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Analytical and numerical research based on one modified refined bending theory

In the article, an analytical and numerical study based on one modified refined bending theory is presented. By the finite difference method, a general numerical calculation algorithm is developed. The solution obtained by the proposed method is compared with the results of known solutions, namely, with the solution of the classical theory, the exact solution, the solution in trigonometric series, as well as with experimental data. Comparison of the results obtained by the method given in the article with the solutions determined by other methods shows sufficient accuracy, which indicates the reliability of the proposed method based on one option of the modified refined bending theory. Classical theory is not applicable to such problems under consideration.

Keywords: modified refined bending theory, finite difference method, lagrange variational principle, differential operator, discretization of a system of equations

Introduction

The rapid development of scientific and technological progress requires the creation and implementation of new progressive materials and structures with predetermined properties. These requirements are fully met by composite materials, in particular, multilayer composites, which have a wide range of performance properties that cannot be achieved using traditional materials.

The use of multilayer composite materials in modern apparatuses and devices required taking into account their structural features, the physical and mechanical properties of the materials used, the number, structure and arrangement of layers for the composite material in mathematical research, as well as the creation of new methods that refine existing theories for the mathematical calculation of the stress-strain state of such structures.

In multilayer composite structures, the layers are made of such a material and these layers are arranged so as to endow the structure with a number of predetermined positive properties. At the same time, the materials are selected in such a way that, in an optimal combination, they give a qualitatively new type of construction. Or, in other words, in multilayer composite structures, the layers are arranged so that, under operational conditions, the structure better corresponds to its functional purpose.

The technical, mathematical and mechanical properties of structures made of multilayer inhomogeneous materials differ significantly in the thickness of their packages. Therefore, the features study for the operation of structures made of multilayer inhomogeneous materials in the thickness of their package by use refined theories is important in the mathematical study and the design of new innovative lightweight structures made of multilayer materials. Bending theories clarifying mathematical and technical theory should take into account the most important operational characteristics of multilayer

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composites, such as strain in the orthogonal direction to the layers, interaction of layers, strength, high resistance to fracture, etc. Each construction of a new multilayer composite that provides an increase in technical characteristics, as a rule, requires the development of new calculation methods based primarily on analytical mathematical research, and later on the numerical implementation of this research and its practical application.

One of the elements for multilayer composite spatial systems is a rectangular plate, which has numerous independent applications. An example of a rectangular plate, clamped with one edge, is a vertical panel, and an example of a plate, elastically clamped with three edges, is the wall of a rectangular reservoir. It should be noted that multilayer plates are a very extensive type of plates and are more often used in many fields of science and technology. The plate can be applied as an independent structure or can be part of the used lamellar system. For example, in the construction plates have all kinds of applications in the form of floorings and wall panels, reinforced concrete slabs to cover industrial and residential buildings, slabs for the foundations of massive structures, etc. Therefore, knowledge of the theory for rectangular plates bending and of classical methods for calculating them is necessary for a modern scientist.

Many analytical and numerical calculation methods are used to study the problems of plate bending [1–12]. An exact solution in analytical form for such problems is possible only in some particular cases for the geometrical type of the plate, the load and the conditions for its fixation on the supports, therefore, for practical applications, numerical, but sufficiently accurate methods for solving the considered problem are of special importance.

When considering the plate bending problems, the finite difference method is the most interesting because of connection with their possible numerical implementation in software package.

1 Initial positions and hypotheses

We consider a rectangular plate made of a multilayer composite material. The sides of the plate are equal to a and b, the thickness of the plate equals h. The study of the deformation of the plate is carried out in a rectangular coordinate system x_1, x_2, $x_3 = z$. The number of layers is arbitrary. The layers of the plate are orthotropic. Orthotropic materials are more difficult to analyze than isotropic materials, because their properties depend on the direction, so we place the directions of the Ox_1 and Ox_2 axes on the axes of the orthotropy of the layers. There is a coordinate plane at an arbitrary height of the plate section. The axes Ox_1 and Ox_2 lie on this coordinate plane.

The total number of layers in the plate is denoted by n. We number the layers as usual, starting from the bottom edge of the plate. The number of an arbitrary layer of the plate is denoted by k. The layer number in the coordinate plane is denoted by m. The totality of all n layers of the plate is called a package of layers.

In the general case, let's assume that the layers of the package have different thicknesses and different stiffness, the mechanical properties of which do not change in their thickness [13]. We suppose that the number of layers and their placement in the package are arbitrary.

During the transition from layer to layer we assume that static conditions

$$\sigma_{i3}^{k} = \sigma_{i3}^{k-1}, \quad \sigma_{33}^{k} = \sigma_{33}^{k-1}$$

and kinematic conditions

$$u_i^k = u_i^{k-1} \quad (i = 1, 2, 3)$$

are fulfilled, where σ_{ij}^k $(i, j = 1, 2, 3)$ are stresses, u_i^k $(i = 1, 2, 3)$ are displacements of the k-th layer. This corresponds to the operation of their layers without slipping and tearing.

Let a normal load $q(x_1, x_2)$ act on the upper surface of the plate. The normal load $q(x_1, x_2)$ varies according to an arbitrary law. The positive direction of the normal load coincides with the direction of the normal axis $x_3 = z$.

On the plate surface, the boundary conditions take the form

$$\sigma_{33}^{n} = q(x_1, x_2), \quad \sigma_{i3}^{n} = 0, \quad \sigma_{j3}^{1} = 0, \quad i = 1, 2, \quad j = 1, 2, 3.$$

The conditions of the deformation continuity for the coordinate surface have the form [13]

$$\chi_{ii,l} - \chi_{12,i} = 0,$$

$$\varepsilon_{11,12} - 2\varepsilon_{12,12} + \varepsilon_{22,11} = 0$$

where $\varepsilon_{i,j}$ is strains, $\chi_{ij,l}$ is the shear function of the coordinate surface.

As the main assumptions for constructing a new refined model of the stress-strain state for a layered plate of an asymmetric structure with orthotropic layers, we accept the following system of hypotheses

$$\sigma_{i3}^{k} = G_{i3}^{k} \psi_{i,3}^{k}(z) \chi_{,i}, \quad \sigma_{33}^{k} = -\sum_{i=1}^{2} \eta_{3i}^{k}(z) \chi_{,ii}, \quad u_3^{n} = W \tag{1}$$

where G_{i3}^{k} is the shear modulus of the material, ψ, η are distribution functions for the k-th layer of the package, W and χ are the sought deflection function and the sought shift function of the coordinate surface, depending on the coordinates x_1 and x_2. The distribution functions depend on the z coordinate.

Hypotheses (1) are derived from the hypotheses made by Prof. A.Sh. Bozhenov [1], with the exception of those components that are not of great importance in calculating the stress-strain state of the plate. Hypotheses (1) guarantee the joint operation of layers without separation from each other and displacement, as well as conditions on the plate surfaces and determine the nonlinear law of variation of transverse shear stresses and normal transverse stresses in the plate thickness. It is assumed that normal displacements are equal to deflections.

For the distribution function in expressions (1), we have the following formulas

$$\psi_{i,3}^{k}(z) = \frac{1}{G_{i3}^{k}} \left[\eta_{2i}^{k}(z) - \eta_{1i}^{k}(z) \delta_i^{*} \right], \quad \eta_{1i}^{k}(z) = \int_{b_{k-1}-\delta_1}^{z} A_i^{k} dz + \sum_{j=1}^{k-1} \int_{b_{j-1}-\delta_1}^{b_j-\delta_1} A_i^{j} dz,$$

$$\eta_{2i}^{k}(z) = \int_{b_{k-1}-\delta_1}^{z} B_i^{k} z dz + \sum_{j=1}^{k-1} \int_{b_{j-1}-\delta_1}^{b_j-\delta_1} B_i^{j} z dz, \quad \eta_{3i}^{k}(z) = \int_{b_{k-1}-\delta_1}^{z} G_{i3}^{k} \psi_{i,3}^{k}(z) dz + C_{3i}^{k}, \tag{2}$$

where δ_1 is the distance from the coordinate plane to the bottom edge of the plate, and the constants have the form

$$C_{3i}^{k} = \sum_{j=1}^{k-1} \int_{b_{j-1}-\delta_1}^{b_j-\delta_1} G_{i3}^{j} \psi_{i,3}^{j}(z) dz.$$

Here and in what follows, the notation introduced in [1] is adopted. For the components in formulas (2), we have the following expressions

$$A_i^{k} = 0.5\{B_{ii}^{k}(1 + \nu_{ii}^{k}) + G_{12}^{k}\}, \quad B_i^{k} = 0.5 B_{ii}^{k}(1 + \nu_{ii}^{k}) + G_{12}^{k}, \quad B_{ii}^{k} = E_i^{k} \nu_0^{k},$$

$$\delta_i^{*} = \frac{\eta_{2i}^{k}}{\eta_{1i}^{k}}, \quad \nu_0^{k} = \left(1 - \nu_{12}^{k} \nu_{21}^{k}\right)^{-1}, \quad B_{i3}^{k} = \left(\nu_{3i}^{k} + \nu_{li}^{k} \nu_{3l}^{k}\right) \nu_0^{k},$$

where E_i^{k} is the modulus of elasticity and ν_{ij}^{k} is the Poisson modulus for the k-th layer of the plate [13].

2 Analytical research

Based on the hypotheses (1) we have adopted, we carry out an analytical study of the stress-strain state for the layer package. First, we present the relationships that we use to derive the calculation formulas for stresses and strains.

We have the relations of Hooke's law

$$\sigma_{ii}^k = B_{ii}^k e_{ii}^k + 2B_{12}^k e_{22}^k + B_{i3}^l e_{33}^k, \quad \sigma_{12}^k = 2G_{12}^k e_{12}^k, \quad \sigma_{i3}^k = 2G_{i3}^k e_{i3}^k. \tag{3}$$

Inverse expressions of Hooke's law have the form

$$e_{ii}^k = \frac{1}{E_i^k}\sigma_{ii}^k - \frac{\nu_{il}^k}{E_l^k}\sigma_{ll}^k - \frac{\nu_{i3}^k}{E_3^k}\sigma_{33}^k,$$

$$e_{33}^k = \frac{1}{E_3^k}\sigma_{33}^k - \frac{\nu_{31}^k}{E_1^k}\sigma_{11}^k - \frac{\nu_{32}^k}{E_2^k}\sigma_{22}^k,$$

$$2e_{i3}^k = \frac{1}{G_{i3}^k}\sigma_{i3}^k, \quad 2e_{12}^k = \frac{1}{G_{12}^k}\sigma_{12}^k.$$

The Cauchy relationships are the following formulas

$$e_{ii}^k = u_{i,i}^k, \quad 2e_{12}^k = u_{1,2}^k + u_{2,1}^k. \quad 2e_{i3}^k = u_{i,3}^k + u_{3,i}^k, \quad e_{33}^k = u_{3,3}^k. \tag{4}$$

We determine the transverse shear strain $e_{i3}^k(x_1, x_2, z)$ from Hooke's law (3) by substituting the hypothesis expression for transverse tangential stresses (1)

$$2e_{i3}^k = \psi_{i,3}^k \chi_{,i}. \tag{5}$$

We find normal transverse strains $e_{33}^k(x_1, x_2, z)$ from the last Cauchy relation taking into account (1)

$$e_{33}^k = 0. \tag{6}$$

Integrating the third Cauchy relation (4) with respect to z, as well as using the relations (1) and (5), we obtain formulas for calculating tangential displacements

$$u_i^k = u_i - zW_{,i} + \psi_{i,i}^k \chi_{,i}, \tag{7}$$

where u_i are tangential displacements and $W_{,i}$ are the sought deflection functions of the coordinate surface, depending on the coordinates x_1 and x_2. Normal displacements are considered equal to deflections.

From the conditions for the joint work of the layers of the package

$$u_i^k = u_i^{k-1} (i = 1, 2, 3)$$

and conditions on the layer located in the coordinate surface

$$u_i^m(x_1, x_2, 0) = u(x_1, x_2)$$

we find the distribution function ψ_i^k in the form of the following expression

$$\psi_i^k = \int_{b_{k-1}-\delta_1}^{z} \psi_{i,3}^k dz + \sum_{j=1}^{k-1} \int_{b_{j-1}-\delta_1}^{b_j-\delta_1} \psi_{i,3}^j dz + \int_0^{b_{m-1}-\delta_1} \psi_{i,3}^m dz - \sum_{j=1}^{m-1} \int_{b_{j-1}-\delta_1}^{b_j-\delta_1} \psi_{i,3}^j dz.$$

Tangential deformations are found from the first Cauchy relations (4), substituting expressions for tangential displacements (7) into them. As a result, tangential strains are expressed by the following formulas

$$\begin{aligned} e_{11}^k &= \varepsilon_{11} - zW_{,11} + \psi_1^k \chi_{,11}, \\ e_{22}^k &= \varepsilon_{22} - zW_{,22} + \psi_2^k \chi_{,22}, \\ e_{21}^k &= \varepsilon_{21} - zW_{,21} + 0.5\big(\psi_2^k + \psi_1^k\big)\chi_{,21}. \end{aligned} \tag{8}$$

Taking into account formulas (1) and expressions for tangential strains (8), the stresses of the generalized Hooke's law (3) are found by the formulas [13]

$$\sigma_{11}^k = B_{11}^k\big(\varepsilon_{11} - zW_{,11} + \psi_1^k\chi_{,11}\big) + B_{12}^k\big(\varepsilon_{22} - zW_{,22} + \psi_2^k\chi_{,22}\big) - B_{13}^k\sum_{i=1}^{2}\eta_{3i}^k\chi_{,ii},$$

$$\sigma_{22}^k = B_{22}^k\big(\varepsilon_{22} - zW_{,22} + \psi_2^k\chi_{,22}\big) + B_{12}^k\big(\varepsilon_{11} - zW_{,11} + \psi_1^k\chi_{,11}\big) - B_{13}^k\sum_{i=1}^{2}\eta_{3i}^k\chi_{,ii},$$

$$\sigma_{12}^k = 2G_{12}^k\big[\varepsilon_{22} + 0.5\big(\psi_1^k + \psi_2^k\big)\chi_{,12} - zW_{,12}\big].$$

Based on formulas for calculating displacements (1), (7) and strains (5), (6), (8) it is possible to determine the components of the stress-strain state of the plate at an arbitrary point of the k-th layer.

Using the Lagrange variational principle and the relationships derived taking into account hypotheses (1), we obtain a system of equations for bending plates made of multilayer composite material with orthotropic layers. We notice that the number and arrangement of layers is arbitrary. Then we introduce force functions into the system of equations and obtain this system of equations in a mixed form

$$\begin{aligned} \triangle_F^2\phi + \triangle_{1S}^2 W - \big(\triangle_{2S}^2 - \triangle_{13}^2\big)\chi &= 0, \\ \triangle_{1S}^2\phi + \big(\triangle_{3S}^2 - \triangle_D^2\big)W + \big(\triangle_p^2 - \triangle_{23}^2 - \triangle_{4S}^2\big)\chi &= -q, \\ \triangle_{2S}^2\phi + \big(\triangle_{5S}^2 - \triangle_p^2\big)W + \big(\triangle_{P1}^2 - \triangle_{33}^2 - \triangle_{P3}\big)\chi &= 0. \end{aligned} \tag{9}$$

This system describes the bending of a multilayer plate with an asymmetric thickness structure with orthotropic layers.

The system of resolving equations of a layered plate is presented in a transformed form in [1].

The general order of the system of equations (9) is equal to twelve. The system of equations (9) takes into account the transverse shear and the interaction of layers. The functions of the coordinate plane, namely the force function ϕ, the deflection function W and the shear function χ are the sought functions in the system of equations (9).

There are differential operators in the system of equations (9). $\triangle$ is a second order differential operator, and $\triangle^2$ is a fourth order differential operator. These differential operators have the following form

$$\begin{aligned} \triangle_f^2 &= A_1^*(...)_{,1111} + A_2^*(...)_{,1122} + A_3^*(...)_{,2222}, \\ \triangle_g &= B_1^*(...)_{,11} + B_2^*(...)_{,22}, \end{aligned} \tag{10}$$

where A_j^* $(j = 1, 2, 3)$ and B_i^* $(i = 1, 2)$ are coefficients in equations (10). These coefficients depend on the stiffness of the package layers.

For different values of f and g, the coefficients of the operators take different values, which are shown in Table 1 [14]. When solving the system of equations (9), one should take into account the boundary conditions for fixing the edges of the plate with respect to the force function, the deflection function, and the shear function [12].

3 Numerical calculation

Using the finite difference method, the system of equations (9) and the boundary conditions of the plate were discretized. [3, 15]. The exclusion of unknown functions of the system of equations (9) outside the grid area of the plate is made in a matrix form.

An algorithm for numerical calculating the bending of multilayer composite plates with orthotropic layers, where the number of layers, their structure and arrangement are arbitrary, was developed by the finite difference method. This algorithm is implemented by a software package on a PC. This software package consists of a head program and several subroutines when using the FortRUN programming language.

The flowchart of the head program is divided into several blocks. Each block is autonomous and designed to perform specific functions. For the convenience of performing calculations, all magnitudes with dimensions are determined in a dimensionless form [12].

Below we describe the functions for these blocks of the flowchart of the head program.

In the first block, all the initial data and parameters of the task are introduced. In the second block, the stiffness characteristics are set for a multilayer composite plate with orthotropic layers. In the third block, systems of equilibrium equations for the plate under consideration are compiled and then solved. In the fourth block, the stress-strain state of the multilayer plate is calculated.

Conclusion

Using hypotheses (1), Lagrange's variational principle, the system of equations of the twelfth order is obtained. This system of equations describes the bending for a multilayer plate of an asymmetric structure in thickness with orthotropic layers. Three functions of the coordinate surface are unknown: the function of forces, the function deflection and the function shear.

The boundary conditions consist of two groups of relations. The first group of boundary conditions is similar in form to the conditions of the classical theory of plate bending and describes the boundary conditions for the coordinate plane of the plate [12]. The second group of equations simulates the type of deformation of the end surface for the plate and assumes the presence of various types of diaphragms at the end of the multilayer plate. The combination of the conditions from the two groups makes it possible to obtain various design features on the contour of the plate, i.e. it allows you to vary the boundary conditions on the edges of the plate.

In Table 2 [14], the solutions calculated by the method described above are checked against the results of solutions determined by known methods, namely, with the solution of the classical theory, the exact solution, the solution in trigonometric series, and the error of the solutions is calculated. In Table 3 [14], a comparison of the obtained solution with experimental data for three-layer plates with different plate parameters is presented.

Comparison of the obtained solution by the finite difference method with solutions determined using known methods, as well as with experimental data, shows a sufficiently acceptable accuracy in solving such problems and indicates the reliability of the proposed relations. It is impossible to apply classical theory for the problems under consideration.

It should be noted that when calculating the multilayer plates with orthotropic layers by analytical methods in the most general formulation: with arbitrary boundary conditions (including elastic), different types of load, complex shapes and different sizes of plates, different thickness of layers and different elastic characteristics, etc., we have to face with great mathematical difficulties, and in most cases to obtain an analytical solution of the problem under consideration is not possible. Such problems can be solved by applying a very efficient finite difference method, which gives a sufficiently high accuracy of solutions.

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Бір модификацияланған нақтыланған иілу теориясы негізінде аналитикалық және сандық зерттеу

Мақалада бір модификацияланған нақтыланған иілу теориясы негізінде аналитикалық және сандық зерттеулер жүргізілген. Ақырлы айырмашылықтар әдісі негізінде сандық есептеудің жалпы алгоритмі жасалған. Ұсынылған әдістеме бойынша алынған шешім белгілі шешімдердің нәтижелерімен, атап айтқанда, классикалық теорияның шешімімен, дәл шешіммен, тригонометриялық қатарлардағы шешіммен, сонымен қатар эксперименттік мәліметтермен салыстырылады. Мақалада көрсетілген әдіспен алынған нәтижелерді басқа әдістермен анықталған шешімдермен салыстыру жеткілікті дәлдікті көрсетеді. Бұл иілудің модификацияланған нақтыланған теориясының бір нұсқасы негізінде ұсынылған әдістің сенімділігін дәлелдейді. Қарастырылып отырған есептер үшін классикалық теория қолданылмайды.

Кілт сөздер: модификацияланған нақтыланған иілу теориясы, ақырлы айырмашылықтар әдісі, Лагранж вариациялық принципі, дифференциалдық оператор, теңдеулер жүйесін дискретизациялау.

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Аналитическое и численное исследование на основе одной модифицированной уточненной теории изгиба

В статье проведено аналитическое и численное исследование на основе одной модифицированной уточненной теории изгиба. На основе метода конечных разностей разработан общий алгоритм численного расчета. Полученное по предложенной методике решение сопоставлено с результатами известных решений, а именно с решением классической теории, с точным решением, с решением в тригонометрических рядах, а также с экспериментальными данными. Сравнение результатов, полученных по данной в статье методике, с решениями, определенными другими методами, показывает достаточную точность, что свидетельствует о достоверности предлагаемой методики на основе одного варианта модифицированной уточненной теории изгиба. Классическая теория для рассматриваемых задач не применима.

Ключевые слова: модифицированная уточненная теория изгиба, метод конечных разностей, вариационный принцип Лагранжа, дифференциальный оператор, дискретизация системы уравнений.

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