DOI 10.31489/2022M3/152-162
UDC 531.31, 519.21

M.I. Tleubergenov[1,2], G.K. Vassilina[1,3,*], S.R. Seisenbayeva[1,2]

[1] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan;*
[2] *Al-Farabi Kazakh National University, Almaty, Kazakhstan;*
[3] *Almaty University of Power Engineering and Telecommunications named after G.Daukeev, Almaty, Kazakhstan*
(E-mail: marat207@mail.ru, v_gulmira@mail.ru, saramatal@mail.ru)

Construction of stochastic differential equations of motion in canonical variables

Galiullin proposed a classification of inverse problems of dynamics for the class of ordinary differential equations (ODE). Considered problem belongs to the first type of inverse problems of dynamics (of the three main types of inverse problems of dynamics): the main inverse problem under the additional assumption of the presence of random perturbations. In this paper Hamilton and Birkhoff equations are constructed according to the given properties of motion in the presence of random perturbations from the class of processes with independent increments. The obtained necessary and sufficient conditions for the solvability of the problem of constructing stochastic differential equations of both Hamiltonian and Birkhoffian structure by the given properties of motion are illustrated by the example of the motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces.

Keywords: stochastic differential equation, class of processes with independent increments, stochastic equations of Hamiltonian and Birkhoffian structures, the main inverse problem.

Introduction

At present, the theory of inverse problems of dynamics in the class of ODEs is fully developed [1–9] and goes back to the Yerugin's fundamental work [10]. In [10], there is constructed a set of ODE that have a given integral curve.

Methods for solving inverse problems of dynamics are generalized to the class of Ito stochastic differential equations in [11–19].

Let the set

$$\Lambda(t) : \lambda(x, \dot{x}, t) = 0, \quad \lambda \in R^m, \ \ x \in R^n, \tag{1}$$

be given. It is required to construct a set of stochastic equations of Hamiltonian and Birkhoffian structure

$$\begin{cases} \dot{q}_k = \dfrac{\partial H}{\partial p_k}, \\ \dot{p}_k = -\dfrac{\partial H}{\partial q_k} + \sigma'_{kj}(q,p,t)\dot{\xi}^j, \quad \left(k = \overline{1,n}\right); \end{cases} \tag{2}$$

$$\left[\frac{\partial R_i(z,t)}{\partial z_l} - \frac{\partial R_l(z,t)}{\partial z_i}\right]\dot{z}_i - \left[\frac{\partial B(z,t)}{\partial z_l} + \frac{\partial R_l(z,t)}{\partial t}\right] = T_{l\mu}\dot{\psi}_\mu, \ \ (i,l = \overline{1,2n}, \ \mu = \overline{1,n+r}) \tag{3}$$

so that the set (1) is an integral manifold of the constructed stochastic equations of the Hamiltonian (2) and Birkhoffian structure (3).

Here $\{\xi_1(t,\omega), \dots, \xi_k(t,\omega)\}$ and $\{\psi_1(t,\omega), ..., \psi_{n+r}(t,\omega)\}$ are systems of random processes with independent increments that can be represented as a sum of Wiener and Poisson processes [20]:

*Corresponding author.
E-mail: v_gulmira@mail.ru

1) $\xi = \xi_0 + \int c(y)P^0(t, dy)$, where ξ_0 is a Wiener process, P^0 is a Poisson process, $P^0(t, dy)$ is the number of the jumps of P^0 in the interval $[0, t]$ that fall onto the set dy, $c(y)$ is a vector function mapping the space R^{2n} into the space R^r of the values of the process $\xi(t)$ for all t;

2) $\psi = \psi_0 + \int \tilde{c}(y)\tilde{P}^0(t, d\gamma)$, ψ_0 is a Wiener process, $\tilde{P}^0$ is a Poisson process, $\tilde{P}^0(t, d\gamma)$ is the number of the jumps of $\tilde{P}^0$ in the interval $[0, t]$ that fall onto the set $d\gamma$, $\tilde{c}(y)$ $c(y)$ is a vector function mapping the space R^{2n} into the space R^{n+r} of the values of the process $\psi(t)$ for all t; $B = B(z, t)$ is a Birkhoff function, $W = (W_{il})$ Birkhoff tensor with components $W_{il} = \left[\frac{\partial R_i(z,t)}{\partial z_l} - \frac{\partial R_l(z,t)}{\partial z_i}\right]$.

The stated problem was solved for the class of ODEs in [21]. In particular, the stochastic Helmholtz problem, i.e., the problem of constructing equivalent stochastic equations of the Lagrangian, Hamiltonian, and Birkhoffian structures by a given second order stochastic Ito equation was considered in [22]. In [23, 24], the above problem of constructing stochastic equations of the form (2) and (3) by a given integral set (1) is considered under the assumption that systems $\{\xi_1(t, \omega), ..., \xi_r(t, \omega)\}$ and $\{\psi_1(t, \omega), ..., \psi_{n+r}(t, \omega)\}$ are systems of independent Wiener processes (as a special case of random processes with independent increments).

Let us give the scheme of solving the set problems: at the first step by the quasi-inversion method [3] in combination with Yerugin's method [10] and by virtue of stochastic differentiation of the complex function in the case of processes with independent increments [20] by the given set (1) the second order Ito differential equation

$$\ddot{x} = f(x, \dot{x}, t) + \sigma(x, \dot{x}, t)\dot{\xi} \tag{4}$$

is constructed so that the set $\Lambda(t)$ is an integral manifold of the constructed equation (4). Further, at the second step, equivalent stochastic equations of Hamiltonian and Birkhoffian structures are constructed by the constructed stochastic equation (4).

1 Construction of stochastic Hamiltonian equation (2) by the given properties of motion (1)

Previously, by virtue of the Ito formula for stochastic differentiation of a complex function, the equation of perturbed motion

$$\dot{\lambda} = \frac{\partial \lambda}{\partial t} + \frac{\partial \lambda}{\partial x}\dot{x} + \frac{\partial \lambda}{\partial \dot{x}}f + S_1 + S_2 + S_3 + \frac{\partial \lambda}{\partial \dot{x}}\sigma\dot{\xi}, \tag{5}$$

is compiled. Here $S_1 = \frac{1}{2}\frac{\partial^2 \lambda}{\partial \dot{x}^2} : \sigma\sigma^T$; following [20], $\frac{\partial^2 \lambda}{\partial \dot{x}^2} : D$, $D = \sigma\sigma^T$ is understood as a vector, the elements of which are the traces of the products of matrices of the second derivatives of the corresponding elements $\lambda_\mu(x, \dot{x}, t)$ of the vector $\lambda(x, \dot{x}, t)$ with respect to the components $\dot{x}$ and the matrix D

$$\frac{\partial^2 \lambda}{\partial \dot{x}^2} : D = \left[tr\left(\frac{\partial^2 \lambda_1}{\partial \dot{x}^2}D\right), \ \ldots \ , tr\left(\frac{\partial^2 \lambda_m}{\partial \dot{x}^2}D\right)\right]^T;$$

$$S_2 = \int \left\{\lambda(x, \dot{x} + \sigma c(y), t) - \lambda(x, \dot{x}, t) + \frac{\partial \lambda}{\partial \dot{x}}\sigma c(y)\right\} dy;$$

$$S_3 = \int [\lambda(x, \dot{x} + \sigma c(y), t) - \lambda(x, \dot{x}, t)]P^0(t, dy).$$

Further, in order for the set (1) to be an integral manifold of equation (4), we introduce arbitrary Yerugin functions [10]: a vector function $A = A(\lambda, x, \dot{x}, t)$ and a matrix $B = B(\lambda, x, \dot{x}, t)$ with properties $A(0, x, \dot{x}, t) \equiv 0$, $B(0, x, \dot{x}, t) \equiv 0$, and such that

$$\dot{\lambda} = A(\lambda, x, \dot{x}, t) + B(\lambda, x, \dot{x}, t)\dot{\xi}. \tag{6}$$

Equations (5) and (6) imply the following equalities

$$\begin{cases} \dfrac{\partial\lambda}{\partial\dot{x}} f = A - \frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3, \\ \dfrac{\partial\lambda}{\partial\dot{x}} \sigma = B. \end{cases} \tag{7}$$

To determine the desired functions f and σ from relations (7), we use the following statement:
Lemma 1 [3; 12–13]. The set of all solutions of the linear system

$$Hv = g, H = (h_{\mu k}), v = (v_k), g = (g_\mu), \mu = \overline{1,m}, k = \overline{1,n}, m \le n, \tag{8}$$

is determined by the expression

$$v = \alpha v^T + v^\upsilon, \tag{9}$$

where the rank of the matrix H equals to m. Here α is a scalar value,

$$v^T = [HC] = [h_1 ... h_m c_{m+1} ... c_{n-1}] = \begin{vmatrix} e_1 & ... & e_n \\ h_{11} & ... & h_{1n} \\ ... & ... & ... \\ h_{m1} & ... & h_{mn} \\ c_{m+1,1} & ... & c_{m+1,n} \\ ... & ... & ... \\ c_{n-1,1} & ... & c_{n-1,n} \end{vmatrix}$$

is the cross product of vectors $h_\mu = (h_{\mu k})$ and arbitrary vectors $c_\rho = (c_{\rho k}), \rho = \overline{m+1, n-1}$; e_k are unit vectors of space R^n, $v^T = (v_k^T)$

$$v_k^T = \begin{vmatrix} 0 & ... & 1 & ... & 0 \\ h_{11} & ... & h_{1k} & ... & h_{1n} \\ ... & ... & ... & ... & ... \\ h_{m1} & ... & h_{mk} & ... & h_{mn} \\ c_{m+1,1} & ... & c_{m+1,n} & ... & c_{m+1,n} \\ ... & ... & ... & ... & ... \\ c_{n-1,1} & ... & c_{n-1,k} & ... & c_{n-1,n} \end{vmatrix}, \quad v^\upsilon = H^+ g,$$

where $H^+ = H^T(HH^T)^{-1}, H^T$ is the matrix transposed to H.

By Lemma 1, using (8), (9) we determine the form of the vector function f and the columns σ_i of the matrix σ

$$f = s_1 \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ \left(A - \frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3\right), \tag{10}$$

$$\sigma_i = s_{2i} \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ B_i, \left(i = \overline{1,r}\right). \tag{11}$$

Here $\sigma_i = (\sigma_{1i}, \sigma_{2i}, \ldots, \sigma_{ni})^T$ denotes the i-th column of the matrix $\sigma = (\sigma_{\nu j})$, $\left(\nu = \overline{1,n},\ \ j = \overline{1,r}\right)$. $B_i = (B_{1i}, B_{2i}, \ldots, B_{mi})^T$ is the i-th column of the matrix $B = (B_{\mu j})$, $\left(\mu = \overline{1,m},\ \ j = \overline{1,r}\right)$. By s_1, s_2 are denoted arbitrary scalar quantities.

The forms of the vector function f (10) and matrix σ (11) imply the general form of the set of second-order Ito differential equations (4) with a given integral manifold (1)

$$\ddot{x} = s_1 \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ \left(A - \frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3\right) +$$

$$+\left(s_{21}\left[\frac{\partial\lambda}{\partial\dot{x}}C\right]+\left(\frac{\partial\lambda}{\partial\dot{x}}\right)^{+}B_1,\ \ldots\ ,\ s_{2r}\left[\frac{\partial\lambda}{\partial\dot{x}}C\right]+\left(\frac{\partial\lambda}{\partial\dot{x}}\right)^{+}B_r\right)\dot{\xi}.$$

To construct the Hamilton function, we first introduce a new variable $y_k = \dot{x}_k$ and rewrite the constructed equation (4) in the form

$$\begin{cases} \dot{x}_k = y_k, \\ y_k = f_k\left(x,y,t\right)+\sigma_{kj}\left(x,y,t\right)\dot{\xi}^j. \end{cases} \tag{12}$$

Here the vector function $f=(f_1,f_2,\ldots,f_n)^T$ and matrix columns $\sigma=(\sigma_1,\sigma_2,\ldots,\sigma_r)$ have the form (10), (11), respectively.

Then, using $z_k=\begin{cases} x_k \\ y_k \end{cases}$, $G_k=\begin{cases} y_k \\ f_k \end{cases}$, $\psi_j=\begin{cases} 0, \text{ for } j=\overline{1,n}\ , \\ \xi^{j-n}, \text{ for } j=n+1,n+2,\ldots,n+m\,, \end{cases}$ $\mu=(\mu_{kj})=\begin{pmatrix} 0_{n\times n} & 0_{n\times m} \\ 0_{n\times n} & \sigma_{n\times m} \end{pmatrix}$, $\sigma=(\sigma_{kj})$ we rewrite equation (12) in the form

$$\dot{z}_k=G_k\left(z,t\right)+\mu_{kj}\left(z,t\right)\dot{\psi}_j. \tag{13}$$

Further, using $\nu_k=\begin{cases} q_k, & k=\overline{1,n} \\ p_{k-n}, & k=n+1,n+2,\ldots 2n \end{cases}$ and $\varphi=(\varphi_{k\nu})=\begin{pmatrix} 0_{n\times n} & I_{n\times n} \\ -I_{n\times n} & 0_{n\times n} \end{pmatrix}$, $p=(p_{kj})=\begin{pmatrix} 0_{n\times n} & 0_{n\times m} \\ 0_{n\times n} & \sigma'_{n\times m} \end{pmatrix}$, and also taking into account $\begin{pmatrix} \frac{\partial H}{\partial p_k} \\ -\frac{\partial H}{\partial q_k} \end{pmatrix}=\left(\varphi_{k\nu}\frac{\partial H}{\partial \nu_\nu}\right)$, we rewrite the stochastic equation of the Hamiltonian structure (2) in the form

$$\dot{\nu}_k-\phi_{k\nu}\frac{\partial H}{\partial\nu_\nu}=p_{kj}\dot{\psi}_j. \tag{14}$$

If we introduce the inverse matrix $(\omega_{k\nu})=(\varphi_{k\nu})^{-1}=\begin{pmatrix} 0_{n\times n} & -I_{n\times n} \\ I_{n\times n} & 0_{n\times n} \end{pmatrix}$ for $(\varphi_{k\nu})$ and vector $z_k\equiv \omega_{k\nu}\nu_\nu=\begin{pmatrix} -p_k, & k=\overline{1,n} \\ q_{k-n}, & k=\overline{n+1,2n} \end{pmatrix}$, then equation (14) is transformed to the equivalent equation

$$\omega_{\nu k}\dot{z}_k-\frac{\partial H}{\partial z_k}=\omega_{\nu k}p_{\nu j}\dot{\psi}_j. \tag{15}$$

Consider the problem of indirect representation of equation (13) in the form of an equation of the Hamiltonian structure (15), i.e., using some matrix $\Gamma=\left(\gamma_\nu^k\right)$, consider the relation

$$\gamma_\nu^k\left(\dot{z}_k-G_k-\mu_{kj}\dot{\psi}_j\right)\equiv\omega_{\nu k}\dot{z}_k-\frac{\partial H}{\partial z_\nu}-\omega_{\nu k}p_{\nu j}\dot{\psi}_j$$

or

$$C_{\nu k}\dot{z}_k-D_\nu(z,t)-\gamma_\nu^k\mu_{kj}\dot{\psi}_j\equiv\omega_{\nu k}\dot{z}_k-\frac{\partial H}{\partial z_\nu}-\omega_{\nu k}p_{\nu j}\dot{\psi}_j\,, \tag{16}$$

where $C_{\nu k}=\gamma_\nu^k;\ D_\nu(z,t)=\gamma_\nu^k G_k$.

To fulfill the identity (16) it is necessary the conditions

$$C_{\nu k}=\omega_{\nu k},\ \ D_\nu(z,t)=-\frac{\partial H}{\partial z_\nu}\,, \tag{17}$$

$$\gamma_\nu^k\mu_{kj}=\omega_{\nu k}p_{\nu j},\ \ \left(\nu,\ k=\overline{1,2n},\ j=\overline{1,n+m}\right), \tag{18}$$

$$\gamma_{\nu}^{k} = \omega_{\nu k} \tag{19}$$

to be satisfied. From relations (16) and conditions (17)–(19), $\mu_{kj} = p_{\nu j}$ follows. This entails the fulfillment of the equality $\sigma_{kj} = \sigma'_{kj}$, $\left(k = \overline{1,n},\ j = \overline{1,m}\right)$.

Theorem 1. For the indirect construction of Hamiltonian structure stochastic equation (2) by the given set (1) so that the set (1) is an integral manifold of equation (15), it is necessary and sufficient that conditions (17)–(19) be satisfied.

2 Construction of the Birkhoffian structure stochastic equation (3) by the given properties of motion (1)

To solve the problem, consider the relation

$$C_{\upsilon k}\dot{z}_k - D_{\upsilon}(z,t) - \mu_{\upsilon j}\dot{\psi}_j \equiv$$

$$\equiv \left[\frac{\partial R_k(z,t)}{\partial z_\nu} - \frac{\partial R_\nu(z,t)}{\partial z_\kappa}\right]\dot{z}_\kappa - \left[\frac{\partial B(z,t)}{\partial z_\nu} + \frac{\partial R_\nu(z,t)}{\partial t}\right] - T_{\nu j}\dot{\psi}_j,\ \ (\nu,\kappa = \overline{1,2n},\ j = \overline{1,n+m}). \tag{20}$$

(20) is fulfilled identically under the following conditions

$$C_{\nu k} = \left[\frac{\partial R_\kappa(z,t)}{\partial z_\upsilon} - \frac{\partial R_\upsilon(z,t)}{\partial z_\kappa}\right],\ \ D_\nu = \left[\frac{\partial B(z,t)}{\partial z_\upsilon} + \frac{\partial R_\upsilon(z,t)}{\partial t}\right],\ \ \mu = T. \tag{21}$$

Theorem 2. To construct the Birkhoffian structure stochastic equation (3) by the given set (1), so that set (1) is an integral manifold of equation (3), it is necessary and sufficient that conditions (21) are satisfied.

3 Example

Consider the stochastic problem of constructing Hamilton and Birkhoff functions for a given property of motion by the example of the motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces [25].

Let the properties of motion

$$\Delta\,(t) : \lambda = \theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2 = 0, \qquad \lambda \in R^1 \tag{22}$$

be given. Then the equation of perturbed motion (5) takes the form

$$\dot{\lambda} = 2\theta\dot{\theta} + 2\alpha_1\dot{\theta}\ddot{\theta} + S_1 + S_2 + S_3 = 2\theta\dot{\theta} + 2\alpha_1\dot{\theta}f + S_1 + S_2 + S_3 + 2\alpha_1\dot{\theta}\sigma\dot{\xi}, \tag{23}$$

where $S_1 = \alpha_1\sigma^2$, $\quad S_2 = \int\left\{2\alpha_1\sigma c(y)[4\dot{\theta} + \sigma c(y)]\right\}dy$, $\quad S_3 = \int\left\{2\alpha_1\sigma c(y)[4\dot{\theta} + \sigma c(y)]\right\}P^0(t,dy)$.

Let us introduce the scalar Yerugin functions $a = a(\lambda,\theta,\dot{\theta},t)$, $b = b(\lambda,\theta,\dot{\theta},t)$ with the property $a(0,\theta,\dot{\theta},t) \equiv b(0,\theta,\dot{\theta},t) \equiv 0$ and such that the relation

$$\dot{\lambda} = a\lambda(\theta,\dot{\theta},t) + b\lambda(\theta,\dot{\theta},t)\dot{\xi} \tag{24}$$

takes place. In our example from relations (23), (24), it follows that a set of equations (4) is written as $\ddot{\theta} = f(\theta,\dot{\theta},t) + \sigma(\theta,\dot{\theta},t)\dot{\xi}$ and it has the integral manifold (22) if f and σ have, respectively, the forms

$$f = \frac{a(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) - 2\theta\dot{\theta} - S_1 - S_2 - S_3}{2\alpha_1\dot{\theta}}, \quad \sigma = \frac{b(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2)}{2\alpha_1\dot{\theta}}. \tag{25}$$

Following [25], we write the equation of motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces in the form

$$\ddot{\theta} = \tilde{f}(\theta, \dot{\theta}) + \tilde{\sigma}(\theta, \dot{\theta})\dot{\xi}, \tag{26}$$

where θ is pitch angle, $\tilde{f}$and $\tilde{\sigma}$ have the forms

$$\tilde{f} = Ql\sin 2\theta - Q[g(\theta) + \eta\dot{\theta}], \quad \tilde{\sigma} = Q\delta[g(\theta) + \eta\dot{\theta}].$$

Before constructing the Hamilton and Birkhoff functions, we first construct the Lagrangian by (26). In (26), relations (25) should be taken into account, which ensure the integrality of the given set (22). From $f = \tilde{f}$, $\sigma = \tilde{\sigma}$, it follows that the four parameters Q, δ, η, l, determining the dynamics of satellite motion, must satisfy the following relations

$$\begin{cases} a(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) - 2\theta\dot{\theta} - S_1 - S_2 - S_3 = 2\alpha_1\dot{\theta}\left\{Ql\sin 2\theta - Q[g(\theta) + \eta\dot{\theta}]\right\}, \\ b(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) = 2\alpha_1\dot{\theta}Q\delta[g(\theta) + \eta\dot{\theta}]. \end{cases}$$

Then, considering definition [26] and the action of random perturbations, equation (26) admits an indirect analytic representation in terms of a stochastic equation with a Lagrangian structure if there exists a function h such that the identity

$$d\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} - \sigma'(\theta, \dot{\theta}, t)\dot{\xi} \equiv h[\ddot{\theta} - f - \sigma\dot{\xi}] \tag{27}$$

takes place. We find the function $h = h(t)$ from the Helmholtz condition [26; 107] $\dfrac{\partial l_2}{\partial\dot{\theta}} = \dfrac{\partial l_1}{\partial t} + \dot{\theta}\dfrac{\partial l_1}{\partial\theta}$, which is necessary and sufficient for constructing the Lagrange equation equivalent to the scalar equation $l_1(\theta, \dot{\theta}, t)\ddot{\theta} + l_2(\theta, \dot{\theta}, t) = 0$. In particular, function $h = e^{-Q\eta t}$ satisfies this condition. Substituting h in (27), we get

$$e^{-Q\eta t}[\ddot{\theta} - f - \sigma\dot{\xi}] = \frac{\partial^2 L}{\partial\dot{\theta}^2}\ddot{\theta} + \frac{\partial^2 L}{\partial\dot{\theta}\partial\theta}\dot{\theta} + \frac{\partial^2 L}{\partial\theta\partial t} - \frac{\partial L}{\partial t}\sigma'\dot{\xi}.$$

Then we construct the desired Lagrangian in the form

$$L = e^{-Q\eta t}[\frac{1}{2}\dot{\theta}^2 - Q(\frac{1}{2}l\cos 2\theta + G)], \quad G = \int g(\theta)d\theta, \tag{28}$$

which provides an indirect representation of equation (27) in the form of the Lagrangian structure equation

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = e^{-Q\eta t}\sigma(\theta, \dot{\theta})\dot{\xi}.$$

Using the Lagrange function (28) and the Legendre transform, we define the Hamilton function as $H = \chi\dot{\theta} - L(\theta, \dot{\theta}, t)\Big|_{\dot{\theta}=\dot{\theta}(\theta,\chi,t)}$. Since $\chi = \dfrac{\partial L}{\partial\dot{\theta}}$, then $\chi = e^{-Q\eta t}\dot{\theta}$ and therefore $\dot{\theta} = e^{Q\eta t}\chi$. Then the canonical equation corresponding to the stochastic Lagrangian structure equation (27) will take the form

$$\begin{cases} \dot{\theta} = \dfrac{\partial H}{\partial\chi}, \\ \dot{\chi} = -\dfrac{\partial H}{\partial\theta} + \widehat{\sigma}\,(\theta, \chi, t)\dot{\xi}, \end{cases} \tag{29}$$

where $\widehat{\sigma} = \sigma'(\theta, \dot{\theta}, t)\Big|_{\dot{\theta}=\dot{\theta}(\theta,\chi,t)}$, and the Hamilton function is defined as

$$H = \frac{1}{2} e^{Q\eta t} \chi^2 e^{-Q\eta t} b(\theta). \tag{30}$$

To solve the problem of representing the Birkhoffian by a given equation (26), we use Theorem 2. According to the above constructed equation (29) and Hamilton function (30) from relations (21) for $C = \begin{pmatrix} \varphi & 0 \\ 0 & \varphi \end{pmatrix}$ functions $R_\upsilon(\upsilon = 1, 2)$, $R = (R_1, R_2)$ and B are defined as $R = \{\chi, (1 + \varphi)\theta\}, B = \frac{1}{2}\varphi e^{Q\eta t}\chi^2 - \varphi e^{-Q\eta t} b(\theta)$, where φ is an arbitrary constant.

Acknowledgments

This research was funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP 09258966).

References

1 Галиуллин А.С. Методы решения обратных задач динамики / А.С. Галиуллин. — М.: РУДН, 1986. — 224 с.

2 Галиуллин А.С. Избранные труды: [В 2-х т.]. — Т. II / А.С. Галиуллин. — М.: РУДН, 2009. — 462 с.

3 Мухаметзянов И.А. Уравнения программных движений / И.А. Мухаметзянов, Р.Г. Мухарлямов. — М.: РУДН, 1986. — 88 с.

4 Mukharlyamov R.G. Differential-algebraic equations of programmed motions of Lagrangian dynamical systems / R.G. Mukharlyamov // Mechanics of Solids. — 2011. — 46. — No. 4. — P. 534–543.

5 Mukharlyamov R.G. Control of system dynamics and constrains stabilization / R.G. Mukharlyamov, M.I. Tleubergenov // Communications in Computer and Information Science. — 2017. — No. 700. — P. 431–442. DOI:10.1007/978-3-319-66836-936.

6 Llibre J. Inverse Problems in Ordinary Differential Equations and Applications / J. Llibre, R. Ramirez. — Switzerland: Springer International Publishing, 2016. — 266 p.

7 Zhumatov S.S. Asymptotic Stability of Implicit Differential Systems in the Vicinity of Program Manifold / S.S. Zhumatov // Ukrainian Mathematical Journal. — 2014. — 66. — No. 4. — P. 625–632. DOI: 10.1007/s11253-014-0959-y.

8 Zhumatov S.S. Exponential Stability of a Program Manifold of Indirect Control Systems / S.S. Zhumatov // Ukrainian Mathematical Journal. — 2010. — 62. — No. 6. — P. 907–915. DOI: 10.1007/s11253-010-0399-2.

9 Zhumatov S.S. Stability of a Program Manifold of Control Systems with Locally Quadratic Relations / S.S. Zhumatov // Ukrainian Mathematical Journal. — 2009. — 61. No. 3. — P. 500–509. DOI: 10.1007/s11253-009-0224-y.

10 Еругин Н.П. Построение всего множества систем дифференциальных уравнений, имеющих заданную интегральную кривую / Н.П. Еругин // Прикладная математика и механика. — 1952. — 10. — Вып. 6. — С. 659–670.

11 Ibraeva G.T. Main Inverse Problem for Differential Systems With Degenerate Diffusion / G.T. Ibraeva, M.I. Tleubergenov // Ukrainian Mathematical Journal. — 2013. — 65. — No. 5. — P. 787–792.

12 Tleubergenov M.I. On the inverse stochastic reconstruction problem / M.I. Tleubergenov // Differential equations. — 2014. — 50. – Issue 2. — P. 274–278. DOI:10.1134/S0012266114020165.

13 Tleubergenov M.I. Stochastic Inverse Problem with Indirect Control / M.I. Tleubergenov, G.T. Ibraeva // Differential equations. — 2017. — 53. — Issue 10. — P. 1387–1391. DOI:10.1134 /S0012266117100172.

14 Vassilina G.K. Solution of the Problem of Stochastic Stability of an Integral Manifold by the Second Lyapunov Method / G.K. Vassilina, M.I. Tleubergenov // Ukrainian Mathematical Journal. – 2016. — 68. — No. 1. – P. 14–28. DOI:10.1007/s11253-016-1205-6.

15 Tleubergenov M.I. On the Solvability of the Main Inverse Problem for Stochastic Differential Systems / M.I. Tleubergenov, G.T. Ibraeva // Ukrainian Mathematical Journal. — 2019. — 71. — No. 1. — P. 157–165. DOI:10.1007/s11253-019-01631-w.

16 Tleubergenov M.I. On the Closure of Stochastic Differential Equations of Motion / M.I. Tleubergenov, G.T. Ibraeva // Eurasian Mathematical Journal. — 2021. — 12. — No. 2. — P. 82–89.

17 Tleubergenov M.I. An Inverse Problem for Stochastic Differential Systems / M.I. Tleubergenov // Differential equations. — 2001. — 37. — Issue 5. — P. 751—753. https://doi.org/10.1023/A:1019 285119532.

18 Tleubergenov M.I. On Stochastic Inverse Problem of Construction of Stable Program Motion / M.I. Tleubergenov, G.K. Vassilina // Open Mathematics. — 2021. — 19. — P. 157–162. https://doi.org/10.1515/math-2021-0005.

19 Tleubergenov M.I. On Construction of a Field of Forces along Given Trajectories in the Presence of Random Perturbations / M.I. Tleubergenov, G.K. Vassilina, G.A. Tuzelbaeva // Bulletin of Karaganda University. Mathematics series. — 2021. — No. 1(101). — P. 98–103. DOI 10.31489/ 2021M1/98-10.

20 Пугачев В.С. Стохастические дифференциальные системы. Анализ и фильтрация / В.С. Пугачев, И.Н. Синицын. — М.: Наука, 1990. — 632 с.

21 Туладхар Б.М. Построение уравнений в форме Лагранжа, Гамильтона и Биркгофа по заданным свойствам движения: автореф. ... канд. физ.-мат. наук / Б.М. Туладхар. — М.: РУДН им. П. Лумумбы, 1983.

22 Тлеубергенов М.И. Обратные задачи стохастических дифференциальных систем: автореф. ... канд. физ.-мат. наук / М.И. Тлеубергенов. — Алматы: Институт математики, 1999.

23 Тлеубергенов М.И. О построении дифференциального уравнения по заданным свойствам движения при наличии случайных возмущений / М.И. Тлеубергенов, Д.Т. Ажымбаев // Изв. НАН РК. Сер. физ.-мат. — 2007. — № 3. — С. 15–20.

24 Tleubergenov M.I. On the construction of a set of stochastic differential equations on the basis of a given integral manifold independent of velocities / M.I. Tleubergenov, D.T. Azhymbaev // Ukrainian Mathematical Journal. — 2010. — 62. – No. 7. — P. 1163–1173. DOI:10.1007/s11253-010-0421-8

25 Сагиров П. Стохастические методы в динамике спутников / П. Сагиров // Механика. Период. сб. переводов ин. ст. — 1974. — № 5 (147). — С. 28–47.

26 Santilli R.M. Foundations of Theoretical Mechanics. 1. The Inverse Problem in Newtonian Mechanics / R.M. Santilli. — New-York: Springer-Verlag, 1978. — 266 p.

М.Ы. Тілеубергенов[1,2], Г.Қ. Василина[1,3], С.Р. Сейсенбаева[1,2]

[1] *Математика және математикалық модельдеу институты, Алматы, Қазақстан;*
[2] *Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан;*
[3] *Ғ. Дәукеев атындағы Алматы энергетика және байланыс университеті, Алматы, Қазақстан*

Канондық айнымалылардағы қозғалыстың стохастикалық дифференциалдық теңдеулерін құру

А.С. Галиуллин динамиканың кері есептерінің классификациясын қарапайым дифференциалдық теңдеулер класында ұсынды. Мақалада қарастырылатын есеп динамиканың кері есептерінің бірінші түріне жатады (динамиканың кері есептерінің негізгі үш түрінің ішінде) кездейсоқ түрткілердің бар болуы туралы қосымша болжамдағы негізгі кері есепке. Сонымен бірге Гамильтон және Бирхофф теңдеулері тәуелсіз өсушелері бар процестер класынан кездейсоқ түрткілер бар болған кезде қозғалыстың берілген қасиеттерінен құрастырылған. Ал алынған қозғалыстың берілген қасиеттері үшін Гамильтондық та, Бирхоффтық та құрылымды стохастикалық теңдеулерін құру есебінің шешімін табу үшін алынған қажетті және жеткілікті шарттары ауырлық күштерінің және аэродинамикалық күштерінің әсерінен Жердің жасанды серігінің қозғалысы мысалында көрсетілген.

Кілт сөздер: стохастикалық дифференциалдық теңдеу, тәуелсіз өсімшелі үдерістер класы, Гамильтондық және Бирхоффтық құрылымды стохастикалық теңдеулер, негізгі кері есеп.

М.И. Тлеубергенов[1,2], Г.К. Василина[1,3], С.Р. Сейсенбаева[1,2]

[1] *Институт математики и математического моделирования, Алматы, Казахстан;*
[2] *Казахский национальный университет имени Аль-Фараби, Алматы, Казахстан;*
[3] *Алматинский университет энергетики и связи имени Г. Даукеева, Алматы, Казахстан*

Построение стохастических дифференциальных уравнений движения в канонических переменных

А.С. Галиуллиным была предложена классификация обратных задач динамики в классе обыкновенных дифференциальных уравнений. И рассматриваемая в настоящей работе задача относится к первому типу обратных задач динамики (из трех основных типов обратных задач динамики) — основной обратной задаче при дополнительном предположении о наличии случайных возмущений. В статье строятся уравнения Гамильтона и Биркгофа по заданным свойствам движения при наличии случайных возмущений из класса процессов с независимыми приращениями. И полученные необходимые и достаточные условия разрешимости задачи построения стохастических уравнений как гамильтоновой, так и биркгофиановой структуры по заданным свойствам движения проиллюстрированы на примере движения искусственного спутника Земли под действием сил тяготения и аэродинамических сил.

Ключевые слова: стохастическое дифференциальное уравнение, класс процессов с независимыми приращениями, стохастическое уравнения гамильтоновой и биркгофиановой структур, основная обратная задача.

References

1 Galiullin, A.S. (1986). *Metody resheniia obratnykh zadach dinamiki [Methods for solving inverse problems of dynamics]*. Moscow: RUDN [in Russian].

2 Galiullin, A.S. (2009). *Izbrannye trudy [Selected works]*. (Vols. 1-2; Vol. II). Moscow: RUDN [in Russian].

3 Mukhametzyanov, I.A., & Mukharlyamov, R.G. (1986). *Uravneniia programmnykh dvizhenii [Equations of program motions]*. Moscow: RUDN [in Russian].

4 Mukharlyamov, R.G. (2011). Differential-algebraic equations of programmed motions of Lagrangian dynamical systems. *Mechanics of Solids, 46*(4), 534–543.

5 Mukharlyamov, R.G., & Tleubergenov, M.I. (2017). Control of system dynamics and constrains stabilization. *Communications in Computer and Information Science, 700*, 431–442. DOI:10.1007/978-3-319-66836-936.

6 Llibre, J., & Ramizer, R. (2016). *Inverse Problems in Ordinary Differential Equations and Applications*. Switzerland: Springer International Publishing.

7 Zhumatov, S.S. (2014). Asymptotic Stability of Implicit Differential Systems in the Vicinity of Program Manifold. *Ukrainian Mathematical Journal, 66*(4), 625–632. DOI: 10.1007/s11253-014-0959-y.

8 Zhumatov, S.S. (2010). Exponential Stability of a Program Manifold of Indirect Control Systems. *Ukrainian Mathematical Journal, 62*(6), 907–915. DOI: 10.1007/s11253-010-0399-2.

9 Zhumatov, S.S. (2009). Stability of a Program Manifold of Control Systems with Locally Quadratic Relations. *Ukrainian Mathematical Journal, 61*(3), 500–509. DOI: 10.1007/s11253-009-0224-y.

10 Yerugin, N.P. (1952). Postroenie vsego mnozhestva sistem differentsialnykh uravnenii, imeiushchikh zadannuiu integralnuiu krivuiu [Construction of the entire set of systems of differential equations with a given integral curve]. *Prikladnaia matematika i mekhanika — Applied Mathematics and Mechanics, 10*(6), 659–670 [in Russian].

11 Ibraeva, G.T., & Tleubergenov, M.I. (2013). Main Inverse Problem for Differential Systems With Degenerate Diffusion. *Ukrainian Mathematical Journal, 65*(5), 787–792.

12 Tleubergenov, M.I. (2014). On the inverse stochastic reconstruction problem. *Differential equations, 50*(2), 274–278. DOI:10.1134/S0012266114020165.

13 Tleubergenov, M.I., & Ibraeva, G.T. (2017). Stochastic Inverse Problem with Indirect Control. *Differential equations, 53*(10), 1387–1391. DOI:10.1134/S0012266117100172.

14 Vassilina, G.K., & Tleubergenov, M.I. (2016). Solution of the Problem of Stochastic Stability of an Integral Manifold by the Second Lyapunov Method. *Ukrainian Mathematical Journal, 68*(1), 14–28. DOI:10.1007/s11253-016-1205-6.

15 Tleubergenov, M.I., & Ibraeva, G.T. (2019). On the Solvability of the Main Inverse Problem for Stochastic Differential Systems. *Ukrainian Mathematical Journal, 71*(1), 157–165. DOI:10.1007/s11253-019-01631-w.

16 Tleubergenov, M.I., & Ibraeva, G.T. (2021). On the Closure of Stochastic Differential Equations of Motion. *Eurasian Mathematical Journal, 12*(2), 82–89.

17 Tleubergenov, M.I. (2001). An Inverse Problem for Stochastic Differential Systems. *Differential equations, 37*(5), 751-–753. https://doi.org/10.1023/A:1019285119532.

18 Tleubergenov, M.I., & Vassilina, G.K. (2021). On Stochastic Inverse Problem of Construction of Stable Program Motion. *Open Mathematics, 19*, 157–162. https://doi.org/10.1515/math-2021-0005

19 Tleubergenov, M.I., Vassilina, G.K., & Tuzelbaeva, G.A. (2021). On construction of a field of forces along given trajectories in the presence of random perturbations. *Bulletin of Karaganda University-Mathematics, 1*(101), 98–103. DOI 10.31489/2021M1/98-10.

20 Pugachev, V.S., & Sinitsyn, I.N. (1990). *Stokhasticheskie differentsialnye sistemy. Analiz i filtratsiia [Stochastic differential systems. Analysis and filtering]*. Moscow: Nauka [in Russian].

21 Tuladkhar, B.M. (1983). *Postroenie uravnenii v forme Lagranzha, Gamiltona i Birkgofa po zadannym svoistvam dvizheniia [Construction of Equations of Lagrange, Hamilton, and Birkhoff Types on the Basis of Given Properties of Motion]. Extended abstract of candidate's thesis.* Moscow [in Russian].

22 Tleubergenov, M.I. (1999). *Obratnye zadachi stokhasticheskikh differentsialnykh sistem [Inverse problems of stochastic differential systems]. Extended abstract of candidate's thesis.* Almaty [in Russian].

23 Tleubergenov, M.I., & Azhymbaev, D.T. (2007). O postroenii differentsialnogo uravneniia po zadannym svoistvam dvizheniia pri nalichii sluchainykh vozmushchenii [On construction of a differential equation on the basis of given properties of motion under random perturbations]. *Izvestiia NAN RK. Seriia fiziko-matematicheskaia — News of the NAS RK. Physico-mathematical series, 3*, 15–20 [in Russian].

24 Tleubergenov, M.I., & Azhymbaev, D.T. (2010). On the construction of a set of stochastic differential equations on the basis of a given integral manifold independent of velocities *Ukrainian Mathematical Journal, 62*(7), 1163–1173. DOI:10.1007/s11253-010-0421-8

25 Sagirov, P. (1974). Stokhasticheskie metody v dinamike sputnikov [Stochastic methods in dynamics of satellites]. *Mekhanika. Periodicheskii sbornik perevodov inostrannykh statei — Mechanics. Periodical Collection of Translations of Foreign Articles, 5*(147), 28–47 [in Russian].

26 Santilli, R.M. (1978). *Foundations of Theoretical Mechanics. 1. The Inverse Problem in Newtonian Mechanics.* New-York: Springer-Verlag.