open.

The proof is similar for BR-soft beta open.

Example 3.20. Consider the topology taken in Example 2.3.

Let $m_k = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4), (3,6)\})\}$ be the BR-soft simply* alpha open set. Then, by taking interior and closure with respect to Proposition 1.3, we have $\overline{BR}_S(\underline{BR}_S(m_k)) = m_E, \overline{BR}_S(\underline{BR}_S(\overline{BR}_S(m_k))) = m_E$. That is, m_k is BR-soft semi open, BR-soft beta open.

Definition 3.21. Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space. For each BR-soft simply* alpha open sets $m_{ki} \subseteq m_k$, the BR-topological approximation operators are defined as follows:

$$\underline{BR}_S(m_{ki}) = \cup\{m_{kj} \in \tau_{BR}; m_{kj} \subseteq m_{ki}\},$$
$$\overline{BR}_S(m_{ki}) = \cap\{m_{kj} \in \tau_{BR}^C; m_{ki} \subseteq m_{kj}\},$$

where m_{kj} is the BR-soft set, $\underline{BR}_S(m_{ki}), \overline{BR}_S(m_{ki})$ are the interior and closure of BR soft simply* alpha open sets in BR-topological rough approximation space respectively.

Theorem 3.22. The collection of all BR soft simply* alpha open sets obtained from BR-topological rough approximation space forms a BR-soft topology τ_{BR}^*.

Proof. It is obvious from *Definition 3.2* and *Proposition 3.12*.

Definition 3.23. Let τ_{BR}^* be the BR-soft topology obtained from the collection of BR soft simply* alpha open sets. For each BR-soft simply* alpha open sets $m_{ki} \subseteq m_k$ and m_{kj}, the approximation operators are defined as follows:

$$\underline{BR}_S(m_{ki}) = \cup\{m_{kj} \in \tau_{BR}^*; m_{kj} \subseteq m_{ki}\},$$
$$\overline{BR}_S(m_{ki}) = \cap\{m_{kj} \in {\tau_{BR}^*}^C; m_{ki} \subseteq m_{kj}\}.$$

Here, $\underline{BR}_S(m_{ki})$ and $\overline{BR}_S(m_{ki})$ are lower approximation (interior) and upper approximation (closure) of BR soft simply* alpha open sets in the BR-soft topology obtained from the collection of BR soft simply* alpha open sets.

The boundary region is the difference between the upper and lower approximation operators.

Concerning the quality of the approximation, accuracy is defined as the ratio of cardinality of the lower approximation (interior) and cardinality of the upper approximation (closure).

Proposition 3.24. Let τ_{BR}^* be a BR-soft topology and m_{ki}, m_{kj} be two BR-soft simply* alpha open subsets of a BR-soft simply* alpha open set m_k, then the BR-topological operators satisfy the following properties:

    i $\underline{BR}_S(\emptyset) = \overline{BR}_S(\emptyset) = \emptyset$,

   ii $\underline{BR}_S(m_{ki}) = \overline{BR}_S(m_{ki}) = m_{ki}$,

  iii If $m_{ki} \subseteq m_{kj}$, then $\underline{BR}_S(m_{ki}) \subseteq \underline{BR}_S(m_{kj})$,

  iv If $m_{ki} \subseteq m_{kj}$, then $\overline{BR}_S(m_{ki}) \subseteq \overline{BR}_S(m_{kj})$,

   v $\underline{BR}_S(m_{ki} \cap m_{kj}) = \underline{BR}_S(m_{ki}) \cap \underline{BR}_S(m_{kj})$,

  vi $\underline{BR}_S(m_{ki}) \cup \underline{BR}_S(m_{kj}) \subseteq \underline{BR}_S(m_{ki}) \cup (m_{kj}))$,

 vii $\overline{BR}_S(m_{ki} \cup m_{kj}) = \overline{BR}_S(m_{ki}) \cup \overline{BR}_S(m_{kj})$,

viii $\overline{BR}_S(m_{ki} \cap m_{kj}) \subseteq \overline{BR}_S(m_{ki}) \cap \overline{BR}_S(m_{kj})$.

Example 3.25. Let $S = \{2, 3, 5\}, T = \{4, 6\}$, and $E = \{e_1, e_2\} = K$. Let $S \times T = \{(2,4), (2,6), (3,4), (3,6), (5,4), (5,6)\}$. Thus, the soft binary relation over $S \times T$ is $m_k = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4), (3,6)\})\}$.

The soft relations induced from soft binary relation are as follows:

$$R(3,4) = \{(e_1, \{(5,4)\})\},$$
$$R(5,4) = \{(e_1, \{(3,4)\})\},$$
$$R(2,4) = \{(e_2, \{(3,6)\})\},$$
$$R(3,6) = \{(e_2, \{(2,4)\})\}.$$

Subbasis $S_B = \{\{(e_1, \{(5,4)\})\}, \{(e_1, \{(3,4)\})\}, \{(e_2, \{(3,6)\})\}, \{(e_2, \{(2,4)\})\}\}$.
The topology obtained by taking the finite intersection of an arbitrary union of elements of subbasis is as follows:

$$\tau_{BR} = \{\emptyset, m_k, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(5,4)\})\}, \{e_1, \{(5,4), (3,4)\}\},$$
$$\{(e_1, \{(5,4)\}), (e_2, \{(3,6)\})\}, \{(e_1, \{(5,4)\}), (e_2, \{(2,4)\})\}, \{(e_1, \{(3,4)\}), (e_2, \{(3,6)\})\}, \{(e_1,$$
$$\{(3,4)\}), (e_2, \{(2,4)\})\}, \{(e_2, \{(2,4), (3,6)\})\}, \{(e_1, \{(5,4), (3,4)\}), (e_2, \{(2,4)\})\},$$
$$\{(e_1, \{(5,4), (3,4)\}), (e_2, \{(3,6)\})\}, \{(e_1, \{(5,4)\}), (e_2, \{(2,4), (3,6)\})\}, \{(e_1, \{(3,4)\}), (e_2,$$
$$\{(2,4), (3,6)\})\}\}.$$

Let $\{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$ be BR-soft nowhere dense set. Then, τ_{BR}^* is the BR-soft topology obtained by taking the union of BR-soft alpha open sets and BR-soft nowhere dense set. Consider BR-soft simply* alpha open sets $m_{k1} = \{(e_1, \{(5,6)\})\}$ and $m_{k2} = \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$ where $m_{k1} \subset m_{k2}$. Therefore, we have,

$$\overline{BR}_S(m_{k1}) = m_{k1}, \underline{BR}_S(m_{k1}) = m_{k1}.$$
$$\overline{BR}_S(m_{k2}) = m_{k2}, \underline{BR}_S(m_{k2}) = m_{k2}.$$
$$\implies \overline{BR}_S(m_{k1}) \subset \overline{BR}_S(m_{k2}) \; and$$
$$\underline{BR}_S(m_{k1}) \subset \underline{BR}_S(m_{k2}).$$

Now, let $m_{k1} = \{(e_1, \{(5,6)\})\}$ and $m_{k2} = \{(e_2, \{(2,6)\})\}$. Then,

$$\overline{BR}_S(m_{k1}) = m_{k1}, \underline{BR}_S(m_{k1}) = m_{k1}.$$
$$\overline{BR}_S(m_{k2}) = m_{k2}, \underline{BR}_S(m_{k2}) = m_{k2}.$$

Thus,

$$m_{k1} \cap m_{k2} = \emptyset, \tag{1}$$
$$\underline{BR}_S(m_{k1} \cap m_{k2}) = \emptyset, \tag{2}$$
$$\underline{BR}_S(m_{k1}) \cap \underline{BR}_S(m_{k2}) = \emptyset. \tag{3}$$

From (1), (2) and (3) we have, $\underline{BR}_S(m_{k1} \cap m_{k2}) = \underline{BR}_S(m_{k1}) \cap \underline{BR}_S(m_{k2})$.
Similarly, $\overline{BR}_S(m_{k1} \cap m_{k2}) = \overline{BR}_S(m_{k1}) \cap \overline{BR}_S(m_{k2})$.

Theorem 3.26. Let m_{ki} and m_{kj} be two BR-soft subsets of BR-topological rough approximation space. If m_{ki} is BR-soft simply* alpha closed, then $\overline{BR}_S(m_{ki} \cap m_{kj}) \subseteq m_{ki} \cap \overline{BR}_S(m_{kj})$.

Proof. Let m_{ki} be BR-soft simply* alpha closed, such that $\overline{BR}_S(m_{ki}) = m_{ki}$. From Proposition 3.24 we have,

$$\overline{BR}_S(m_{ki} \cap m_{kj}) \subseteq \overline{BR}_S(m_{ki}) \cap \overline{BR}_S(m_{kj})$$
$$\subseteq m_{ki} \cap \overline{BR}_S(m_{kj}).$$

Example 3.27. Consider the topology taken in Example 2.3. Let $m_{k1} = \{(e_1, \{(3,4),(5,4)\})\}$ be a BR-soft simply* alpha closed and $m_{k2} = \{(e_1, \{(3,4)\})\}$ be BR-soft subset. Then,

$$m_{k1} \cap m_{k2} = m_{k2}, \tag{4}$$

$$\overline{BR}_S(m_{k1} \cap m_{k2}) = m_{k2}, \tag{5}$$

$$\overline{BR}_S(m_{k2}) = \{(e_1, \{(3,4),(5,6)\}, (e_2, \{(2,6)\})\}. \tag{6}$$

From (4),(5),(6) we have, $\overline{BR}_S(m_{k1} \cap m_{k2}) = m_{k1} \cap \overline{BR}_S(m_{k2})$.

Theorem 3.28. Let m_{ki} be a BR-soft subset of BR-topological rough approximation space, then $BR_S S^* \alpha d(m_{ki}) = \emptyset$.

Proof. Let $(s,t) \in (S, T, R_{m(s,t)}, \tau_{BR})$ and BR-topological rough approximation space be discrete, then every BR-soft subset is BR-soft open and BR-soft simply* alpha open. Thus, every (s_i, t_j) is BR-soft simply* alpha open. Let $m_{kj} = (s,t)$. Then, $m_{ki} \cap m_{kj} = m_{ki} \cap (s,t) \subseteq (s,t)$.

Hence, (s,t) is not a BR-soft simply* alpha limit point of m_{ki} which implies $BR_S S^* \alpha d(m_{ki}) = \emptyset$.

Theorem 3.29. For any BR-soft simply* alpha subsets m_{ki} of $(S, T, R_{m(s,t)}, \tau_{BR})$, $\overline{BR}(m_{ki}) = m_{ki} \cup BR_S S^* \alpha d(m_{ki})$.

Proof. Let $(s,t) \in \overline{BR}(m_{ki})$. Assume $(s,t) \notin m_{ki}$ and $m_{kj} \in \tau_{BR}^*$ with $(s,t) \in m_{kj}$. Then, $(m_{ki} \cap m_{kj}) - (s,t) \neq \emptyset$ implies $(s,t) \in BR_S S^* \alpha d(m_{ki})$. Hence, $\overline{BR}(m_{ki}) \subseteq m_{ki} \cup BR_S S^* \alpha d(m_{ki})$.

Let $(s,t) \in m_{ki} \cup BR_S S^* \alpha d(m_{ki})$ implies $m_{ki} \subseteq \overline{BR}(m_{ki})$. Since, all BR-soft simply* alpha limit points of m_{ki} are soft preclosure of m_{ki}. $m_{ki} \cup BR_S S^* \alpha d(m_{ki}) \subseteq \overline{BR}(m_{ki})$.
Hence, $\overline{BR}(m_{ki}) = m_{ki} \cup BR_S S^* \alpha d(m_{ki})$.

4 Application

To observe the accuracy of the proposed method, two examples have been demonstrated in this section.

Example 4.1. Decision making on the infections of COVID-19 in humans is taken as an application of our approach. Since we use soft binary relation, this method helps to find people affected by COVID and the reasons for getting affected at the same time.

Let $S = \{S_1, S_2, S_3, S_4\}$ be four people considered and $T = \{T_1, T_2, T_3, T_4\}$ be the reasons for getting affected where,
$T_1 =$ stay at home.
$T_2 =$ go out and contact infected people.
$T_3 =$ low immunity; rarely go out.
$T_4 =$ Stay at home but any one in family go out.

Let $E = \{e_1 \ (fever), e_2 \ (fatigue), e_3 \ (loss \ of \ smell/taste), e_4 \ (Cough)\}$ be the parameter set and $A = \{e_1, e_3\}$ subset of E.

$S \times T = \{(S_1, T_1), (S_1, T_2), (S_1, T_3), (S_1, T_4), (S_2, T_1), (S_2, T_2), (S_2, T_3), (S_2, T_4), (S_3, T_1), (S_3, T_2), (S_3, T_3),$
$(S_3, T_4), (S_4, T_1), (S_4, T_2), (S_4, T_3), (S_4, T_4)\}.$

The following table (Table 1) represents the BR-soft set over $S \times T$ with respect to E.

Soft matrix

$S \times T$	e_1	e_2	e_3	e_4	Yes/No
(S_1, T_1)	1	0	0	1	0
(S_1, T_2)	1	1	1	0	1
(S_1, T_3)	0	1	0	1	0
(S_1, T_4)	1	0	1	1	1
(S_2, T_1)	0	1	0	0	0
(S_2, T_2)	0	1	1	1	1
(S_2, T_3)	1	1	0	0	0
(S_2, T_4)	0	1	0	1	0
(S_3, T_1)	1	0	0	0	0
(S_3, T_2)	0	0	1	0	1
(S_3, T_3)	0	0	0	0	0
(S_3, T_4)	1	0	1	0	1
(S_4, T_1)	0	0	0	1	0
(S_4, T_2)	1	1	1	1	1
(S_4, T_3)	0	0	0	0	0
(S_4, T_4)	1	0	0	0	0

Let the soft set over the parameter set A be

$$F(e_1) = \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\},$$
$$F(e_3) = \{(S_1, T_2), (S_1, T_4), (S_2, T_2), (S_3, T_2), (S_3, T_4), (S_4, T_2)\}$$

in which the BR-soft set represents the people infected with COVID and their reason.

Let the BR-soft subset be

$$m_{ki} = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_2, T_3), (S_4, T_2)\}), (e_3, \{(S_1, T_2), (S_3, T_2), (S_4, T_2)\})\}.$$

According to Feng's method,

$$\underline{S}_{apr}(m_{ki}) = \{(s, t) \in S \times T : for\ every\ k \in K, (s, t) \in m(k) \subseteq S \times T\},$$
$$\overline{S}_{apr}(m_{ki}) = \{(s, t) \in S \times T :\ for\ every\ k \in K, (s, t) \in m(k) \cap S \times T \neq \emptyset\},$$

where $\underline{S}_{apr}(m, k_i), \overline{S}_{apr}(m, k_i)$ are lower and upper approximation operators. Thus we have,

$$\underline{S}_{apr}(m_{ki}) = \emptyset,$$
$$\overline{S}_{apr}(m_{ki}) = \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_2), (S_2, T_3), (S_3, T_1),$$
$$(S_3, T_2), (S_3, T_4), (S_4, T_2), (S_4, T_4)\}.$$

Accuracy = cardinality of $\underline{S}_{apr}(m, k_i)/$ cardinality of $\overline{S}_{apr}(m, k_i) = 0/10 = 0$ which implies that no patient is infected with COVID which contradicts the data given in Table 1.

To find the approximation operators of the proposed method, the subbase are obtained from Soft

relations as follows:

$$R(S_1, T_3) = R(S_2, T_1) = R(S_2, T_4) = R(S_3, T_3) = R(S_4, T_1) = R(S_4, T_3) = \emptyset,$$
$$R(S_1, T_1) = \{(e_1, \{(S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\})\},$$
$$R(S_1, T_2) = \{(e_1, \{(S_1, T_1), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\},$$
$$(e_3, \{(S_1, T_4), (S_2, T_2), (S_3, T_2), (S_3, T_4), (S_4, T_2)\})\},$$
$$R(S_1, T_4) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\},$$
$$(e_3, \{(S_1, T_2), (S_2, T_2), (S_3, T_2), (S_3, T_4), (S_4, T_2)\})\},$$
$$R(S_2, T_2) = \{(e_3, \{(S_1, T_2), (S_1, T_4), (S_3, T_2), (S_3, T_4), (S_4, T_2)\})\},$$
$$R(S_2, T_3) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\})\},$$
$$R(S_3, T_1) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_4), (S_4, T_2), (S_4, T_4)\})\},$$
$$R(S_3, T_2) = \{(e_3, \{(S_1, T_2), (S_1, T_4), (S_2, T_2), (S_3, T_4), (S_4, T_2)\})\},$$
$$R(S_3, T_4) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_4, T_2), (S_4, T_4)\},$$
$$(e_3, \{(S_1, T_2), (S_2, T_2), (S_3, T_2), (S_4, T_2)\})\},$$
$$R(S_4, T_2) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_4)\},$$
$$(e_3, \{(S_1, T_2), (S_1, T_4), (S_2, T_2), (S_3, T_2), (S_3, T_4)\})\},$$
$$R(S_4, T_4) = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2)\})\}.$$

Thus, topological rough approximation space τ_{BR} with soft binary relation over S, T as subbase is obtained by taking an arbitrary union of finite intersection of elements of a subbasis.
According to proposed method,

$$\underline{\tau}_{BR}(m_{ki}) = \{(e_1, \emptyset), (e_3, \{(S_1, T_2), (S_3, T_2), (S_4, T_2)\})\},$$
$$\overline{\tau}_{BR}(m_{ki}) = m_k.$$

Accuracy = cardinality of $\underline{\tau}_{BR}(m_{ki})/$ cardinality of $\overline{\tau}_{BR}(m_{ki}) = 3/5 = 0.6$.
  If the people infected with COVID and their reasons be

$$m_{kj} = \{(e_1, \{(S_1, T_1), (S_1, T_2), (S_1, T_4), (S_2, T_3), (S_3, T_1), (S_3, T_4), (S_4, T_2), (S_4, T_4)\}),$$
$$(e_3, \{(S_1, T_2), (S_1, T_4), (S_2, T_2), (S_3, T_2), (S_3, T_4), (S_4, T_2)\})\}.$$

Then, according to Feng's method,

$$\underline{S}_{apr}(m_{kj}) = S \times T,$$
$$\overline{S}_{apr}(m_{kj}) = S \times T.$$

Accuracy is one.
  Similarly, according to the proposed method,

$$\underline{\tau}_{BR}(m_{kj}) = m_{kj},$$
$$\overline{\tau}_{BR}(m_{kj}) = m_{kj}.$$

Accuracy is one.
  From the above two cases, it is obvious that, in the case of soft topological approximation space, accuracy of the proposed method is higher than Feng's method.

Example 4.2. Consider *Example 2.3* where S is the set of all prime numbers less than or equal to 6, T is the set of all composite numbers less than or equal to 6. The soft relation induced from soft binary relation are as follows:

$$R(3,4) = \{(e_1, \{(5,4)\})\},$$
$$R(5,4) = \{(e_1, \{(3,4)\})\},$$
$$R(2,4) = \{(e_2, \{(3,6)\})\},$$
$$R(3,6) = \{(e_2, \{(2,4)\})\}.$$

Subbasis $S_B = \{\{(e_1, \{(5,4)\})\}, \{(e_1, \{(3,4)\})\}, \{(e_2, \{(3,6)\})\}, \{(e_2, \{(2,4)\})\}\}$.

The topology obtained by taking the finite intersection of an arbitrary union of elements of subbasis is as follows:

$$\tau_{BR} = \{\emptyset, m_k, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(3,4)\})\}, \{(e_2, \{(3,6)\})\}, \{(e_2, \{(2,4)\})\}, \{e_1, \{(5,4), (3,4)\}\},$$
$$\{(e_1, \{(5,4)\}), (e_2, \{(3,6)\})\}, \{(e_1, \{(5,4)\}), (e_2, \{(2,4)\})\}, \{(e_1, \{(3,4)\}), (e_2, \{(3,6)\})\},$$
$$\{(e_1, \{(3,4)\}), (e_2, \{(2,4)\})\}, \{(e_2, \{(2,4), (3,6)\})\}, \{(e_1, \{(5,4), (3,4)\}), (e_2, \{(2,4)\})\},$$
$$\{(e_1, \{(5,4), (3,4)\}), (e_2, \{(3,6)\})\}, \{(e_1, \{(5,4)\}), (e_2, \{(2,4), (3,6)\})\},$$
$$\{(e_1, \{(3,4)\}), (e_2, \{(2,4), (3,6)\})\}\},$$

where

$$m_k = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4), (3,6)\})\},$$
$$m_E = \{(e_1, \{(3,4), (5,4), (5,6)\}), (e_2, \{(2,4), (2,6), (3,6)\})\}.$$

Then, $(S, T, R_{m(s,t)}, \tau_{BR})$ is BR-topological rough approximation space.

$$\tau_{BR}^C = \{m_E, \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\}, \{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,4), (2,6), (3,6)\})\}),$$
$$\{(e_1, \{(5,4), (5,6)\}), (e_2, \{(2,4), (2,6), (3,6)\})\}, \{(e_1, \{(3,4), (5,4), (5,6)\}), (e_2,$$
$$\{(2,4), (2,6)\})\}, \{(e_1, \{(3,4), (5,4), (5,6)\}), (e_2, \{(2,6), (3,6)\})\}, \{(e_1, \{(5,6)\}),$$
$$(e_2, \{(2,4), (2,6), (3,6)\})\}, \{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,4), (2,6)\})\}, \{(e_1, \{(5,4),$$
$$(3,4)\}), (e_2, \{((2,6), (3,6)\})\}, \{(e_1, \{(5,4), (5,6)\}), (e_2, \{(2,4), (2,6)\})\}, \{(e_1, \{(5,4),$$
$$(5,6)\}), (e_2, \{(2,6), (3,6)\})\}, \{(e_1, \{(3,4), (5,4), (5,6)\}), (e_2, \{(2,6)\})\}, \{(e_1, \{(5,6)\}),$$
$$(e_2, \{(2,4), (2,6)\})\}, \{(e_1, \{(5,6)\}), (e_2, \{(2,6), (3,6)\})\}, \{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,6)\})\},$$
$$\{(e_1, \{(5,4), (5,6)\}), (e_2, \{(2,6)\})\}\}.$$

The BR-soft nowhere dense sets are $\emptyset, \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}, \{(e_1, \{(5,6)\})\}$ and $\{(e_2, \{(2,6)\})\}$. Then, the collection of BR-soft simply* alpha open sets are as follows:
$\tau_{BR}' = \tau_{BR}$, when $\emptyset$ is the BR-soft nowhere dense set (Since all BR-soft open sets are BR-soft alpha open sets).

$\tau_{BR}'' = \tau_{BR} \cup \{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$.

$\tau_{BR}''' = \tau_{BR} \cup \{(e_1, \{(5,6)\})\}$.

$\tau_{BR}'''' = \tau_{BR} \cup \{(e_2, \{(2,6)\})\}$.

$\tau_{BR}^* = \cup\{\tau_{BR}^i\}$ where $i = ', '', ''', ''''$.
The accuracy of BR-soft subsets of $S \times T$ is obtained by the Pawlak accuracy measure as follows:

Table 2 describes the accuracy of BR-soft subsets in τ_{BR} containing BR-soft open sets based on Yao's method where accuracy = cardinality of $int(m_k)/$ cardinality of $cl(m_k)$.

Table 3 describes the accuracy of BR-soft subsets in τ^*_{BR} containing BR-soft simply* alpha open sets. Accuracy = cardinality of $\underline{BR}_S/$ cardinality of $\overline{BR}_S$, $\underline{BR}_S$ and $\overline{BR}_S$ are lower and upper approximation operators.

T a b l e   2

Accuracy of BR-soft subsets

BR-soft subsets	Yao's method		Accuracy
	int	cl	
$A = \{(e_1, \{(5,6)\})\}$	$\emptyset$	$\{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$	0
$B = \{(e_2, \{(2,6)\})\}$	$\emptyset$	$\{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$	0
$C = \{(e_1, \{(2,4)\})\}$	C	$\{(e_1, \{(5,6)\}), (e_2, \{(2,4), (2,6)\})\}$	0.3
$D = \{(e_1, \{(3,4)\}), (e_2, \{(2,4)\})\}$	D	$\{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,4), (2,6)\})\}$	0.5
$E = \{(e_1, \{(3,4)\}), (e_2, \{(2,6)\})\}$	$\{(e_1, \{(3,4)\})\}$	$\{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,6)\})\}$	0.3
$F = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4)\})\}$	F	$\{(e_1, \{(3,4), (5,4), (5,6)\}), (e_2, \{(2,6)\})\}$	0.75
$G = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4), (3,6)\})\}$	G	m_E	0.7
$H = \{(e_1, \{(3,4)\}), (e_2, \{(2,4), (3,6)\})\}$	H	$\{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,4), (3,6), (2,6)\})\}$	0.6
$I = \{(e_1, \{(5,4), (5,6)\}), (e_2, \{(3,6)\})\}$	$\{(e_1, \{(5,4)\}), (e_2, \{(3,6)\})\}$	$\{(e_1, \{(5,4), (5,6)\}), (e_2, \{(3,6), (2,6)\})\}$	0.5
$J = \{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,6)\})\}$	$\{(e_1, \{(3,4)\})\}$	J	0.3
$K = m_E$	m_k	m_E	0.7

T a b l e   3

Accuracy of BR-soft subsets

BR-soft subsets	Proposed method		Accuracy
	$\underline{BR}_S$	$\overline{BR}_S$	
$A = \{(e_1, \{(5,6)\})\}$	A	A	1
$B = \{(e_2, \{(2,6)\})\}$	B	B	1
$C = \{(e_1, \{(2,4)\})\}$	C	C	1
$D = \{(e_1, \{(3,4)\}), (e_2, \{(2,4)\})\}$	D	D	1
$E = \{(e_1, \{(3,4)\}), (e_2, \{(2,6)\})\}$	E	E	1
$F = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4)\})\}$	F	F	1
$G = \{(e_1, \{(3,4), (5,4)\}), (e_2, \{(2,4), (2,6)\})\}$	G	G	1
$H = \{(e_1, \{(3,4)\}), (e_2, \{(2,4), (3,6)\})\}$	H	H	1
$I = \{(e_1, \{(5,4), (5,6)\}), (e_2, \{(3,6)\})\}$	I	I	1
$J = \{(e_1, \{(3,4), (5,6)\}), (e_2, \{(2,6)\})\}$	J	J	1
$K = m_E$	m_E	m_E	1

From the above tables (Table 2 and 3), it is obvious that the accuracy of the proposed method is higher than that of Yao's model.

Conclusion

In the current research, new classes of BR-soft open sets are introduced in BR-topological rough approximation space and their properties are studied. The accuracy measure of BR-soft subsets in BR-soft topology obtained from the collection of BR-soft simply* alpha open sets is evaluated. It is shown that the accuracy of the proposed method is high in comparison with the methods proposed by Feng and Yao. From Example 4.2, it is observed that, by using the proposed method, properties of the missing elements can also be studied. This gives a new view on solving decision making problems for a reliable solution.

Further to this work, efforts are being taken to study other topological properties like continuity, compactness, filters etc. Statistical properties of the defined set are being studied and attempts are made

to develop new methods for attribute reduction. In addition, the proposed method can be extended to other areas like fuzzy, intuitionistic fuzzy, hesitant fuzzy etc., and their properties can be studied in advanced topological areas. The proposed method can also be implemented for problems with missing information.

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Ч.Р. Парвати, А. София

PSGR Кришнаммал Қыздар колледжі, Тамил Наду, Индия

Аппроксимациясы шамамен алынған топологиялық кеңістігінде бинарлы жұмсақ жиындардың жаңа кластарын зерттеу

Жұмсақ бинарлы қатынас жұмсақ жиындардың жаңа кластарын анықтау үшін қолданылады, атап айтқанда, екі түрлі универсумның аппроксимациясы шамамен алынған топологиялық кеңістігінде BR-жұмсақ қарапайым ашық жиынды және BR-жұмсақ қарапайым альфа ашық жиынын анықтау

үшін қолданылады. Анықталған жиын BR бағдарламалық құрал жинағының жетіспейтін элемент-тері туралы ақпаратты береді және шешім қабылдауды жеңілдетуге көмектеседі. Аппроксимация операторлары анықталып, ұсынылған жиындардың сипаттамалары мысалдар арқылы зерттелген. Анықталатын жиындар мен басқа жұмсақ жиындар арасында байланыс табылды. Ұсынылған әдісті басқа әдістермен салыстыру үшін дәлдік сынағы жүргізілді. Ұсынылған әдістің дәлірек екені анық-талды.

Кілт сөздер: жұмсақ жиын, шамамен алынған жиын, қарапайым ашық, жуықтау кеңістігі, топологиялық кеңістік.

Ч.Р. Парвати, А. София

Женский колледж PSGR Кришнаммал, Тамил Наду, Индия

Исследование новых классов бинарных мягких множеств в топологическом пространстве грубой аппроксимации

Мягкое бинарное отношение используется для определения новых классов мягких множеств, а имен-но BR-мягкого просто открытого множества и BR-мягкого просто* альфа открытого множества, в топологическом пространстве грубой аппроксимации двух разных универсумов. Определенный на-бор предоставляет информацию о недостающих элементах программного набора BR и может помочь упростить принятие решений. Определены операторы аппроксимации и на примерах изучены харак-теристики предложенных множеств. Выявлена связь между определимыми множествами и другими мягкими множествами. Проведена проверка точности для сравнения предложенного метода с други-ми методами. Установлено, что предложенный метод является более точным.

Ключевые слова: мягкое множество, грубое множество, просто открытое, аппроксимационное про-странство, топологическое пространство.