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On recognizing groups by the bottom layer

The article discusses the possibility of recognizing a group by the bottom layer, that is, by the set of its elements of prime orders. The paper gives examples of groups recognizable by the bottom layer, almost recognizable by the bottom layer, and unrecognizable by the bottom layer. Results are obtained for recognizing a group by the bottom layer in the class of infinite groups under some additional restrictions. The notion of recognizability of a group by the bottom layer was introduced by analogy with the recognizability of a group by its spectrum (the set of orders of its elements). It is proved that all finite simple non-Abelian groups are recognizable by spectrum and bottom layer simultaneously in the class of finite simple non-Abelian groups.

Keywords: group, layer-finite group, bottom layer, spectrum, recognizability.

Introduction

The article discusses the possibility of restoring groups by the bottom layer under additional conditions.

The bottom layer of a group is the set of its elements of prime orders.

A group is called *recognizable by the bottom layer under additional conditions* if it is uniquely reconstructed by the bottom layer under these conditions.

A group G is called *almost recognizable by the bottom layer under additional conditions* if there exists a finite number of pairwise non-isomorphic groups satisfying the same conditions, with the same bottom layer as the group G.

A group G is called *unrecognizable by the bottom layer under additional conditions* if there exists an infinite number of pairwise non-isomorphic groups satisfying the same conditions, with the same bottom layer as the group G.

Many results for groups with a given bottom layer describe some of the properties of the groups. For example, V.D. Mazurov proved that a group with a bottom layer consisting of elements 2, 3, 5, in which all other non-identity elements are of order 4, is locally finite [1]. If the bottom layer of finite group consists of elements of orders 2, 3, 5 and the group contains no non-identity elements of other orders, then W. Shi proved that this is a group of even permutations on five elements [2].

The results on group recognition by the bottom layer were reported at the conferences [3–5] and published in journals [6–8].

Main part

Let us give an example of a group recognizable by the bottom layer in the class of finite groups. If the bottom layer of group G consists of elements of order 2 and the group contains no non-identity elements of other orders, then G is an elementary Abelian 2-group. That is, group G is recognizable by the bottom layer in the class of finite groups.

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An example of unrecognizability by the bottom layer in the class of finite groups is given by the following infinite series of groups: in the infinite row of the cycle groups of the orders p, p^2, $p^3, \ldots$ for some prime p the same bottom layer consisting of $p-1$ elements of order p. In this example, the groups are unrecognizable by the bottom layer in the class of finite groups.

Recall that a group G is called *layer-finite* if it has a finite number of elements of each order. This term was introduced by S.N. Chernikov. The definition of a layer-finite group arose in connection with the study of infinite locally finite p-groups provided that the center of the group $Z(G)$ has a finite index [9].

The groups in the following example are almost recognizable groups by the bottom layer in the class of infinite layer-finite groups. V.P. Shunkov proved that if the bottom layer in an infinite layer-finite group consists of one element of order 2, then the group G is either quasi-cyclic or an infinite generalized quaternion group. The groups from the result of V.P. Shunkov are almost recognizable by the bottom layer in the class of infinite layer-finite groups.

Earlier the recognizability of a complete group with a layer-finite center and a periodic quotient group by it is obtained in the class of infinite groups:

Let G be a complete group in which $Z(G)$ is layer-finite and $G/Z(G)$ is a periodic group. If the bottom layer of group G consists of an element p^{n-1} of order p, then group G is recognizable by the bottom layer in the class of groups satisfying these conditions [6].

Let us recall some results on the recognizability of groups in some classes of groups obtained earlier by the authors.

If G is a complete group in which $Z(G)$ is layer-finite and $G/Z(G)$ is a periodic group containing for each prime p only a finite number of p-elements, then group G is recognizable by the bottom layer among groups with such properties [6].

Definition 1. Layer-finite group is called a *thin layer-finite group* if all of its Sylow subgroups are finite.

Let G be a group in which the center contains a complete layer-finite subgroup R such that the factor group G/R is a thin layer-finite group. The group G is recognizable by the bottom layer among groups with such properties [6].

Let G be a complete nilpotent p-group with a finite bottom layer. Then group G is recognizable by the bottom layer among groups with such properties [6].

Let G be a complete periodic group in which for each prime p there is only a finite number of Sylow p-subgroups and for every prime p there is at least, one Sylow p-subgroup in G, which is a layer-finite group. Then the group is recognizable by the bottom layer among groups with such properties [6].

A complete nilpotent p-group with a finite bottom layer is recognizable by the bottom layer in the class of groups satisfying these conditions [6].

In articles [7, 8], the recognizability by the bottom layer of the complete group is considered under slightly different conditions: layer finiteness of the group or the existence of a layer finite subgroup S in the center of the group G such that G/S is layer finite group. In the same papers, it was proved that a group is recognizable by the bottom layer among locally solvable group without involutions with the minimality condition.

It is convenient to consider the recognition of groups by the bottom layer in the class of layer-finite groups. However, we can also consider other classes of groups.

Now we consider under which conditions it is possible to recognize groups by the bottom layer in the class of infinite groups.

Periodic complete Abelian groups are not necessarily layer-finite. The next theorem establishes the recognizability of a group by the bottom layer in this class of groups.

Theorem 1. Group G is recognizable by the bottom layer among periodic complete Abelian groups.

Proof. Indeed, let group G satisfy the indicated conditions. By Proposition 1, the complete Abelian group G decomposes into a direct sum of subgroups isomorphic to the additive group of rational numbers or to quasi-cyclic groups, possibly for different prime numbers. There can be no rational groups in this extension, since G is a periodic group and, therefore, there are no elements of infinite order in it. Since the direct product of quasi-cyclic groups is restored from the bottom layer, the group G is recognizable by the bottom layer among groups with the properties as in the theorem. The theorem is proved.

Definition 2. A group is called *radically complete* if for any of its elements a and for each natural number n the equation $x^n = a$ has at least one solution in it [10].

Theorem 2. Group G is recognizable by the bottom layer among periodic radically complete groups satisfying the normalizing condition.

Proof. Indeed, let group G satisfy the indicated conditions. By Proposition 2 the elements of finite order of the radically complete group satisfying the normalizing condition G form a complete Abelian group. As G is periodic, such a group G by Theorem 1 is recognizable by the bottom layer among periodic complete Abelian groups. So G is recognizable by the bottom layer among periodic radically complete groups satisfying the normalizing condition. The theorem is proved.

Radically complete groups are not necessaryily layer-finite. For example, direct product of infinite number of quasi-cyclic groups for the same prime number is radically complete, but it is not a layer-finite group.

The notion of recognizability of a group by the bottom layer was introduced by analogy with the recognizability of a group by its spectrum.

The spectrum of a finite group is a set of orders of its elements. A finite group G is called *recognizable by spectrum* if any finite group which has the spectrum coinciding with the spectrum of G is isomorphic to G. A group G is called *almost recognizable by its spectrum* if there are finitely many pairwise non-isomorphic groups with the same spectrum as the group G. A group G is called *spectrum-unrecognizable* if there are infinitely many pairwise non-isomorphic groups with the same spectrum as G.

Results on groups recognizable by spectrum could be found in the works of A.V. Vasil'ev, V.D. Mazurov, A.M. Staroletov, A.A. Buturlakin, M.A. Grechkoseeva, and others [11–21].

An example of a group that is not recognizable by spectrum is group A_6 with the spectrum 2, 3, 4, 5, 8, 9 (there are infinitely many groups, one of which is group A_6) [12]. Also the group $L_3(3)$ with the spectrum 2, 3, 4, 8, 9, 13, 16, 27 is unrecognizable by spectrum [12].

It is proved in [14] that the symmetric groups S_n are recognizable by spectrum for $n \notin \{2, 3, 4, 5, 6, 8, 10, 15, 16, 18, 21, 27, 33, 35, 39, 45\}$. In 1994, W. Shi and R. Brandl proved the recognizability of an infinite series of simple linear groups $L_2(q)$, $q \neq 9$ [15, 16].

A.V. Vasil'ev established the result on the almost spectrum recognition of the group $U_4(5)$ in the class of finite groups:

Let G be a finite simple group $U_4(5)$ and H be a finite group with the property $\omega(H) = \omega(G)$. Then $H \cong G$ or $H \cong G(\gamma)$, where γ is a field automorphism of the group G of order 2. In particular, $h(G) = 2$.

By $h(G)$ it is denoted the number of pairwise non-isomorphic finite groups G with the same spectrum [17].

Thus, the group $U_4(5)$ is almost recognizable by its spectrum in the class of finite groups.

We established previously [6] that the group $U_4(5)$ is recognizable by both the spectrum and the bottom layer in the class of finite groups:

If G is a finite simple group $U_4(5)$, H is a finite group with the property $\omega(H) = \omega(G)$ and the bottom layer is the same as for the group $U_4(5)$, then $H \cong G$. That is, $U_4(5)$ is the only finite group with such a spectrum and such a bottom layer.

Almost all finite simple non-Abelian groups are recognized by their spectrum in the class of finite simple non-Abelian groups. However, there are some exceptions: different groups of this set have the same spectra.

Theorem 3. All finite simple non-Abelian groups are simultaneously recognizable by spectrum and bottom layer in the class of finite simple non-Abelian groups.

Proof. Let us show the possibility of recognizing by the bottom layer such finite simple non-Abelian groups with the same spectrum using the example of the groups $S_6(2)$ and $O_8^+(2)$.

The group $O_8^+(2)$ is simple, has order $174182400 = 2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$. With the help of the GAP package, it was established that there are 69615 involutions (elements of order 2) and 24883200 elements of order 7 in it.

The group $S_6(2)$ has order $1451520 = 2^9 \cdot 3^4 \cdot 5 \cdot 7$. Using the GAP package, it was found that it contains 5103 involutions (elements of order 2) and 207360 elements of order 7.

Thus, the groups $S_6(2)$ and $O_8^+(2)$ have different numbers of elements of the second and seventh orders on the bottom layer and thus are recognized simultaneously by the spectrum and the bottom layer in the class of finite simple non-Abelian groups.

In paper [11], it was established that among the finite simple non-Abelian groups, apart from the groups $S_6(2)$ and $O_8^+(2)$, there is only one more pair of almost unrecognizable by spectrum groups $O_7(3)$ and $O_8^+(3)$.

The first group $O_7(3)$ from this pair is simple non-Abelian, has order $4585351680 = 2^9 \cdot 3^9 \cdot 5 \cdot 7 \cdot 13$. Using the GAP package, it was established that there are 38211264 elements of the fifth order in it.

The second considered group $O_8^+(3)$ has the order $4952179814400 = 2^{12} \cdot 3^{12} \cdot 5^2 \cdot 7 \cdot 13$. Using the GAP package, it was found that it contains 8253633024 elements of the fifth order.

Thus, the groups $O_7(3)$ and $O_8^+(3)$ have different numbers of fifth-order elements in the bottom layer and thus are recognized simultaneously by the spectrum and the bottom layer in the class of finite simple non-Abelian groups. The theorem is proved.

In proving the results of the paper, we used the following theorems, which were referred to as propositions with the corresponding number.

Proposition 1 (Theorem 9.1.6 from [22]). A nonzero complete Abelian group can be decomposed into a direct sum of subgroups isomorphic to the additive rational group or quasi-cyclic groups, may be for different prime numbers.

Proposition 2 (Theorem 2.8 from [10]). If a radically complete group satisfies the normalizing condition, then the elements of its finite order form a complete Abelian subgroup.

Conclusion

The possibilities of recognizing of some finite and infinite layer-finite groups by the bottom layer are considered. Results are obtained for recovering groups by the bottom layer in the class of infinite groups with some additional conditions. It is proved that all simple non-Abelian groups are simultaneously recognizable by spectrum and bottom layer in the class of finite simple non-Abelian groups.

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Төменгі қабат бойынша группаларды тану туралы

Мақалада группаны төменгі қабаттан, яғни оның элементтерінің жай реттері жиынын қалпына келтіру мүмкіндігі қарастырылған. Жұмыста төменгі қабат арқылы танылатын, төменгі қабат арқылы дерлік танылатын және төменгі қабат арқылы танылмайтын топтардың мысалдары келтірілген. Шексіз группалар класындағы төменгі қабаттан группаны қайта құру нәтижелері кейбір қосымша шектеулер бойынша алынды. Төменгі қабат бойынша группаны тану түсінігі спектр бойынша группаларды тануға (оның элементтерінің қатарларының жиынтығы) ұқсас енгізілді. Барлық ақырлы жай абельдікемес группалардың спектрі және төменгі қабаты бойынша танылуы ақырлы қарапайым абельдікемес группалар класында бір уақытта дәлелденген.

Кілт сөздер: группа, қабатты ақырлы группа, төменгі қабат, спектр, танымдылық.

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О распознавании групп по нижнему слою

В статье обсуждена возможность восстановления группы по нижнему слою, то есть по множеству её элементов простых порядков. Приведены примеры распознаваемых по нижнему слою, почти распознаваемых по нижнему слою и нераспознаваемых по нижнему слою групп. Получены результаты восстановления группы по нижнему слою в классе бесконечных групп при некоторых дополнительных ограничениях. Понятие распознаваемости группы по нижнему слою введено по аналогии с распознаваемостью группы по спектру (множеству порядков её элементов). Доказана распознаваемость всех конечных простых неабелевых групп по спектру и нижнему слою одновременно в классе конечных простых неабелевых групп.

Ключевые слова: группа, слойно конечная группа, нижний слой, спектр, распознаваемость, конечные простые неабелевы группы.

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