DOI 10.31489/2022M/34-41

UDC 517.9

A.Kh. Attaev[1], M.I. Ramazanov[2,*], M.T. Omarov[2]

[1] *Institute of Applied Mathematics and Automation of Kabardin-Balkar Scientific Center of RAS, Nalchik, Russia;*
[2] *Karagandy University of the name of academician E.A. Buketov, Karaganda, Kazakhstan*
(E-mail: attaev.anatoly@yandex.ru, ramamur@maill.ru, kazahazavod1997@gmail.com)

On the Correctness of Boundary Value Problems for the Two-Dimensional Loaded Parabolic Equation

The paper studies the problems of the correctness of setting boundary value problems for a loaded parabolic equation. The feature of the problems is that the order of the derivative in the loaded term is less than or equal to the order of the differential part of the equation, and the load point moves according to a nonlinear law. At the same time, the distinctive characteristic is that the line, on which the loaded term is set is at the zero point. On the basis of the study the authors proved the theorems about correctness of the studied boundary value problems.

Keywords: loaded differential equations, parabolic type equations, uniqueness, existence, boundary problem, loading, perturbation.

Introduction

The steadily growing interest in the study of loaded differential equations is explained by the expanding scope of their applications and the fact that loaded equations constitute a special class of equations with their own specific problems. The main questions that arise in the theory of boundary value problems for partial differential equations remain the same for boundary value problems for loaded equations. However, the presence of a loaded operator does not always allow one to apply the well-known theory of boundary value problems for loaded equations without changes. For example, the question of the functional spaces correct choice for solving problems is relevant.

Loaded differential equations are differential equations containing values of the unknown function and its derivatives at some fixed points of the domain or on some manifolds of nonzero measure. General boundary value problems consisting of general boundary conditions and so-called differential boundary equations (loaded differential equations) were studied by many researchers in the last century, for example, the review article by Kraal [1] and the literature cited therein. Recently, there has been renewed interest in the study of these kinds of problems, for instance, [2–6]. Because of their complexity, numerical methods and, in particular, finite difference methods are mainly used to solve these general boundary value problems [7–10].

Loaded differential equations also arise in applied mathematics, where mathematical problems are modeled by simpler ones that are easier to solve. As such an example, let us mention the case of the Fredholm integro-differential equations, where the integral term is replaced by an approximate quadrature rule, leading to loaded differential equations [11]. Then these equations are solved directly or, in most cases, they are discretized using various difference schemes for the derivatives, leading to loaded difference equations or systems of loaded difference equations. This procedure has recently been implemented to solve linear boundary value problems for first order integro-differential Fredholm equations [12].

Boundary value problems for loaded differential equations, in some cases, are correct in natural classes of functions, that is, in this case, the loaded term is interpreted as a weak perturbation. If

*Corresponding author.
E-mail: ramamur@maill.ru

the uniqueness of the solution of the boundary value problem is violated, then the loading can be interpreted as a strong perturbation. It turns out here that the character of the load is a perturbation (weak or strong perturbation) [13–20].

1 *Problem setting*

In the domain

$$\Omega = \{(r,t)\,, r>0, t>0\}$$

consider the following problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1-2\beta}{r}\dfrac{\partial u}{\partial r} - \lambda \left.\dfrac{\partial^k u}{\partial r^k}\right|_{r=t^\alpha} + f(r,t) & (1) \\ u(r,0)=0; u(0,t)=0, & (2) \end{cases}$$

where $0<\beta<1$, $\lambda \in R$ is a spectral parameter, $f(r,t)\in M(\Omega)$ is a given function,

$$f_k(t) = \left.\left(\frac{\partial^k}{\partial r^k}\int_0^t\int_0^\infty \left[\frac{r^\beta \xi^{1-\beta}}{2(t-\tau)}\exp\left[-\frac{r^2+\xi^2}{4(t-\tau)}\right] I_\beta\left(\frac{r\xi}{2(t-\tau)}\right)\right]\cdot f(\xi,\tau)d\xi d\tau\right)\right|_{r=t^\alpha} \in M(0,\infty)$$

$$M(\Omega) = L_\infty(\Omega)\cap C(\Omega), M(0,\infty) = L_\infty(0,\infty)\cap C(0,\infty).$$

The purpose is to determine for which integer values $k=0,1,2$ and for which values $\alpha>0,\ 0<\beta<1$ the problem (1)–(2) will be correct in other words have a unique solution.

2 *Main part*

Remark 1. Obviously, this problem at $\lambda=0$ has a unique solution due to the lack of a loaded term

$$u(r,t) = \int_0^t\int_0^\infty \left[\frac{r^\beta \xi^{1-\beta}}{2(t-\tau)}\exp\left[-\frac{r^2+\xi^2}{4(t-\tau)}\right] I_\beta\left(\frac{r\xi}{2(t-\tau)}\right)\right]\cdot f(\xi,\tau)d\xi d\tau).$$

By means of this solution, we invert the differentiated part of the problem. In order to invert it, we transfer the loaded term $\lambda\frac{\partial^k u}{\partial r^k}\Big|_{r=t^\alpha}$ to the right-hand side and consider it temporarily known, we obtain

$$u(r,t) = \lambda\int_0^t \left.\frac{\partial^k u}{\partial \xi^k}\right|_{\xi=\tau^\alpha} d\tau \int_0^\infty \left[\frac{r^\beta \xi^{1-\beta}}{2(t-\tau)}\exp\left[-\frac{r^2+\xi^2}{4(t-\tau)}\right] I_\beta\left(\frac{r\xi}{2(t-\tau)}\right)\right] d\xi + f_0(r,t), \qquad (3)$$

where

$$f_0(r,t) = \int_0^t\int_0^\infty \left[\frac{r^\beta \xi^{1-\beta}}{2(t-\tau)}\exp\left[-\frac{r^2+\xi^2}{4(t-\tau)}\right] I_\beta\left(\frac{r\xi}{2(t-\tau)}\right)\right]\cdot f(\xi,\tau)d\xi d\tau).$$

In equality (3) we calculate the inner integral:

$$Q(r,t-\tau) = \int_0^\infty \left[\frac{r^\beta \xi^{1-\beta}}{2(t-\tau)}\exp\left[-\frac{r^2+\xi^2}{4(t-\tau)}\right] I_\beta\left(\frac{r\xi}{2(t-\tau)}\right)\right] d\xi =$$
$$= \frac{r^\beta}{2(t-\tau)}\exp\left[-\frac{r^2}{4(t-\tau)}\right]\int_0^\infty \xi^{1-\beta}\exp\left[-\frac{\xi^2}{4(t-\tau)}\right].$$

Using the following substitution

$$\frac{r}{2(t-\tau)}\xi = \eta; \qquad \xi = \frac{2(t-\tau)}{r}\eta; \qquad d\xi = \frac{2(t-\tau)}{r}d\eta$$

we obtain

$$Q(r,t-\tau) = \frac{r^\beta}{2(t-\tau)} \exp\left[-\frac{r^2}{4(t-\tau)}\right] \int_0^\infty \frac{2^{1-\beta}(t-\tau)^{1-\beta}}{r^{1-\beta}} \eta^{1-\beta} \exp\left[-\frac{t-\tau}{r^2}\eta^2\right] I_\beta(\eta) \cdot \frac{2(t-\tau)}{r} d\eta =$$
$$= \exp\left[-\frac{r^2}{4(t-\tau)}\right] \frac{2^{1-\beta}(t-\tau)^{1-\beta}}{r^{2-2\beta}} \int_0^\infty \exp\left[-\frac{t-\tau}{r^2}\eta^2\right] \eta^{1-\beta} I_\beta(\eta) d\eta =$$
$$= \exp\left[-\frac{r^2}{4(t-\tau)}\right] \frac{2^{1-\beta}(t-\tau)^{1-\beta}}{r^{2-2\beta}} \cdot \exp\left[\frac{r^2}{4(t-\tau)}\right] \frac{2^{\beta-1}(t-\tau)^{\beta-1}}{\Gamma(\beta) r^{2\beta-2}} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right) =$$
$$= \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right),$$

where $\gamma(\beta, z)$ is an incomplete gamma function.

Therefore, we are able to express $Q(r, t-\tau)$ in the following way:

$$Q(r,t-\tau) = \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right).$$

Then the integral representation of solution (3) takes the form

$$u(r,t) = \lambda \int_0^t \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right) \cdot \left.\frac{\partial^k u}{\partial \xi^k}\right|_{\xi=\tau^\alpha} d\tau + f_0(r,t) \tag{4}$$

it is apparent from (4) that in order to find a solution to problem (1)–(2), it is sufficient to find the value of the loaded term $\left.\frac{\partial^k u}{\partial r^k}\right|_{r=t^\alpha}$.

I. Let $k = 0$ then relation (4) takes the following form

$$u(r,t) = \lambda \int_0^t \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right) \cdot u(\xi,\tau)|_{\xi=\tau^\alpha} d\tau + f_0(r,t). \tag{5}$$

Assuming $r = t^\alpha$ in both parts of the equality (5) and introducing the notation $\mu_0(t) = u(r,t)|_{r=t^\alpha}$ we obtain the following integral equation with respect to the unknown function:

$$\mu_0(t) = \lambda \int_0^t K_0(t,\tau) \cdot \mu_0(\tau) d\tau + f_0(t),$$

where

$$K_0(t,\tau) = \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right), f_0(t) = f_0(r,t)|_{r=t^\alpha},$$

which solution $\forall\lambda \in R$, $\forall\alpha > 0$, $\forall f_0(t) \in M(0,\infty)$ can be found by the method of successive approximations. Here we take into account that $K_0(t,\tau) \le 1$, and is continuous $\forall(t,\tau)$, $0 < \tau < t$. This implies that problem (1)–(2) has a unique solution.

Theorem 1. For $k = 0$ and $\forall\lambda \in R$, $\forall\alpha > 0$, $\forall f_0(t) \in M(0,\infty)$ the boundary value problem (1)–(2) has a unique solution.

II. Let us assume that $k = 1$. Then (5) takes the following form:

$$u(r,t) = \lambda \int_0^t \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right) \cdot \left.\frac{\partial u}{\partial \xi}\right|_{\xi=\tau^\alpha} d\tau + f_1(r,t).$$

In order to determine the loaded term $\frac{\partial u}{\partial \xi}\Big|_{\xi=\tau^\alpha}$ considering it is possible, we differentiate both parts of this equality by the variable r and take following consideration $r = t^\alpha$. Yet it would be convenient to calculate beforehand following:

$$\frac{\partial}{\partial r}\gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right)\Bigg|_{r=t^\alpha} = \frac{1}{\Gamma(\beta)} \cdot \left[\frac{r^2}{4(t-\tau)}\right]^{\beta-1} \cdot \frac{r}{2(t-\tau)} \exp\left[-\frac{r^2}{4(t-\tau)}\right]\Bigg|_{r=t^\alpha} =$$

$$= \frac{1}{\Gamma(\beta)} \frac{1}{2^{2\beta-1}} \cdot \frac{t^{(2\beta-1)\alpha}}{(t-\tau)^\beta} \cdot \exp\left[-\frac{t^{2\alpha}}{4(t-\tau)}\right].$$

By denoting $\mu_1(t) = \frac{\partial u}{\partial r}\big|_r = t^\alpha$, we obtain the following integral equation with respect to the unknown function $\mu_1(t)$

$$\mu_1(t) = \lambda \int_0^t K_1(t,\tau) \cdot \mu_1(\tau) + f_1(t), \tag{6}$$

where

$$K_1(t,\tau) = \frac{1}{\Gamma(\beta)} \frac{1}{2^{2\beta-1}} \cdot \frac{t^{(2\beta-1)\alpha}}{(t-\tau)^\beta} \cdot \exp\left[-\frac{t^{2\alpha}}{4(t-\tau)}\right],$$

$$f_1(t) = \frac{\partial}{\partial r} f_1(r,t)\big|_{r=t^\alpha}.$$

If $0 < \beta \leq 1/2$, then the kernel $K_1(t,\tau)$ has a weak singularity $\forall \alpha > 0$, yet if $1/2 < \beta < 1$, then in order to have a unique solution the condition must be satisfied for the integral equation (6):

$$0 < \alpha < \frac{2-\beta}{1-2\beta}.$$

Consequently, the theorem is valid.

Theorem 2. If $k = 1$, then for $0 < \beta \leq 1/2$ and $\forall \lambda \in R$, $\forall \alpha > 0$, $\forall f_1(t) \in M(0,\infty)$ the boundary value problem (1)–(2) has a unique solution $u(r,t) \in M(\Omega)$. As for $1/2 < \beta < 1$ in order for the boundary value problem (1)–(2) to have a unique solution $u(r,t) \in M(\Omega)$ the following condition must be satisfied:

$$0 < \alpha < \frac{2-\beta}{1-2\beta}.$$

Remark 2. Thus, for $k = 0$ and for $k = 1$ under the conditions of Theorem 2 the loaded term $\frac{\partial u}{\partial \xi}\Big|_{\xi=\tau^\alpha}$ in equation (1) of problem (1)–(2) can be interpreted as weak perturbation.

III. Let us assume that $k = 2$. Then (5) takes the following form:

$$u(r,t) = \lambda \int_0^t \frac{1}{\Gamma(\beta)} \cdot \gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right) \cdot \frac{\partial^2 u}{\partial \xi^2}\Bigg|_{\xi=\tau^\alpha} d\tau + f_2(r,t). \tag{7}$$

In order to determine the loaded term $\frac{\partial^2 u}{\partial \xi^2}\Big|_{\xi=\tau^\alpha}$ let us differentiate both parts of equality (7) twice with respect to the variable r and take following consideration $r = t^\alpha$. Yet it would be convenient to calculate beforehand next expression:

$$\frac{\partial^2}{\partial r^2}\gamma\left(\beta, \frac{r^2}{4(t-\tau)}\right)\Bigg|_{r=t^\alpha} = \frac{2\beta-1}{2^{2\beta-1}\Gamma(\beta)} \cdot \frac{r^{2\beta-2}}{(t-\tau)^\beta} \cdot \exp\left[-\frac{r^2}{4(t-\tau)}\right]\Bigg|_{r=t^\alpha} -$$

$$- \frac{1}{2^{2\beta}\Gamma\beta} \cdot \frac{r^{2\beta}}{(t-\tau)^{\beta+1}} \exp\left[-\frac{r^2}{4(t-\tau)}\right]\Bigg|_{r=t^\alpha} =$$

$$= \frac{1}{\Gamma(\beta)}\left\{\frac{2\beta-1}{2^{2\beta-1}} \cdot \frac{1}{2^{2\alpha(1-\beta)}(t-\tau)^{\beta+1}} - \frac{1}{2^{2\beta}} \cdot \frac{t^{2\alpha\beta}}{(t-\tau)^{\beta+1}}\right\} \cdot \exp\left[-\frac{t^{2\alpha}}{4(t-\tau)}\right].$$

By denoting

$$\mu_1(t) = \left.\frac{\partial^2 u}{\partial r^2}\right|_r = t^\alpha,$$

we obtain the following integral equation with respect to the unknown function $\mu_2(t)$:

$$\mu_2(t) - \lambda \int_0^t K_2(t,\tau) \cdot \mu_2(\tau) = f_2(t), \tag{8}$$

where

$$K_2(t,\tau) = \frac{1}{\Gamma(\beta)} \left\{ \frac{1}{2^{2\beta}} \cdot \frac{t^{2\alpha\beta}}{(t-\tau)^{\beta+1}} - \frac{2\beta-1}{2^{2\beta-1}} \cdot \frac{1}{2^{2\alpha(1-\beta)}(t-\tau)^{\beta+1}} \right\} \cdot \exp\left[-\frac{t^{2\alpha}}{4(t-\tau)}\right].$$

$$f_2(t) = \left.\frac{\partial^2}{\partial r^2} f_0(r,t)\right|_{r=t^\alpha}.$$

Let us study the kernel $K_2(t,\tau)$ of this equation. Initially, we calculate the following integral:

$$\int_0^t K_2(t,\tau) = \frac{1}{\Gamma(\beta)}\Gamma\left(\beta, \frac{t^{2\alpha-1}}{4}\right) - \frac{2\beta-1}{2\Gamma(\beta)}\Gamma\left(\beta-1, \frac{t^{2\alpha-1}}{4}\right). \tag{9}$$

Further, using the following equality:

$$\Gamma(\alpha+1, x) = \alpha\Gamma(\alpha, x) + x^\alpha e^{-x},$$

we obtain

$$\Gamma\left(\beta-1, \frac{t^{2\alpha-1}}{4}\right) = -\frac{1}{1-\beta}\Gamma\left(\beta, \frac{t^{2\alpha-1}}{4}\right) + \frac{1}{1-\beta}\frac{4^{1-\beta}t^{1-\beta}}{t^{2\alpha(1-\beta)}} \cdot \exp\left[-\frac{t^{2\alpha-1}}{4}\right].$$

Hence it follows that

$$\begin{aligned}\int_0^t K_2(t,\tau) =& \frac{1}{\Gamma(\beta)}\Gamma\left(\beta, \frac{t^{2\alpha-1}}{4}\right) + \\ &+ \frac{2\beta-1}{2\Gamma(\beta)}\frac{1}{1-\beta}\Gamma\left(\beta, \frac{t^{2\alpha-1}}{4}\right) - \\ &- \frac{2\beta-1}{2\Gamma(\beta)}\frac{4^{1-\beta}}{1-\beta} \cdot t^{(1-2\alpha)(1-\beta)} \cdot \exp\left[-\frac{t^{2\alpha-1}}{4}\right] = \\ &= \frac{\Gamma\left(\beta, \frac{t^{2\alpha-1}}{4}\right)}{\Gamma(\beta)}\left[1 + \frac{2\beta-1}{2(1-\beta)}\right] - \frac{2\beta-1}{2^{2\beta-2}(1-\beta)}\frac{1}{\Gamma(\beta)} \cdot t^{(1-2\alpha)(1-\beta)} \exp\left[-\frac{t^{2\alpha-1}}{4}\right].\end{aligned} \tag{10}$$

This implies that the inhomogeneous integral equation (8) has a unique solution $\forall\beta \in (0,1)$ if the condition $\alpha \in \left(0, \frac{1}{2}\right)$ is satisfied.

Theorem 3. If $k = 2$, then for the condition $0 < \alpha < 1/2$ the boundary value problem (1)–(2) has a unique solution $u(r,t) \in M(\Omega)$ for $\forall\lambda \in R,\ \forall\beta \in (0,1),\ \forall f_2(t) \in M(0,\infty)$.

3 Conclusion

From (9)–(10) it follows that for $\beta = 1/2, \alpha > 1/2$ we have the following equality:

$$\lim_{t\to 0}\int_0^t K_2(t,\tau)d\tau = 1.$$

In summation, this implies that the Volterra type integral equation of the second kind (8) cannot be solved by the method of successive approximations. Moreover, the corresponding homogeneous integral equation for $\lambda \geq 1$ will have nonzero solutions, thus the inhomogeneous integral equation has a non-unique solution. Then from relation (3) it will follow that the boundary value problem (1)–(2) will be incorrect, since it has a non-unique solution.

As noted in [21–24], the corresponding boundary problems may turn out to be Noetherians with both positive and negative indices. Further investigations of boundary problems of type (1)–(2) for different laws of motion of the load point will be continued.

Acknowledgments

The research is funded by the Science Committee of the Ministry of Higher Education and Science of the Republic of Kazakhstan (Grants no. AP09259780, 2021-2023, Grants no. AP09258892, 2021-2023).

References

1 Kraal, A.M. (1975). The development of general differential and general differential-boundary systems. *Rocky Mountain Journal of Mathematics, 5*(4). https://doi.org/10.1216/rmj-1975-5-4-493.

2 Nakhushev, A.M. (1983). Loaded equations and their applications. *Differ. Uravn., 19*(1), 86–94.

3 Amangalieva, M.M., Akhmanova, D.M., Dzhenaliev M.T., & Ramazanov, M.I. (2011). Boundary value problems for a spectrally loaded heat operator with load line approaching the time axis at zero or infinity. *Diff. Equat., 47,* 231–243. https://doi.org/10.1134/ S0012266111020091.

4 Baltayeva, U.I., & Islomov, B.I. (2011). Boundary value problems for the loaded third order equations of the hyperbolic and mixed types. *Ufa Math. J., 3,* 15–25.

5 Fedorov, V.E., & Borel, L.V. (2015). Solvablity of loaded linear evolution equations with a degenerate operator at the derivative. *St. Petersburg Math. J., 26,* 487–497.

6 Lomov, I.S. (2014). Loaded differential operators: Convergence of spectral expansions. *Diff. Equat., 50,* 1070–1079. https://doi.org/10.1134/S0012266114080060.

7 Abdullayev, V.M., & Aida-zade, K.R. (2014). Numerical method of solution to loaded nonlocal boundary value problems for ordinary differential equations. *Computational Mathematics and Mathematical Physics, 54,* 1096–1109. https://doi:10.1134/S0965542514070021.

8 Abdullayev, V.M., & Aida-zade, K.R. (2016). Finite-difference methods for solving loaded parabolic equations. *Computational Mathematics and Mathematical Physics, 56*(1), 93–105. https://doi.org/10.1134/s0965542516010036.

9 Alikhanov, A.A., Berezgov, A.M., & Shkhanukov-Lafishev, M.X. (2008). Boundary value problems for certain classes of loaded differential equations and solving them by finite difference methods. *Computational Mathematics and Mathematical Physics, 48,* 1581–1590. https://doi:10.1134/S096554250809008X.

10 Bondarev, E.A., & Voevodin, A.F. (2000). A finite-difference method for solving initial-boundary value problems for loaded differential and integro-differential equations. *Diff. Equat., 36,* 1711–1714. https://doi:10.1007/BF02757374.

11 Nakhushev, A.M. (1982). An approximate method for solving boundary value problems for differential equations and its application to the dynamics of ground moisture and ground water. *Differ. Uravn., 18,* 72–81.

12 Dzhumabaev, D. (2017). Computational methods of solving the boundary value problems for the loaded differential and Fredholm Integro-differential equations. *Mathematical Methods in the Applied Sciences, 41*(4), 1439–1462. https://doi.org/10.1002/mma.4674.

13 Kheloufi, A., & Sadallah, B.K. (2011). On the regularity of the heat equation solution in non-cylindrical domains: Two approaches. *Applied Mathematics and Computation, 218*(5), 1623–1633. https://doi.org/10.1016/j.amc.2011.06.042.

14 Kheloufi, A. (2013). Existence and uniqueness results for parabolic equations with Robin type boundary conditions in a non-regular domain of R3. *Applied Mathematics and Computation, 220,* 756–769. https://doi.org/10.1016/j.amc.2013.07.027.

15 Cherfaoui, S., Kessab, A., & Kheloufi, A. (2017). Well-posedness and regularity results for a 2m-th order parabolic equation in symmetric conical domains of RN+1. *Mathematical Methods in the Applied Sciences, 40*(16), 6035–6047. https://doi.org/10.1002/mma.4451.

16 Kheloufi, A. (2014). On a fourth order parabolic equation in a nonregular domain of R3. *Mediterranean Journal of Mathematics, 12*(3), 803–820. https://doi.org/10.1007/s00009-014-0429-7.

17 Kheloufi, A., & Sadallah, B.-K. (2014). Study of the heat equation in a symmetric conical type domain of RN+1. *Mathematical Methods in the Applied Sciences, 37*(12), 1807–1818. https://doi.org/10.1002/mma.2936.

18 Chapko, R., Johansson, B.T., & Vavrychuk, V. (2014). Numerical solution of parabolic cauchy problems in planar corner domains. *Mathematics and Computers in Simulation, 101*, 1–12. https://doi.org/10.1016/j.matcom.2014.03.001.

19 Wang, Y., Huang, J., & Wen, X. (2021). Two-dimensional euler polynomials solutions of two-dimensional Volterra integral equations of fractional order. *Applied Numerical Mathematics, 163,* 77–95. https://doi.org/10.1016/j.apnum.2021.01.007.

20 Dehbozorgi, R., & Nedaiasl, K. (2021). Numerical solution of nonlinear weakly singular Volterra integral equations of the first kind: An HP-version collocation approach. *Applied Numerical Mathematics, 161,* 111–136. https://doi.org/10.1016/j.apnum.2020.10.030.

21 Ramazanov, M.I., Kosmakova, M.T., & Tuleutaeva, Zh.M. (2021). On the Solvability of the Dirichlet Problem for the Heat Equation in a Degenerating Domain. *Lobachevskii Journal of Mathematics, 42*(15), 3715–3725.

22 Ramazanov, M.I., & Gulmanov, N.K. (2021). Solution of two dimensional boundary value problem of heat conduction in a degenerating domain. *Bulletin of the Karaganda University. Mathematics series, 3*(111), 65–78.

23 Jenaliyev, M.T., Kosmakova, M.T., Tanin, A.O. & Ramazanov, M.I. (2019). To the solution of one pseudo-Volterra integral equation. *Bulletin of the Karaganda University. Mathematics series, 1*(93), 19–30

24 Ramazanov, M.I., Kosmakova, M.T., Romanovsky, V.G., Zhanbusinova, B.H., & Tuleutaeva, Zh.M. (2018) Boundary value problems for essentially-loaded parabolic equation. *Bulletin of the Karaganda University. Mathematics series, 4*(92), 79–86.

А.Х. Аттаев[1], М.И. Рамазанов[2], М.Т. Омаров[2]

[1] *РҒА Кабардин-Балқар ғылыми орталығының Қолданбалы математика және автоматика институты, Нальчик, Ресей;*
[2] *Академик Е.А.Бөкетов атындағы Қарағанды университеті, Қарағанды, Қазақстан*

Екі өлшемді жүктелген параболалық теңдеу үшін шекаралық есептердің дұрыс қойылуы

Мақалада жүктелген параболалық теңдеу үшін шекаралық есептерді дұрыс қоюдың сұрақтары зерттелген. Есептердің ерекшелігі - жүктелген мүшедегі туындының реті теңдеудің дифференциалдық бөлігінің ретінен кіші және оған тең, ал жүктеме нүктесі сызықты емес заң бойынша қозғалады. Бұл жағдайда ерекшеленетін белгі — жүктелген термин көрсетілген қарастырылатын сызық нөлдік нүктеде орналасқан. Зерттеу негізінде авторлар зерттелетін шекаралық есептердің дұрыс қойылғандығы туралы теоремаларды дәлелдеді.

Кілт сөздер: жүктелген дифференциалдық теңдеулер, параболалық типті теңдеулер, бірегейлік, бар болу, шекаралық есеп, жүктеме, ауытқу.

А.Х. Аттаев[1], М.И. Рамазанов[2], М.Т. Омаров[2]

[1] *Институт прикладной математики и автоматизации Кабардино-Балкарского научного центра РАН, Нальчик, Россия;*
[2] *Карагандинский университет имени академика Е.А. Букетова, Караганда, Казахстан*

О корректности краевых задач для двумерного нагруженного параболического уравнения

В статье исследованы вопросы корректности постановок краевых задач для нагруженного параболического уравнения. Особенностью задач является то, что порядок производной в нагруженном слагаемом меньше и равен порядку дифференциальной части уравнения, и при этом точка нагрузки движется по нелинейному закону. Кроме того, отличительной чертой является то, что рассматриваемая линия, на которой задается нагруженное слагаемое, расположена в точке нуль. На основе исследования авторы доказали теоремы о корректности исследуемых краевых задач.

Ключевые слова: нагруженные дифференциальные уравнения, уравнения параболического типа, единственность, существование, граничная задача, нагрузка, возмущение.