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Existentially prime Jonsson quasivarieties and their Jonsson spectra

This article is devoted to the study of Jonsson quasivarieties in a signature enriched with new predicate and constant symbols. New concepts of semantic Jonsson quasivariety and fragment-conservativeness of the center of the Jonsson theory are introduced. The cosemanticness classes of the Jonsson spectrum constructed for a semantic Jonsson quasvariety are considered. In this case, the Kaiser hull of the semantic Jonsson quasivariety is assumed to be existentially prime. By constructing a central type for classes of theories from the Jonsson spectrum, the following results are formulated and proved. In the first main result, the necessary and sufficient condition is given for the center of the cosemanticness class of an existentially prime semantic Jonsson quasivariety to be λ-stable. The second result is the criterion for the center of the class of theories to be ω-categorical in the enriched language. The obtained theorems can be useful in continuing studies of various Jonsson algebras, in particular, Jonsson quasivarieties.

Keywords: Jonsson theory, perfect Jonsson theory, variety, quasivariety, semantic Jonsson quasivariety, Jonsson spectrum, existentially prime theory, central type, orbital type, central-orbital type, fragments of Jonsson sets.

Introduction

It is well-known fact that the greatest part of considered objects in Model Theory is connected with the study of incomplete theories. Many classical algebras, such as groups, fields, R-modules and many others, are axiomatized by incomplete theories. Nevertheless, this class of theories is too vast and, consequently, complicated for considering in detail. This is the reason why we need to introduce some conditions that clarify the subject of our research and allow studying various algebras, as well as their syntactic and semantic properties.

Thus, a subclass of incomplete theories where we do our research is Jonsson theories. One can find basic material in [1, 2] and more specific information on the connection between Jonsson theories, for example, in [3–5]. In this paper we mainly deal with semantic Jonsson quasivarieties and central-orbital type that play a significant role in the apparatus of Jonsson theories. In Section 1, necessary information on Jonsson theories is given. Section 2 is devoted to considering some specific properties of the Jonsson spectra of semantic Jonsson quasivarieties in the case of existential primeness. The main results are connected with constructing of the central type and stability and categoricity of cosemanticness classes.

All definitions that are not given in this article can be found in [2].

1 Preliminary information

We start with the main definitions and facts concerning the subject of the study. Recall the definitions of Jonsson theory and related concepts.

We are working within the framework of the following definition of Jonsson theory published in the Russian edition of [1].

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Definition 1. [1; 80] A theory T is called Jonsson if the following conditions hold for T:
1. T has at least one infinite model;
2. T is an inductive theory;
3. T has the amalgam property (AP);
4. T has the joint embedding property (JEP).

Classical examples of Jonsson theories include:
1) group theory;
2) the theory of abelian groups;
3) the theory of Boolean algebras;
4) the theory of linear orders;
5) field theory of characteristic p, where p is zero or a prime number;
6) the theory of ordered fields;
7) the theory of modules.

The following notions and facts form a necessary apparatus for studying Jonsson theories.

Definition 2. [2; 155] Let T be a Jonsson theory. A model C_T of power $2^{|T|}$ is called to be a semantic model of the theory T if C_T is a $|T|^+$-homogeneous $|T|^+$-universal model of the theory T.

Theorem 1. [2; 155] T is Jonsson iff it has a semantic model C_T.

The following definition was introduced by T.G. Mustafin.

Definition 3. [2; 155] A Jonsson theory T is called perfect if its semantic model C_T is saturated.

Definition 4. [2; 161] The elementary theory of a semantic model of the Jonsson theory T is called the center of this theory. The center is denoted by T^*, i.e. $Th(C) = T^*$.

The following theorem represents one of the most considerable facts describing perfect Jonsson theories.

Theorem 2. [2; 162] Let T be a perfect Jonsson theory. Then the following statements are equivalent:
1) T^* is the model companion of T;
2) $ModT^* = E_T$;
3) $T^* = T^f$, where T^f is a forcing companion of the theory T.

Some classical examples of perfect Jonsson theories one can find in [4], while non-perfect Jonsson theories are considered in [5].

Theorem 3. [5] Let T be a Jonsson theory. Then for any model $A \in E_T$ the theory $T^0(A)$ is Jonsson, where $T^0(A) = Th_{\forall\exists}(A)$.

We can see that in the case of perfectness of T its center T^* is also a perfect Jonsson theory.

The following definition will help us to specify the class of Jonsson theories which we will deal with in this paper.

Definition 5. [6; 120] A Jonsson theory is said to be hereditary if, in any of its permissible enrichment, it preserves the Jonssonness.

Unfortunately, there is no complete description of this notion. However, one can find some useful information in [7] on hereditary Jonsson theories.

One more specific notion that is widely used in the study of Jonsson theories is a Jonsson set. Recall its definition.

Definition 6. [8] Let T be a Jonsson theory and C be its semantic model. A Σ-definable subset of C is called a Jonsson set for the theory T, if $dcl(X) = M$, $M \in E_T$. A theory $Th_{\forall\exists}(M)$ is called a fragment of the Jonsson set X.

Some research methods, where this notion is used, are revealed in [9].

The following class of theories was specified by Yeshkeyev A.R.

Definition 7. [10] A theory T is called existentially prime, if $AP_T \cap E_T \neq \emptyset$, where AP_T is the class of algebraically prime models of T.

Let T be a Jonsson theory and $S^J(Y)$ be a set of all existentially complete n-types over Y that are consistent with T, for any finite n.

Definition 8. [11] A Jonsson theory T is $J-\lambda$-stable if for any T-existentially closed model A and for any subset Y of A from the inequality $|Y| \leq \lambda$ it follows that $|S^J(Y)| \leq \lambda$.

In [11], the authors proved the following result that shows the connection between Jonsson stability and stability in the classical sense.

Theorem 4. [11] Let T be a perfect Jonsson theory and let T be complete for existential sentences. Let $\lambda \geq \omega$. Then the following statements are equivalent:

1) T is $J-\lambda$-stable;

2) T^* is λ-stable, where $T^* = Th(C)$, C is a semantic model of T.

Let L be a first-order language of a signature σ and let K be a class of L-structures. Then we can consider a Jonsson spectrum for K, which can be defined as follows.

Definition 9. [5] A set $JSp(K)$ of Jonsson theories of L, where

$$JSp(K) = \{T \mid T \text{ is a Jonsson theory and } K \subseteq Mod(T)\},$$

is called a Jonsson spectrum of K.

Jonsson spectra are well-described in [12]

Let T_1 and T_2 be Jonsson theories, T_1^* and T_2^* be their centres, respectively.

Definition 10. [2; 40] T_1 and T_2 are said to be cosemantic Jonsson theories (denoted by $T_1 \bowtie T_2$), if $T_1^* = T_2^*$.

It is easy to see that the relation of cosemanticness between two Jonsson theories is an equivalence relation:

1) this relation is reflexive since for every Jonsson theory T the equation $T^* = T^*$ holds,

2) it is symmetric as soon as, for any Jonsson theories T_1 and T_2, if $T_1^* = T_2^*$ then $T_2^* = T_1^*$,

3) finally, " $\bowtie$ " is transitive, that follows from the fact that, for any Jonsson theories T_1, T_2 and T_3, if $T_1^* = T_2^*$, $T_2^* = T_3^*$ then $T_1^* = T_3^*$.

This means that, when introducing the relation of cosemanticness on the Jonsson spectrum $JSp(K)$, we get a partition of $JSp(K)$ into cosemanticness classes. The obtained factor-set is denoted by $JSp(K)_{/\bowtie}$.

Now let us consider the notion of a semantic Jonsson quasivariety. One should note that this concept differs significantly from the concept of a Jonsson quasivariety introduced in [13].

Let K be a quasivariety in the usual sense as in [14; 269]. We construct a set $\forall\exists(K)$, where $\forall\exists(K)$ is a set of Jonsson theories and obtained as follows:

$$\forall\exists(K) = \{Th(K) \cup \varphi \,|\, \varphi \text{ is an } \forall\exists\text{-sentence and } \varphi \cup Th(K) \text{ is consistent}\}. \tag{1}$$

In other words, the set $\forall\exists(K) = \{T_1, T_2, ...\}$ is a list of all Jonsson theories that satisfy Condition 1. Then C_i is a semantic model of T_i from this list. Let us consider the following set:

$$JK = \{C_i \,|\, C_i \text{ is a semantic model of } T_i,\, T_i \in \forall\exists(K)\}.$$

Definition 11. The set JK is a semantic Jonsson quasivariety, if the theory $T^0(JK) = Th_{\forall\exists}(JK)$ is Jonsson.

The theory $T^0(JK)$ is called a Kaiser hull of the class JK.

Definition 12. A set of theories $JSp(JK)$, where

$$JSp(JK) = \{T^0(JN) \mid N \text{ is a subquasivariety of } K\},$$

is said to be a Jonsson spectrum of a semantic Jonsson quasivariety JK.

2 Central types for cosemanticness classes of $JSp(JK)$

In this section, we consider the Jonsson spectrum of a semantic Jonsson quasivariety from a position of central-orbital types and existential primeness. The main definitions and facts related to central type can be found in the papers of the first author Yeshkeyev A.R., for example [15–17]. A special role is played by the work [18] where the author defined the notion of central-orbital type for the Jonsson case by analogy with [19]. Here we apply the results of [18] to semantic Jonsson quasivarieties.

Currently, the class of Jonsson quasivarieties is not studied well enough. Generally speaking, in contrast to the case of complete theories, the apparatus for studying incomplete theories (including Jonsson ones) is not developed at a sufficient level. This is why we have to restrict this research by introducing some specific conditions.

First of all, we have to refine that all the Jonsson theories in this section are hereditary (Definition 5) by our assumption.

Another necessary restriction is formulated by the following definition.

Definition 13. The center T^* of a Jonsson theory T is said to be fragment-conservative if the semantic model of any fragment of T^* is an existentially closed submodel of the semantic model C of T.

Further in this paper, we work with Jonsson theories whose centers are fragment-conservative.

We work in a first-order language L of a signature σ. Let JK be an existentially prime semantic Jonsson quasivariety, which means that the theory $T^0(JK)$ from Definition 11 is existentially prime as it is described in Definition 7. Let $JSp(JK)$ be a Jonsson spectrum of JK. We introduce the relation of cosemanticness on $JSp(JK)$. In this manner, we have a factor-set $JSp(JK)_{/\bowtie}$ consisting of all Jonsson theories that satisfy Definition 12. Let us consider some class $[T]_{/\bowtie} \in JSp(JK)_{/\bowtie}$. Let C be a semantic model of each theory $T \in [T]_{/\bowtie}$, $X \subseteq C$ be a Jonsson set.

To consider the properties of JK and $JSp(JK)$ through constructing the central type for the cosemanticness classes, firstly we need to enrich the signature σ by new constant c and predicate P symbols as follows.

Let $\sigma_\Gamma(X) = \sigma \cup \{c_a, a \in X\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$. We consider a class of theories $[T_X^C]$ in the new enriched signature $\sigma_\Gamma(X)$ for each cosemanticness class $[T]_{/\bowtie}$, where $T_X^C \in [T_X^C]$ is constructed as follows:

$$T_X^C = T \cup Th_{\forall\exists}(C, a)_{a\in X} \cup \{P(c_a), a \in X\} \cup \{P(c)\} \cup \{P, \subseteq\}.$$

Here P is a new 1-ary predicate symbol interpretations of which are an existentially closed submodel M of the semantic model C, i.e. $P(C) = M, M \in E_T$, $T \in [T]_{/\bowtie}$.

As soon as any theory $T \in [T]_{/\bowtie}$ is hereditary by our assumption and the introduced enrichments are permissible, every theory T_X^C in the class $[T_X^C]$ is also Jonsson. Therefore, there is a semantic model C' for $[T_X^C]$. It is easy to see, that the semantic models of the theories from $[T_X^C]$ coincide, so we denote it by C'. Let $T' = Th(C')$ be a center for the class $[T_X^C]$. Now we will consider the theory T' in a restricted signature $\sigma_\Gamma(X)\backslash\{c\}$ so that T' becomes a complete type of c.

Definition 14. A complete type described above is called a central type for the Jonsson theory T with respect to the Jonsson set X (denoted by p_X^C).

In case when a central type coincides with an orbital type of a Jonsson theory the obtained type is called a central-orbital type. Some properties related to central-orbital types of a Jonsson theory are considered in [18]. Since a central-orbital type is central and orbital at the same time, all statements that are connected with central types and mentioned in this section can be considered in terms of central-orbital types.

Here we work with the cosemanticness classes of theories, not single theories. Taking into consideration this fact and the results from [18], we can get the following theorems.

Theorem 5. Let JK be an existentially prime semantic Jonsson quasivariety, $JSp(JK)$ be its Jonsson spectrum, $[T]_{/\bowtie} \in JSp(JK)_{/\bowtie}$, $T_i \in [T]_{/\bowtie} (i \in I)$, T^* is a center for the class $[T]_{/\bowtie}$. Let X_i be Jonsson sets for the theories T_i respectively, $dcl(X_i) = M_i$, $M_i \in E_{T_i}$. $[T_X^C]$ is the class of the theories in the enriched signature as it is described above. If $\lambda \geq \omega$, then T^* is $J-\lambda$-stable if and only if S is λ-stable for any theory $S \in [T_X^C]$.

Proof. The proof follows from Theorem 4 from Section 1 and Theorem 1 from [18], applying this result to each arbitrary theory from $[T_X^C]$.

The following result demonstrates the connection between the categoricity of a center for the cosemanticness class of Jonsson fragments and the categoricity of the corresponding theories in the enriched signature. Here we need to introduce the following notation. Let a cosemanticness class $[T]_{/\bowtie}$ consist of theories $T_i, i \in I$. As soon as all theories are inductive, for any T_i there exists a non-empty class of existentially closed models E_{T_i}. For each i, we consider a Jonsson set X_i such that a model $M_i \in E_{T_i}$ is a definable closure of X_i, i.e. $dcl(X_i) = M_i$. After this, we can construct a theory $Th_{\forall\exists}(M_i)$, which is called a Jonsson fragment of the Jonsson set X_i, for each theory T_i. Thus we get a class of all Jonsson fragments for the corresponding cosemanticness class $[T]_{/\bowtie}$. We denote the obtained class by $[T_X]$. Note that every theory in this class is Jonsson, which means that it has a semantic model. It is easy to see that the semantic models for each $T_X \in [T_X]$ coincide, so let us denote the center for this class by T_X^*.

The following lemma is true for $[T_X]$ because of heredity.

Lemma 1. Every theory $T_X \in [T_X]$ is Jonsson in the new signature $\sigma_\Gamma(X)$.

Theorem 6. Let K be an existentially prime semantic Jonsson quasivariety, $JSp(JK)$ be its Jonsson spectrum, $[T]_{/\bowtie} \in JSp(JK)_{/\bowtie}$, $T_i \in [T]_{/\bowtie} (i \in I)$, T^* is a center for the class $[T]_{/\bowtie}$, and let $[T_X]$ be as it is describe above. Then T_X^* is ω-categorical if and only if each $S \in [T_X^C]$ is ω-categorical.

Proof. The proof can be obtained by applying Theorem 2 of [18] to arbitrary theories of the mentioned classes.

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Экзистенциалды жай йонсондық квазикөптүрліліктер және олардың йонсондық спектрлері

Мақала жаңа предикаттық және тұрақты символдармен байытылған сигнатурадағы йонсондық квазикөптүрліліктерді зерттеуге арналған. Семантикалық йонсондық квазикөптүрліліктер мен йонсондық теорияның фрагмент-консервативтілігі туралы жаңа түсініктер енгізілді. Йонсондық квазикөптүрліліктер үшін құрылған йонсондық спектрдің косемантты кластары қарастырылған. Бұл жағдайда йонсондық квазикөптүрліліктің Кайзер қабықшасы экзистенциалды жай деп ұйғарылады. Йонсондық спектрдің теория кластары үшін центрлік типті құру арқылы келесі нәтижелер тұжырымдалды және дәлелденді. Бірінші негізгі нәтижеде экзистенциалды жай йонсондық квазикөптүрліліктің косемантты класының центрі Λ-тұрақты болуы үшін қажетті және жеткілікті шарттар келтірілген. Екінші нәтиже, байытылған тілдің теориялар класы центрінің ω-категориялығының критерийі болып табылады. Алынған теоремалар әртүрлі йонсондық алгебраларды, атап айтқанда, йонсондық квазикөптүрліліктерді зерттеуді жалғастыру үшін пайдалы болуы мүмкін.

Кілт сөздер: йонсондық теория, кемел йонсондық теория, көптүрлілік, квазикөптүрлілік, семантикалық йонсондық квазикөптүрлілік, йонсондық спектр, экзистенциалды жай спектр, центральді тип, орбитальді тип, центральді-орбитальді тип, йонсондық жиынның фрагменттері.

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Экзистенциально простые йонсоновские квазимногообразия и их йонсоновские спектры

Статья посвящена изучению йонсоновских квазимногообразий в сигнатуре, обогащенной новым предикатным и константным символами. Введены новые понятия семантического йонсоновского квазимногообразия и фрагмент-консервативности центра йонсоновской теории. Рассмотрены классы косемантичности йонсоновского спектра, построенного для йонсоновского квазимногообразия. При этом оболочка Кайзера йонсоновского квазимногообразия предполагается экзистенциально простой. С помощью построения центрального типа для классов теорий из йонсоновского спектра формулируются и доказываются следующие результаты. В первом основном результате приведено необходимое и достаточное условие для того, чтобы центр класса косемантичности экзистенциально простого йонсоновского квазимногообразия являлся λ-стабильным. Второй результат является критерием ω-категоричности центра класса теорий обогащенного языка. Полученные теоремы могут быть полезны для продолжения исследований различных йонсоновских алгебр, в частности, йонсоновских квазимногообразий.

Ключевые слова: йонсоновская теория, совершенная йонсоновская теория, многообразие, квазимногообразие, семантическое йонсоновское квазимногообразие, йонсоновский спектр, экзистенциально простая теория, центральный тип, орбитальный тип, центрально-орбитальный тип, фрагменты йонсоновского множества.

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