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Forcing companions of Jonsson AP-theories

This article is devoted to the study of the forcing companions of the Jonsson AP-theories in the enriched signature. It is proved that the forcing companion of the theory does not change when expanding the theories under consideration, which have some properties, by adding new predicate and constant symbols to the language. The model-theoretic results obtained in this paper in general form are supported by examples from differential algebra. An approach in combining a Jonsson and non-Jonsson theories is demonstrated. In this paper, for the first time in the history of Model Theory. This will allow us to further develop the methods of research of Jonsson theories and expand the apparatus for studying incomplete theories.

Keywords: Jonsson theory, perfect Jonsson theory, AP-theory, forcing, forcing companion, enrichment of a signature, expanding theory, differential field, differentially closed field, differentially perfect field.

Introduction

In recent years, Model Theory has increasingly revealed its potential in solving important problems from various areas of mathematics. Thus, many significant facts concerning differential algebras, namely differential fields of zero and positive characteristic, were established through the use of model-theoretic methods in the studies of D. Marker, L. Blum, K. Wood, and others. At the same time, there is an increasing need to develop their own apparatus of Model Theory, especially in the study of incomplete theories. In the 1980s, among inductive theories, a special subclass of Jonsson theories was singled out, which are incomplete. Examples of Jonsson theories are the theories of well-known classical algebras, such as group theory, fixed characteristic field theory, linear order theory, etc. are provided. The methods used in the study of this class largely demonstrate their usefulness due to the numerous results obtained by B. Poizat, T.G. Mustafin, A.R. Yeshkeyev, E.T. Mustafin.

In [1], the authors began the study of the Jonsson differential algebras: results were obtained for differential fields of characteristic 0 and p. Here we continue to develop this direction while expanding the language of these theories and considering forcing companions in a new enrichment.

In the framework of the study of Jonsson theories, earlier works [2–4] considered theories obtained as constructions of Jonsson theories. In this paper, we work with a theory that is a union of two theories, where the first one is Jonsson and the other is not.

1 Preliminary information

We start with the main definitions and facts concerning the subject of the study. Recall the definitions of a model companion and a forcing companion.

Definition 1. [5; 156] Let T and T_{MC} be some L-theories. The theory T_{MC} is called a model completion of the theory T if:

1) T and T_{MC} are mutually model consistent, i.e., any model of the theory T is embedded in the model of the theory T_{MC} and vice versa;

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2) T_{MC} is a model complete theory;

3) if $A \models T$, then $T_{MC} \cup D(A)$ is a complete theory. The theory T_{MC} is called a model companion if conditions 1) and 2) hold.

Definition 2. [6; 129] Let T be a theory of the language L. A forcing companion of the theory T is a theory T^f which is the set of all sentences of the language L weakly forced by $\emptyset$.

The following results were proved by J. Barwise and A. Robinson:

Theorem 1. [6; 133] Let T_1 and T_2 be the theories of the language L. Then T_1 and T_2 are mutually model consistent if and only if $T_1^f = T_2^f$.

Theorem 2. [6; 134] Let T be mutually model consistent with some inductive theory T'. Then $T' \subseteq T^f$. Therefore, if T is an inductive theory then $T \subseteq T^f$.

Definition 3. [5; 80] A theory T has the joint embedding property (JEP) if for any models U, B of the theory T there exists a model M of the theory T and isomorphic embeddings $f : U \to M$, $g : B \to M$.

Definition 4. [5; 68] A theory T has the amalgam property (AP) if for any models U, B_1, B_2 of the theory T and isomorphic embeddings $f_1 : U \to B_1$, $f_2 : U \to B_2$ there are $M \models T$ and isomorphic embeddings $g_1 : B_1 \to M$, $g_2 : B_2 \to M$ such that $g_1 \circ f_1 = g_2 \circ f_2$.

Since the work relates mainly to the study of the Jonsson theories, we will give the main definitions concerning them. More detailed information about the Jonsson theories can be found mainly in [7]. In works [8–11], newer and more specific results have been published, and the apparatus for studying Jonsson theories has been expanded.

We are working within the framework of the following definition of Jonsson theory published in the Russian edition of [5].

Definition 5. [5; 80] A theory T is called Jonsson if:
1. the theory T has at least one infinite model;
2. T is an inductive theory;
3. T has the amalgam property (AP);
4. T has the joint embedding property (JEP).

Many classical objects from Algebra satisfied such conditions, and these theories are Jonsson
1) group theory;
2) theory of abelian groups;
3) theory of boolean algebras;
4) theory of linear orders;
5) field theory of characteristic p, where p is zero or a prime number;
6) theory of ordered fields;
7) theory of modules.

The following concepts and facts play a crucial role in the construction of a model-theoretic apparatus associated with the study of Jonsson theories.

Definition 6. [7; 155] Let T be a Jonsson theory. A model C_T of power $2^{|T|}$ is called to be a semantic model of the theory T if C_T is a $|T|^+$-homogeneous $|T|^+$-universal model of the theory T.

Theorem 3. [7; 155] T is Jonsson if it has a semantic model C_T.

The following definition was introduced by T.G. Mustafin.

Definition 7. [7; 155] A Jonsson theory T is called perfect if its semantic model C_T is saturated.

Definition 8. [7; 161] The elementary theory of a semantic model of the Jonsson theory T is called the center of this theory. The center is denoted by T^*, i.e. $Th(C) = T^*$.

Theorem 4. [7; 158] Let T be an arbitrary Jonsson theory. Then the following conditions are equivalent:

1) the theory T is perfect;
2) $T^* = Th(C)$ is the model companion of the theory T.

The following theorem is of particular importance for this study:

Theorem 5. [7; 162] Let T be a perfect Jonsson theory. Then the following statements are equivalent:

1) T^* is the model companion of T;
2) $ModT^* = E_T$;
3) $T^* = T^f$, where T^f is a forcing companion of the theory T.

Theorem 6. [12; 1243] Let T be a Jonsson theory. Then for any model $A \in E_T$ theory $T^0(A)$ is Jonsson, where $T^0(A) = Th_{\forall\exists}(A)$.

We can see that in the case of the perfectness of T its center T^* is also a Jonsson theory.

The following definition will help us to specify the class of Jonsson theories which we will deal with in this paper.

Definition 9. [13; 120] A Jonsson theory is said to be hereditary if, in any of its permissible enrichment, it preserves the Jonssonness.

As mentioned before, Jonsson theories should have joint embedding and amalgam properties. At the same time, it is known from [14; 270] that these two properties are generally independent of each other. However, theories with AP and JEP form special subclasses among inductive theories that are of interest for studying the internal structure of their model classes. In work [1], A.R. Yeshkeyev introduced the following concepts:

Definition 10. [1; 130] A theory T is called an AP-theory if, from the fact that it has the amalgam property, it follows that T also has the joint embedding property, i.e. $AP \to JEP$.

Definition 11. [1; 130] A theory T is called a JEP-theory if T has the joint embedding property and this implies the presence of the amalgam property, i.e. $JEP \to AP$.

Definition 12. [1; 130] We call a theory T an AJ-theory if the properties of the amalgam and the joint embedding are equivalent for T, i.e. $AP \leftrightarrow JEP$.

Examples are various classes of unars [14; 270]. In addition, in [1], it is shown that the theory of differential fields of characteristic 0 and the theory of differentially perfect fields of characteristic p, which will be discussed later, are AP-theories.

2 Forcing companions of theories in an enriched signature

Now we move on to the problem statement. We consider the theories Δ_1, Δ_2, Δ_3 satisfy the following conditions:

1) Δ_1 is an inductive theory that is not a Jonsson theory but has a model companion which is the theory Δ_3,
2) Δ_2 is a hereditary Jonsson AP-theory that has a model companion, which is also Δ_3.

Based on the conditions set, we can draw the following conclusions. All three theories are mutually model consistent, because Δ_3 is mutually model consistent with both Δ_1 and Δ_2, for which Δ_3 is the model companion, which means that Δ_1 and Δ_2 are mutually model consistent with each other. At the same time, according to Theorem 1, the forcing companions of mutually model consistent theories must coincide, which means that $\Delta_1^f = \Delta_2^f$. Δ_2 is a perfect Jonsson theory, while $\Delta_2^* = Th(C) = \Delta_3$, C is a semantic model of Δ_2, which follows from Theorem 4. In addition, Theorem 5 gives us reason to assert that Δ_3 is also a forcing companion of Δ_2, i.e. $\Delta_3 = \Delta_2^f$. So we get $\Delta_1^f = \Delta_2^f = \Delta_3$.

Consider the following extensions of the theories Δ_1, Δ_2, Δ_3 in various language enrichment L by adding new constant and predicate symbols c and P. Let $\overline{\Delta_1}$ be a theory extending Δ_1 by enriching the language L with the predicate symbol P as follows:

$$\overline{\Delta_1} = \Delta_1 \cup \Delta_1^f \cup \{P, \subseteq\},$$

where $\{P, \subseteq\}$ is an infinite list of $\exists$-sentences and interpretation of P is an existentially closed submodel in model of Δ_1.

Let $\overline{\Delta_2}$ be a theory that extends Δ_2 when a new constant symbol c is added to the language L and defined as follows:

$$\overline{\Delta_2} = \Delta_2 \cup \Delta_2^f \cup Th_{\forall\exists}(C, c),$$

where C is a semantic model of Jonsson theory Δ_2. Since Δ_2 is a hereditary Jonsson theory, $\overline{\Delta_2}$ is also a Jonsson theory.

Here we pose two questions:

1) How will the addition of new symbols P and c to the language L and the subsequent expansion of Δ_1 and Δ_2 affect the forcing companion of the received theories?

2) When combining the theories $\overline{\Delta_1}$ and $\overline{\Delta_2}$, can a consistent theory be obtained and what will be its forcing companion?

The answer to the first question is the following theorem.

Theorem 7. $\overline{\Delta_1}^f = \Delta_1^f$.

Proof. According to Theorem 2, because Δ_1 is an inductive theory, $\Delta_1 \subseteq \Delta_1^f$. This means that $\Delta_1 \cup \Delta_1^f = \Delta_1^f = \Delta_3$. Therefore, $\overline{\Delta_1}$ can be written as $\Delta_3 \cup \{P, \subseteq\}$. Since the set $\{P, \subseteq\}$ consists only of existential formulas, theories Δ_3 and $\overline{\Delta_1}$ do not differ in universal formulas, which means they are mutually model consistent. As is known from Theorem 1, the forcing companions in this case of these two theories must be equal. At the same time, Δ_3, which is a forcing companion of Δ_1 and Δ_2, is forcing-complete, because $\Delta_3^f = (\Delta_1^f)^f = \Delta_1^f = \Delta_3$. Hence, $\Delta_3^f = \overline{\Delta_1}^f = \Delta_3$, and $\overline{\Delta_1}^f = \Delta_1^f$.

Thus, we can conclude that the forcing companion of the inductive theory Δ_1 does not change when enriching the language of this theory with a new predicate symbol P.

Theorem 8. $\overline{\Delta_2}^f = \Delta_2^f$.

Proof. The proof is similar to the proof of Theorem 7. Since Δ_2 is a Jonsson theory, it is inductive, which means by Theorem 2 $\Delta_2 \subseteq \Delta_2^f$ and $\Delta_2 \cup \Delta_2^f = \Delta_2^f = \Delta_3$. So $\overline{\Delta_2} = \Delta_3 \cup Th_{\forall\exists}(C, c)$. All the sentences in $Th_{\forall\exists}(C, c)$ are $\forall\exists$-formulas, which means that theories Δ_3 and $\overline{\Delta_2}$ do not differ in universal formulas, i.e., they are mutually model consistent. We can conclude from this that their forcing companions are equal, with $\Delta_3^f = \overline{\Delta_2}^f = \Delta_3$, and $\overline{\Delta_2}^f = \Delta_2^f$.

This means that the addition of the new constant c to language L did not affect the forcing companion when expanding theory Δ_2 to $\overline{\Delta_2}$.

To answer the second question, we recall the Robinson's consistency theorem.

Theorem 9. [5; 77] Let T be a complete theory of language L, languages L_1 and L_2 are extensions of language L such that $L_1 \cap L_2 = L$, and theories T_1 and T_2 are consistent extensions of theory T in languages L_1 and L_2 respectively. Then $T_3 = T_1 \cup T_2$ is a consistent theory.

Now we can formulate and prove the following result.

Theorem 10. i) The theory $\overline{\Delta_1} \cup \overline{\Delta_2}$ is consistent.

ii) $(\overline{\Delta_1} \cup \overline{\Delta_2})^f = \Delta_1^f = \Delta_2^f$

Proof. i) As noted above, $\overline{\Delta_1} = \Delta_3 \cup \{P, \subseteq\}$ and $\overline{\Delta_2} = \Delta_3 \cup Th_{\forall\exists}(C, c)$. Applying Theorem 9, we will consider Δ_3 as the theory T, $\overline{\Delta_1}$ as the theory T, acting as an extension of Δ_3 by adding a new predicate symbol P to the language, and T_2 as the theory $\overline{\Delta_2}$, which is an extension of Δ_3 by adding the constant symbol c to the language. In this case, $L = L_1 \cap L_2$, where L_1 is the language of theory $\overline{\Delta_1}$, L_2 is the language of theory $\overline{\Delta_2}$. Therefore, the theory obtained as the union of $\overline{\Delta_1} \cup \overline{\Delta_2}$ is consistent.

ii) Obviously, $\overline{\Delta_1} \cup \overline{\Delta_2} = \Delta_3 \cup \{P, \subseteq\} \cup Th_{\forall\exists}(C, c)$. Theorems 7 and 8 allow us to assert that the forcing companion of theories $\Delta_3 \cup \{P, \subseteq\} \cup Th_{\forall\exists}(C, c)$ and Δ_3 is theory Δ_3. Hence, $(\overline{\Delta_1} \cup \overline{\Delta_2})^f = \Delta_1^f = \Delta_2^f$.

3 Application of the result to differential algebra

The results formulated above, described for the general situation in model theory, can be interpreted using examples of differential algebra, namely, when considering the theory of differential fields of characteristic 0, the theory of differentially closed fields of characteristic 0, the theory of differential fields of characteristic p, the theory of differentially closed fields of characteristic p. First, we will give the basic definitions and theorems concerning these theories. All concepts whose definitions are not given here can be found in [1].

We use the following notation: DF for the theory of differential fields, DPF for the theory of differentially perfect fields, DCF for the theory of differentially closed fields. The lower index 0 or p indicates the corresponding characteristic of the underlying field.

Definition 13. [15; 7] The differentiation of the ring A is called the mapping $a \to D(a)$ rings A into itself satisfying the relations

$$D(x+y) = D(x) + D(y),$$

$$D(xy) = xDy + yDx.$$

Definition 14. [15; 8] A differential ring is a commutative ring with a unit in which some differentiation is given.

In the case where the differential ring is a field F, we will talk about a differential field. Differential fields are models of the theory of differential fields DF, given by the axioms of field theory and the following two sentences:

$$\forall x \forall y \;\; D(x+y) = D(x) + D(y),$$

$$\forall x \forall y \;\; D(xy) = xD(y) + yD(x),$$

where $x, y \in F$.

The language used to study differential fields is the language $L = \{+, -, \cdot, D, 0, 1\}$. Here the differentiation operator D plays the role of a single functional symbol.

The concept of a differentially closed field was first proposed by A. Robinson [16; p. 2]. However, A. Robinson did not formulate axioms for the theory of differentially closed fields, which was corrected later by L. Blum for the case of characteristic 0. The situation with characteristic p was studied in detail by C. Wood and looks similar.

Definition 15. [17; 9] A differential field F is called differentially closed if whenever $f(x), g(x) \in F\{X\}$, $g(x)$ is nontrivial, has a nonzero value and the order of $f(x)$ is greater than the order of $g(x)$, there exists $a \in F$ such that $f(a) = 0$ and $g(a) \neq 0$.

Thus, the theory of differentially closed fields DCF is a theory consisting of the axioms DF and the following two axioms:

1) Each nonconstant polynomial from one variable has a solution.

2) If $f(x)$ and $g(x)$ are differential polynomials such that the order of $f(x)$ is greater than the order of $g(x)$, $g(x)$ is nontrivial, then $f(x)$ has a solution not being the solution of $g(x)$.

The following are some basic facts about the theories of differential fields and differentially closed fields of various characteristics.

Theorem 11. [18; 581] DCF_p is complete and model-complete.

Theorem 12. [19; 131] DF_0 has the joint embedding and amalgam properties.

Theorem 13. [19; 128] The DCF_0 theory is a model completion of the DF_0 theory.

Theorem 14. [18; 578] The theory DF_p of differential fields of characteristic p does not admit the amalgam property.

The author notes that the main reason is the absence of roots of the p-th degree in some constant elements of the field in the general case.

Theorem 15. [20; 92] DF_p has a model companion, but does not have a model completion.

Definition 16. [20; 92] A differential field F is called differentially perfect if any of its extensions is separable.

Theorem 16. [20; 92] In order for the differential field F of characteristic p to be differentially perfect, it is necessary and sufficient that $p = 0$ or $p > 0$ and $F^p = C$.

Thus, the theory DPF differentially perfect fields of characteristic p is given by the axioms DF and the following axiom:

$$\forall x \exists y \ \ (D(x) = 0 \rightarrow y^p = x).$$

Theorem 17. [18; 579] DPF_p is a model consistent extension of DF_p.

Based on this fact, it is easy to see that theories DPF_p and DF_p are mutually model consistent, since each differentially perfect field is a model of theory DF_p and there will always be some model of theory DPF_p, in which any differential field of characteristic p can be embedded.

Theorem 18. [18; 578] The theory DPF_p of differentially perfect fields of characteristic p admits the amalgam property.

Theorem 19. [18; 581] The theory DCF_p of differentially closed fields of characteristic p is the model companion of the theory DF_p differential fields of characteristic p and the model completion for the theory DPF_p of differentially perfect fields of characteristic p.

In work [1], the following statements related to the theories described above were proved.

Theorem 20. [1; 131] DF_0 is a perfect Jonsson theory.

Theorem 21. [1; 131] DCF_0 is the center of the Jonsson theory DF_0.

Theorem 22. [1; 131] DF_p is not a Jonsson theory.

Theorem 23. [1; 132] DPF_p is a perfect Jonsson theory.

Theorem 24. [1; 132] DCF_p is the center of the Jonsson theory DPF_p.

In addition, DF_0 and DPF_p are strongly convex theories in the classical Robinson sense, which allows us to state the following:

Theorem 25. [1; 132] DF_0 and DPF_p are Jonsson AP-theories.

Due to the above facts, we can project the results described in the previous paragraph to the case of differentially closed fields of zero and positive characteristic. However, while in the case of characteristic 0 the results are trivial by virtue of Theorem 20, the situation with differential fields of characteristic p is of greater interest. As the theory Δ_1, we can consider DF_p, which is not Jonsson, as stated in Theorem 22, but inductive (because of universality) and has a model companion according to Theorem 19, which is DCF_p. The role of the theory Δ_2 will be played by the Jonsson AP-theory DPF_p, whose model completion (and, consequently, model companion) is DCF_p. Δ_3 is replaced by

DCF_p, which is the center and the forcing companion of DPF_p. We additionally impose a condition on DCF_p, considering it to be hereditary Jonsson theories with respect to enrichment with a new constant symbol c. Since DCF_p is the center of DPF_p, and also due to the saturation of the semantic model C of DPF_p, the heredity of DCF_p is sufficient for DPF_p to be a hereditary Jonsson theory as well. According to Theorem 17, DF_p and DPF_p are mutually model consistent (which is also clear from the fact that they have a common model companion). We obtain that, by virtue of mutual model consistency, the forcing companions of the theories of differential fields and differentially perfect fields of the characteristics of p are equal and represent DCF_p:

$$DF_p^f = DPF_p^f = DCF_p.$$

Since we are going to add a new predicate symbol P later, it will not affect the mutual model compatibility of these theories in any way, because P does not generate new elements in the models DPF_p and DCF_p. The situation is similar with the new constant c: since the constant can be represented as a single predicate symbol, mutual model compatibility is preserved for the new specified theories.

Finally, by enriching the language of differential field theory with the new predicate symbol and constant, as was done in Section 2, we can obtain the following theories:

$$\overline{DF_p} = DF_p \cup DF_p^f \cup \{P, \subseteq\}, \tag{1}$$

$$\overline{DPF_p} = DPF_p \cup DPF_p^f \cup Th_{\forall\exists}(C, c). \tag{2}$$

Note that the equalities (1) and (2) can be written as

$$\overline{DF_p} = DCF_p \cup \{P, \subseteq\},$$

$$\overline{DPF_p} = DCF_p \cup Th_{\forall\exists}(C, c).$$

Thus, based on the reasoning and conclusions of the previous section, we can draw the following conclusions:

Theorem 26. $\overline{DF_p}^f = DF_p^f$.

Theorem 27. $\overline{DPF_p}^f = DPF_p^f$.

Theorem 28. i) $\overline{DF_p} \cup \overline{DPF_p}$ is consistent.
ii) $(\overline{DF_p} \cup \overline{DPF_p})^f = DF_p^f = DPF_p^f$.

In the future, the authors plan to continue the study of theory $\overline{\Delta_1} \cup \overline{\Delta_2}$ obtained within the framework of constructing the central types in the Jonsson theory and the Jonsson spectrum in the sense of the works [21–23].

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Мақала йонсондық AP-теорияларының форсинг компаньондерін байытылған сигнатурада зерттеуге арналған. Теорияның форсинг-компаньоны тілге жаңа предикаттық және тұрақты символдарын қосу арқылы белгілі бір қасиеттері бар қарастырылып отырған теориялардың кеңеюінде өзгермейтіні дәлелденді. Осы жұмыста жалпы түрде алынған модельді-теоретикалық нәтижелер дифференциалды алгебраның мысалдарымен расталады. Сонымен қатар модельдер теориясының тарихында алғаш рет йонсондық және йонсондық емес теорияларды біріктіруге деген көзқарас көрсетілген. Бұл йонсондық теорияларды зерттеу әдістерін одан әрі дамытуға және толық емес теорияларды зерттеуге арналған аппаратты кеңейтуге мүмкіндік береді.

Кілт сөздер: йонсондық теория, кемел йонсондық теория, AP-теория, форсинг, форсинг-компаньон, сигнатураны байыту, теорияны кеңейту, дифференциалдық өріс, дифференциалды тұйық өріс, дифференциалды кемел өріс.

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Форсинг-компаньоны йонсоновских AP-теорий

Статья посвящена изучению форсинг-компаньонов йонсоновских AP-теорий в обогащённой сигнатуре. Доказано, что форсинг-компаньон теории не меняется при расширении рассматриваемых теорий, обладающих некоторыми свойствами, с помощью добавления в язык новых предикатного и константного символов. Теоретико-модельные результаты, полученные в данной работе в общем виде, подкреплены примерами из дифференциальной алгебры. Авторами статьи впервые в истории теории моделей продемонстрированы подход к комбинированию йонсоновской и нейонсоновской теорий. Это позволит в дальнейшем развить методы исследования йонсоновских теорий и расширить аппарат для изучения неполных теорий.

Ключевые слова: йонсоновская теория, совершенная йонсоновская теория, AP-теория, форсинг, форсинг-компаньон, обогащение сигнатуры, расширение теории, дифференциальное поле, дифференциально замкнутое поле, дифференциально совершенное поле.

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