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A.R. Yeshkeyev, A.R. Yarullina*, S.M. Amanbekov, M.T. Kassymetova

Karaganda University of the name of academician E.A. Buketov, Institute of Applied Mathematics, Karaganda, Kazakhstan
(E-mail: aibat.kz@gmail.com, linka14221@mail.ru, amanbekovsmath@gmail.com, mairushaasd@mail.ru)

On Robinson spectrum of the semantic Jonsson quasivariety of unars

Given article is devoted to the study of semantic Jonsson quasivariety of universal unars of signature containing only unary functional symbol. The first section of the article consists of basic necessary concepts. There were defined new notions of semantic Jonsson quasivariety of Robinson unars $J\mathbb{C}_U$, its elementary theory and semantic model. In order to prove the main result of the article, there were considered Robinson spectrum $RSp(J\mathbb{C}_U)$ and its partition onto equivalence classes $[\Delta]$ by cosemanticness relation. The characteristic features of such equivalence classes $[\Delta] \in RSp(J\mathbb{C}_U)$ were analysed. The main result is the following theorem of the existence of: characteristic for every class $[\Delta]$ the meaning of which is Robinson theories of unars; class $[\Delta]$ for any arbitrary characteristic; criteria of equivalence of two classes $[\Delta]_1, [\Delta]_2$. The obtained results can be useful for continuation of the various Jonsson algebras' research, particularly semantic Jonsson quasivariety of S-acts over cyclic monoid.

Keywords: Jonsson theory, unars, universal theory, Robinson theory, quasivariety, semantic Jonsson quasivariety, Jonsson spectrum, Robinson spectrum, equivalence class, cosemanticness.

Introduction

The study of model-theoretic relations of classical algebras and their syntactic properties from the Jonsson theories consideration, which are, generally speaking, incomplete, allows one to describe quite broad classes of theories. The article is a continuation of the work [1]. The authors of this article aimed at deepening of the universal unar's semantic model's characteristic study and strengthening the existing result by considering new and more general notion of semantic Jonsson quasivariety, and also by defining the notion of Robinson spectrum and its equivalence classes for unars.

The first section of the article gives the required notions of Jonsson theories, particularly Jonsson spectrum and its related notions. The second is devoted to the definitions connected with Jonsson universal unars and their semantic model's characteristic. The main section contains the definition of arbitrary characteristic and the main theorem on cosemanticness classes of factor-set $RSp(J\mathbb{C})_{/\bowtie}$, obtained during research conduction. All necessary base definitions can be found in [2], definitions and notions concerning Jonsson theories in [3–18].

All definitions that were not given in the current article can be extracted from [3].

1 Semantic Jonsson quasivariety

One of the important definitions, used by the authors of given article, is the definition of Jonsson theory. Let us recall the conditions, that should be satisfied in order for a theory to be Jonsson.

Definition 1. [3; 80] A theory T is said to be Jonsson, if:
1) T has at least one infinite model;
2) T is $\forall\exists$-axiomatising;
3) T has JEP property;
4) T has AP property.

*Corresponding author.
E-mail: linka14221@mail.ru

$\forall$-axiomatizing Jonsson theory is called the Robinson theory.

Let us recall some necessary notions from Jonsson model theory.

Theorem 1. [3; 155] T is Jonsson iff it has a semantic model $\mathfrak{C}_T$.

The definition of Jonsson theory's semantic model.

Definition 2. [3; 155] Let T be a Jonsson theory. A model $\mathfrak{C}_T$ of power $2^{|T|}$ is called to be a semantic model of the theory T if $\mathfrak{C}_T$ is a $|T|^+$-homogeneous $|T|^+$-universal model of the theory T.

The next definition was introduced by T.G. Mustafin.

Definition 3. [3; 161] The elementary theory of a semantic model of the Jonsson theory T is called the center of this theory. The center is denoted by T^*, i.e. $Th(C) = T^*$.

Since the current research is connected with consideration of Robinson spectrum for classes of algebras, let us give the following conditions of Jonsson theories' cosemanticness.

Definition 4. [3; 40] Let T_1 and T_2 be Jonsson theories, T_1^* and T_2^* be their centres, respectively. T_1 and T_2 are said to be cosemantic Jonsson theories (denoted by $T_1 \bowtie T_2$), if $T_1^* = T_2^*$.

Theorem 2. [3; 176] Let T_1 and T_2 be Jonsson theories, $\mathfrak{C}_{T_1}$ and $\mathfrak{C}_{T_2}$ be their semantic models, respectively. Then the next conditions are equivalent:
1) $\mathfrak{C}_{T_1} \bowtie \mathfrak{C}_{T_2}$;
2) $\mathfrak{C}_{T_1} \equiv_J \mathfrak{C}_{T_2}$;
3) $\mathfrak{C}_{T_1} = \mathfrak{C}_{T_2}$.

Let K be a class of models of fixed signature σ. Then we can consider Jonsson spectrum for K, which can be defined as follows.

Definition 5. [5] A set $JSp(K)$ of Jonsson theories of signature σ, where

$$JSp(K) = \{T \mid T \text{ is Jonsson theory and } K \subseteq Mod(T)\}$$

is called the Jonsson spectrum for class K.

Hence, in the particular case, when the Jonsson theory is $\forall$-axiomatising we get the concept of the Robinson theory, respectively, the notion of the Jonsson spectrum allows us to consider the Robinson spectrum.

Definition 6. A set $RSp(K)$ of Robinson theories of signature σ, where

$$RSp(K) = \{T \mid T \text{ is Robinson theory and } \forall A \in K,\ A \models T\}$$

is called the Robinson spectrum for class K.

Definition 4 states, that two Jonsson theories are cosemantic ($T_1 \bowtie T_2$), if their centres are equal. It is easy to check, that such cosematicness relation, given on a set of Jonsson theories, will be an equivalence relation. The proof of this fact one can find in detail in [4]. Hence, based on theorem 2, we can consider the cosemanticity relation on Jonsson spectrum $JSp(K)$ and obtain a partition of $JSp(K)$ onto equivalence classes. We get a factor-set, denoted as $JSp(K)_{/\bowtie}$. The factor-set $RSp(K)_{/\bowtie}$ will be obtained correspondingly.

According to A.I. Malcev [2], quasivarieties of algebras are the classes of algebras, that can be set by means of collection of quasi-identities (conditional identities). Quasi-identities are $\forall$-formulas, and quasivarieties are presented as particular types of universally axiomatising classes of algebras. A class $\mathfrak{R}$ of algebraic system is called a quasivariety if there is such collection of quasi-identities of signature σ that this algebraic system consists of those and only those systems of signature σ, in which all formulas from σ are true [2].

We want to define semantic Jonsson quasivariety as follows. Let K be a class of quasivariety in the sense of [2] of first-order language L, $L_0 \subset L$, where L_0 is the set of sentences of language L. Let us consider the elementary theory $Th(K)$ of such class K. By adding to $Th(K)$ $\forall\exists$ sentences of language L, that are not contained in the $Th(K)$, we can consider the set of Jonsson theories $J(Th(K))$ defined as follows.

Denotation 1. A set $J(Th(K)) = \{\Delta \mid \Delta-\text{Jonsson theory}, \ \Delta = Th(K)\cup\{\varphi^i\}\}$, where $\varphi^i \in \forall\exists(L_0)$ and $\varphi^i \notin Th(K)$ for some $i \in \{0,1\}$, $Th(K)$ is elementary theory of class of quasivariety K, $\forall\exists(L_0)$ is a set of all $\forall\exists$ sentences of language L.

According to theorem 1 the theory is Jonsson iff it has a semantic model. Hence every Jonsson theory $\Delta \in J(Th(K))$ has its own semantic model $\mathfrak{C}_\Delta$. Let us consider the set of such semantic models and denote it as $J\mathbb{C}$.

Denotation 2. A set $J\mathbb{C} = \{\mathfrak{C}_\Delta \mid \Delta \in J(Th(K)), \mathfrak{C}_\Delta \text{ is semantic model of } \Delta\}$.

We will call the set $J\mathbb{C}$ semantic Jonsson quasivariety of class K if its elementary theory $Th(J\mathbb{C})$ is Jonsson theory.

2 Robinson spectrum of semantic Jonsson quasivariety of Robinson unars

We will consider some basic definitions, denotations, properties of arbitrary Jonsson universals, necessary for proofing the main result of the article.

Denotation 3. [1] 1) If Γ is collection or type of the sentences, then T_Γ is following set of formulas $\{\psi \in T : \{\varphi \in \Gamma : T \vdash \varphi\} \vdash \psi\}$;
2) ∇ is $\Pi_1 \cup \Sigma_1$, that is ∇ is a collection of all universal and existential formulas.

Here, in the second item, Π_1 denotes universal formulas, Σ_1 denotes existential ones.

Definition 7. [1] 1) If $T = T_\forall$, then $T_\forall$ is said to be universal;
2) If $T = T_\nabla$, then the theory T is called primitive.

Thus, by the universal we call a set of all universal conclusions of Jonsson theory T. The next proposition plays an important role in the proof of the obtained main theorem of the article.

Proposition 1. [1] Let T_1, T_2 be Jonsson universals. Then the following conditions are equivalent:
1) $T_1 = T_2$;
2) $\mathfrak{C}_{T_1} \simeq \mathfrak{C}_{T_2}$;
3) $T_1^* = T_2^*$.

$\mathfrak{C}_{T_1}$ and $\mathfrak{C}_{T_2}$ are semantic models of Jonsson theories T_1, T_2 respectively. Each model U of Jonsson theory of unars T is an unar. Consequently, the following fact is true.

Lemma 1. [1] For any unar U the following is satisfied

$$U \models T \Leftrightarrow U \text{ embeds in } \mathfrak{C}.$$

The following definitions are necessary for the construction of semantic model of cosemanticness classes of Robinson spectrum for semantic Jonsson quasivariety of Robinson unars.

Definition 8. [1] 1) If $A \subseteq \mathfrak{C}, a \in \mathfrak{C}$, then $[A, a]$ denotes sub-unar, generated by subset $A \cup \{a\}$.
2) We will write $tp^{\mathfrak{C}}_{at}(a, A) = tp^{\mathfrak{C}}_{at}(b, A)$ if there is such isomorphism $\varphi : [A, a] \simeq [A, b]$, that $\varphi(c) = c, \forall c \in A$, and $\varphi(a) = b$.

Definition 9. [1] 1) If H is sub-unar $\mathfrak{C}, f^n(a) = h \in H, f^k(a) \notin H$ for all $k < n$, then the element h will be called input element from a in H, and number n will be called the distance from a to H. In this case we will use denotation $h = \text{input}(a, H), n = \rho(a, H)$. We will write $\rho(a, H) = \infty$, if $f^n(a) \notin H, \forall n < \omega$.

2) $\chi(a) = \begin{cases} \omega, \text{if } f^n(a) \neq f^k(a), \forall n < k < \omega \\ < n, m >, \text{if } < n, m >= min\{< n, m >: f^n(a) = f^{n+m}(a)\}. \end{cases}$

Definition 10. [1] If $a \in \mathfrak{C}$, then

$$k(a) \ |\{b \in \mathfrak{C} : f(b) = a\}|.$$

Definition 11. [1] A set $\{a_1, ..., a_m\}$ of elements $\mathfrak{C}$ will be called m-loop, if $a_i \neq a_j, f(a_i) = a_{i+1}$ for all $1 \leq i < j \leq m$ and $f(a_m) = a_1$.

The next definition determines the characteristic of semantic model of Robinson unar's Jonsson theory.

Definition 12. [1] A fourset $(\Omega, \nu, \mu, \varepsilon)$ will be called a characteristic $\mathfrak{C}$ and denoted as $char(\mathfrak{C})$, if
$\Omega = \{\chi(a) : a \in \mathfrak{C}\}$,
$\nu \ \omega \setminus \{0\} \to \omega \cup \{\infty\}$ such that $\forall m > 0$,
$\nu(m) = \begin{cases} k, \text{if the quantity } m - \text{loops in } \mathfrak{C} \text{ is equal to } k < \omega, \\ \infty, \text{otherwise;} \end{cases}$
$\mu : \Omega \to \omega \cup \{\infty\}$ such that if $\alpha \in \Omega$ and $\alpha \in \chi(a)$, then $\mu(\alpha) = k(a)$, if $k(a) < \omega$ and $\mu(\alpha) = \infty$, if $k(a) = |\mathfrak{C}|$;
$\varepsilon = \begin{cases} 0, \text{if } |\{a \in \mathfrak{C} : \chi(a) = \omega\}| = 0, \\ \infty, \text{otherwise.} \end{cases}$

The next lemma gives some useful specification to the definition of above-mentioned fourset.

Lemma 2. [1] If $char(\mathfrak{C}) = (\Omega, \nu, \mu, \varepsilon)$, then
1°. $\varnothing \neq \Omega \subseteq \{\omega\} \cup (\omega \times \omega)$;
2°. $(n, m) \in \Omega \& 0 \leq k < n \Rightarrow (k, m) \in \Omega$;
3°. $\nu(m) > 0 \Leftrightarrow (0, m) \in \Omega$;
4°. $\omega \in \Omega \Leftrightarrow \varepsilon = \infty$;
5°. $|\Omega| = \omega \Rightarrow \omega \in \Omega$;
6°. $(n, m) \in \Omega \Rightarrow ((n + 1, m) \notin \Omega \Leftrightarrow \mu^*(n, m) = 0$;
7°. $\omega \notin \Omega \& |\Omega| < \omega \Rightarrow \exists m < \omega(\nu(m) = \infty) \vee \exists n < \omega, m < \omega((n, m) \in \Omega \& \mu(n, m) = \infty)$;
8°. $|\Omega| = \omega \Rightarrow \begin{cases} \mu(\omega) \geq k, \text{if } \exists k, l < \omega(k = max\{\mu(n, m) \in \Omega, n + m \geq l\}); \\ \mu(\omega) = \infty, \text{ otherwise.} \end{cases}$

3 Main result

The theory $Th_{\forall}(U)$ of all universal sentences, true in U is the Jonsson theory. This statement was proven in the work [1]. By virtue of $\forall$-axiomatisability of elemantary theory of unars, $Th_{\forall}(U)$ is the Robinson theory of unars.

Thus, we use the denotation 2 of semantic Jonsson quasivariety of class K and consider a set $J\mathbb{C}_U = \{\mathfrak{C}_\Delta \mid \Delta \in J(Th(K)), \mathfrak{C}_\Delta \text{ is a semantic model } \Delta\}$ of signature $\sigma_U =< f >$, where Δ is a Robinson theory of unars, f is unary functional symbol. Such $J\mathbb{C}_U$ defines semantic Jonsson quasivariety of Robinson unars.

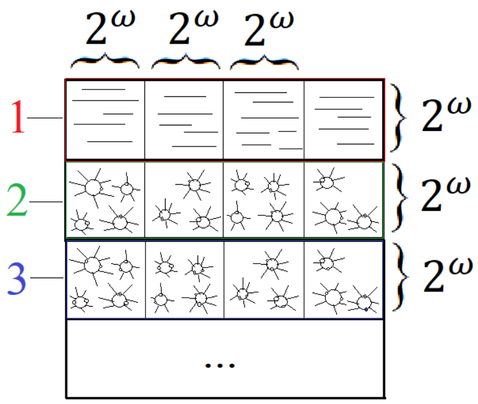

Figure 1. Semantic Jonsson quasivariety of Robinson unars $J\mathbb{C}_U$

We can see on the figure 1, that 1, 2, 3 are $\mathfrak{C}_{\Delta_1}$, $\mathfrak{C}_{\Delta_2}$, $\mathfrak{C}_{\Delta_3}$, which are semantic models of $[\Delta_1]$, $[\Delta_2]$, $[\Delta_3]$ respectively. The semantic models consist of unars of length 0, 1, 2 and so on.

Let us define the Robinson spectrum of the set $J\mathbb{C}_U$ as follows.

Definition 13. A set $RSp(J\mathbb{C}_U)$ of Robinson theories of signature σ_U, where

$$RSp(J\mathbb{C}_U) = \{\Delta \mid \Delta \text{ is Robinson theory of unars and } \forall \mathfrak{C}_\Delta \in J\mathbb{C}_U, \mathfrak{C}_\Delta \models \Delta\}$$

is called the Robinson spectrum for class $J\mathbb{C}_U$, where $J\mathbb{C}_U$ is semantic Jonsson quasivariety of Robinson unars.

Further we can consider the notion of cosemanticness relation on Robinson spectrum $RSp(J\mathbb{C}_U)$ and get the partition $RSp(J\mathbb{C}_U)$ on equivalence classes. As a result we obtain a factor-set, denoted as $RSp(J\mathbb{C}_U)_{/\bowtie}$ and consisted of equivalence classes parted by cosemanticness relation $[\Delta] \in RSp(J\mathbb{C}_U)_{/\bowtie}$.

Remark 1. Everywhere in this section $[\Delta]$ denotes an equivalence class of Robinson theories of unars parted by cosemanticness relation on Robinson spectrum $RSp(J\mathbb{C}_U)$, $\mathfrak{C}_{[\Delta]}$ denotes this class's semantic model.

According to theorem 2 and definition 6 we can deduce the conclusion, that $char(\mathfrak{C}_{[\Delta]})$ defines similarly to $char(\mathfrak{C})$ from definition 12 for every semantic model $\mathfrak{C}_{[\Delta]}$ of every class $[\Delta]$.

Definition 14. As a characteristic $Char([\Delta])$ we will understand $Char(\mathfrak{C}_{[\Delta]})$.

Lemma 3. For classes $[\Delta_1], [\Delta_2]$ of Robinson theories of unars the following conditions are equivalent:
1) $[\Delta_1]$ is equivalent to $[\Delta_2]$;
2) $Char([\Delta_1]) = Char([\Delta_2])$.

Proof. 1) $\Rightarrow$ 2) According to the theorem 2 if two classes are equivalent, then their semantic models will be equal to each other. Therefore, the characteristics of those models will also be equal to each other.

2) $\Rightarrow$ 1) follows from the fact that $Char(\mathfrak{C}_{[\Delta]})$ defines the semantic models $\mathfrak{C}_{[\Delta]}$ up to isomorphism. Hence $\mathfrak{C}_{[\Delta]_1} \simeq \mathfrak{C}_{[\Delta]_2}$, according to proposition 1.

Definition 15. [1] An arbitrary fourset $(\Omega^{**}, \nu^{**}, \mu^{**}, \varepsilon^{**})$ will be called a characteristic if the following conditions are satisfied:
1) $\varnothing \neq \Omega^{**} \subseteq \{\omega\} \cup (\omega \times \omega)$;
2) $\nu^{**} : \omega \setminus \{0\} \to \omega \cup \{\infty\}$;
3) $\mu^{**} : \Omega^{**} \to \omega \cup \{\infty\}$;
4) $\varepsilon^{**} = 0$ or $\varepsilon^{**} = \infty$;

5) $\omega \in \Omega^{**} \Leftrightarrow \varepsilon = \infty$;
6) $(n,m) \in \Omega^{**} \cap (\omega \times \omega) \& 0 \leq k < n \Rightarrow (k,m) \in \Omega^{**}$;
7) $\nu^{**}(m) > 0 \Leftrightarrow (0,m) \in \Omega{**}$;
8) $|\Omega^{**}| = \omega \Rightarrow \omega \in \Omega^{**}$;
9) $(n,m) \in \Omega^{**} \cap (\omega \times \omega) \Rightarrow (\mu(n,m) = 0 \Leftrightarrow (n+1,m) \neq \Omega^{**})$;
10) $\omega \notin \Omega^{**} \& |\Omega^{**}| < \omega \Rightarrow \exists m < \omega (\nu(m) = \infty) \vee \exists n < \omega, m < \omega \ \ ((n,m) \in \Omega^{**} \& \mu(n,m) = \infty)$;
11) $|\Omega^{**}| = \omega \Rightarrow \begin{cases} \mu(\omega) \geq k, \text{if } \ \exists k, l < \omega (k = max\{\mu(n,m) : (n,m) \in \Omega^{**}, n+m \geq l\}); \\ \mu(\omega) = \infty, \text{otherwise}; \end{cases}$
12) $\mu^{**}(\omega) > 0$.

Lemma 4. $Char(\mathfrak{C}_{[\Delta]})$ is characteristic.

Proof. Follows immediately from lemma 2.

Theorem 3 (Main theorem). 1) Every class $[\Delta]$ has a characteristic.
2) For any characteristic π there is a class $[\Delta]$, that has characteristic π.
3) Two classes $[\Delta_1], [\Delta_2]$ are equivalent iff their characteristics are equal.

Proof. Items 1) and 3) are proven in lemmas 2 and 3 respectively.

2) Let us consider given arbitrary characteristic $\pi = (\Omega^{**}, \nu^{**}, \mu^{**}, \varepsilon^{**})$. We need to define class $[\Delta]_\pi$, that is the equivalence class of Robinson theories of unars parted by cosemanticness relation on Robinson spectrum $RSp(J\mathbb{C}_U)$ of characteristic π. Let us start from the denotation of collection of universal sentences of unars' language

$Q_{k,n,m} = \forall x (f^n(x) = f^{n+m}(x) \wedge (\&_{0 \leq i < j < n+m} f^i(x) \neq f^j(x)) \rightarrow \forall y_1, ..., y_{k+1} (\wedge_{i=1}^{k+1} f(y_i) = (x) \rightarrow \&_{1 \leq i < j \leq k+1} y_i = y_j)$.

$Q_{k,n,m}$ expresses "$\chi(x) = (n,m) \Rightarrow k(x) \leq k$".

$P_{l,m}$ is $\forall x_1, ..., x_{l+1} ([\wedge_{i+1}^{l+1} (f^m(x_i) = x_i \wedge \wedge_{m-1}^{j=1} f^j(x_i)) \neq x_i)] \rightarrow \&_{1 \leq i < j < l+1_{0 \leq k, n \leq m-1}} f^k(x_i) = f^m(x_j))$.

$P_{l,m}$ states that the quantity of m-loops is no more then l.

R_m is $\forall x \neg (x = f^m(x) \wedge \wedge_{i=1}^{m-1} x \neq f^i(x))$.

R_m expresses the absence of m-loops.

Φ_m is $\forall x (f^m(x) \neq x)$. No comments needed here.

F_r is $\forall x \forall y_1, ..., y_{r+1} (\wedge_{i=1}^{r+1} f(y_i) = x \rightarrow \vee_{1 \leq i < j \leq r+1} y_i = y_j)$.

$F_r \Leftrightarrow \forall \alpha \in \Omega^{**} (\mu^{**}(\alpha) \leq r) \Leftrightarrow \forall x (k(x) \leq r)$.

$E_{r,m}$ is $\forall x (\wedge_{0 \leq i < j \leq m} f^i(x) \neq f^j(x) \rightarrow \forall y_1, ..., y_{r+1} (\wedge_{i=1}^{r+1} f(y_i) = x \rightarrow \vee_{1 \leq i < j \leq r+1} y_i = y_j)$.

$E_{m,r}$ states that if x is not an elemnt of s-loop for all $s \leq m$, then $K(x) \leq r$.

If $|\Omega^{**}| < \omega$ and $\omega \notin \Omega^{**}$, then D_Ω^{**} is $\forall x \vee_{(n,m) \in \Omega^{**}} (\vee_{0 \leq i < j \leq n+m-1} f^i(x) \neq f^j(x) \wedge f^n(x) = f^{n+m}(x))$.

In this case $D_\Omega^{**} \Leftrightarrow \forall x (\chi(x) \in \Omega^{**})$.

Let us move on to definition of $[\Delta]_\pi$.

Case 1. $\varepsilon^{**} = \infty$.

By the condition 5) of definition 15 it is equivalent to $\omega \in \Omega^{**}$. By the condition 12) $\mu^{**}(\omega) > 0$.

Case 1.1. $\Omega^{**} \setminus \{\omega\} \neq \varnothing$.

Case 1.1.1. $\mu^{**}(\omega) = \infty$.

Let $\theta_{\Omega^{**}, \nu^{**}, \mu^{**}}$ be $\{Q_{k,n,m} : (n,m) \in \Omega^{**} \setminus \{\omega\}, k = \mu^{**}(n,m)\} \cup \{P_{l,m} : 0 < m < \omega, 1 \leq l = \nu^{**}(m) < \omega\} \cup \{R_m : 0 < m < \omega, \nu^{**}(m) = 0\}$.

We suppose $[\Delta]_\pi = \theta_{\Omega^{**}, \nu^{**}, \mu^{**}}$

Case 1.1.2. $\mu^{**}(\omega) = r < \omega$.

Let $[\Delta]_\pi = \theta_{\Omega^{**}, \nu^{**}, \mu^{**}} \cup \{F_r\}$.

Case 1.2. $\Omega^{**} = \{\omega\}$.

Case 1.2.1. $\mu^{**}(\omega) = \infty$

By definition $[\Delta]_\pi = \{\Phi_m : 0 < m < \omega\}$.

Case 1.2.2. $\mu^{**}(\omega) = r$.

By definition $[\Delta]_\pi = \{\Phi_m : 0 < m < \omega\} \cup \{F_r\}$.

Case 2. $\varepsilon^{**} = 0$.

Note, that in this case by conditions 5) and 8) $\omega \notin \Omega^{**}$ and $|\Omega^{**}| < \omega$. Let us suppose $[\Delta]_\pi = \{Q_{k,n,m} : (n,m) \in \Omega^{**}, k = \mu^{**}(n,m)\} \cup \{P_{l,m} : 0 < m < \omega, 1 \leq l = \nu^{**}(m) < \omega\} \cup \{D^{**}_{\Omega}\}$. It is not hard to check, that in every case $[\Delta]_\pi$ is the equivalence class of Robinson theories of unars parted by cosemanticness relation on Robinson spectrum $RSp(J\mathbb{C}_U)$ and $Char(\mathfrak{C}_{[\Delta]_\pi}) = \pi$. The theorem is proven.

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А.Р. Ешкеев, А.Р. Яруллина, С.М. Аманбеков, М.Т. Касыметова

Академик Е.А. Бөкетов атындағы Қарағанды университеті, Қолданбалы математика институты, Қарағанды, Қазақстан

Унарлардың семантикалық йонсондық квазикөптүрліліктерінің робинсондық спектрі

Мақала сигнатурасы тек бір орынды функционалдық символдан тұратын, универсалды унарлардың семантикалық йонсондық квазикөптүрліліктерін зерттеуге арналған. Мақаланың бірінші бөлімі негізгі қажетті ұғымдардан тұрады. Сонымен қатар $J\mathbb{C}_U$ робинсондық унарлардың семантикалық йонсондық квазикөптүрліліктерінің, оның элементарлы теориясы мен семантикалық моделінің жаңа түсініктері анықталды. Мақаланың негізгі нәтижесін дәлелдеу үшін $RSp(J\mathbb{C}_U)$ робинсондық спектр және оның косемантты қатынас арқылы $[\Delta]$ эквиваленттік кластарға бөлінуі қарастырылған. Мұндай $[\Delta] \in RSp(J\mathbb{C}_U)$ эквиваленттік кластардың сипаттамалық ерекшеліктері талданған. Мәні унарлардың робинсондық теориялары болатын әрбір $[\Delta]$ үшін кездейсоқ сипаттаманың; кез келген кездейсоқ сипаттама үшін $[\Delta]$ класының; екі $[\Delta]_1, [\Delta]_2$ кластарының эквиваленттілік критерийінің бар болу теоремасы негізгі нәтиже болып табылады. Алынған нәтижелер әртүрлі йонсондық алгебраларды, атап айтқанда, циклді моноид арқылы анықталған полигондардың семантикалық йонсондық квазикөптүрліліктерді зерттеуді жалғастыру үшін пайдалы болуы мүмкін.

Кілт сөздер: йонсондық теория, унарлар, универсалды теория, робинсондық теория, квазикөптүрлілік, семантикалық йонсондық квазикөптүрлілік, йонсондық спектр, робинсондық спектр, эквиваленттік класс, косеманттылық.

А.Р. Ешкеев, А.Р. Яруллина, С.М. Аманбеков, М.Т. Касыметова

Карагандинский университет имени академика Е.А. Букетова, Институт прикладной математики, Караганда, Казахстан

Робинсоновский спектр семантического йонсоновского квазимногообразия унаров

Статья посвящена изучению семантического йонсоновского квазимногообразия универсальных унаров сигнатуры, содержащей единственный функциональный символ. Первый раздел статьи состоит из базовых необходимых понятий. Были определены новые понятия семантического йонсоновского квазимногообразия робинсоновских унаров $J\mathbb{C}_U$, его элементарной теории и семантической модели. Для того чтобы доказать главный результат статьи, были рассмотрены робинсоновский спектр $RSp(J\mathbb{C}_U)$ и его разбиение на классы эквивалентности $[\Delta]$ с помощью отношения косемантичности. Проанализированы характерные особенности таких классов эквивалентностей $[\Delta] \in RSp(J\mathbb{C}_U)$. Основным результатом является следующая теорема о существовании: произвольной характеристики для каждого $[\Delta]$, значение которого-робинсоновские теории унаров; класс $[\Delta]$ для любой произвольной характеристики; критерий эквивалентности классов $[\Delta]_1, [\Delta]_2$. Полученные результаты могут быть полезны в продолжении исследования различных йонсоновских алгебр, в частности, семантического йонсоновского квазимногообразия полигонов над циклическим моноидом.

Ключевые слова: йонсоновская теория, унары, универсальная теория, робинсоновская теория, квазимногообразие, семантическое йонсоновское квазимногообразие, йонсоновский спектр, робинсоновский спектр, класс эквивалентности, косемантичность.

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