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Boundary control problem for a hyperbolic equation loaded along one of its characteristics

This paper investigates the unique solvability of the boundary control problem for a one-dimensional wave equation loaded along one of its characteristic curves in terms of a regular solution. The solution method is based on an analogue of the d'Alembert formula constructed for this equation. We point out that the domain of definition for the solution of DE, when the initial and final Cauchy data given on intervals of the same length is a square. The side of the squire is equal to the interval length. The boundary controls are established by the components of an analogue of the d'Alembert formula, which, in turn, are uniquely established by the initial and final Cauchy data. It should be noted that the normalized distribution and centering are employed in the final formulas of sought boundary controls, which is not typical for initial and boundary value problems initiated by equations of hyperbolic type.

Keywords: hyperbolic equation, distributed oscillatory system, damping problem, gas/liquid flows, loaded equation, initial conditions, boundary conditions, analogue of the d'Alembert formula, boundary controls, normal distribution, distribution function.

Introduction

Let an oscillatory system be described by equation

$$u_{xx} - u_{tt} = \lambda\, u\left(\frac{x+t}{2},\, \frac{x+t}{2}\right) \tag{1}$$

with the following initial and boundary conditions

$$u(x,\, 0) = \varphi_0(x), \quad u_t(x,\, 0) = \psi_0(x), \qquad 0 \leqslant x \leqslant l, \tag{2}$$

$$u(0,\, t) = \mu(t), \quad u(l,\, t) = \nu(t), \qquad 0 \leqslant t \leqslant T, \tag{3}$$

where λ is an arbitrary real number.

The boundary control problem involves searching admissible boundary values $\mu(t)$ and $\nu(t)$, that in a minimum time interval move the oscillatory system from the initial (2) to a predetermined final phase

$$u(x,\, T) = \varphi_1(x), \quad u_t(x,\, T) = \psi_1(x), \qquad 0 \leqslant x \leqslant l. \tag{4}$$

Control (1) belongs to the class of loaded differential equations [1]. The point $\left(\frac{x_0+t_0}{2},\, \frac{x_0+t_0}{2}\right)$ lies on the characteristic curve $x - y = 0$ of equation (1), for an arbitrary point $(x_0,\, t_0) \in \mathbb{R}^2$. The point $(x_0,\, t_0)$ also moves to the point $\left(\frac{x_0+t_0}{2},\, \frac{x_0+t_0}{2}\right)$ along the characteristic curve $x + y = x_0 + y_0$ of (1). The boundary control problem for the equation (1) with the right-hand side of the form $\lambda\, u_{tt}(x_0,\, t)$, which, according to [2], is called an essentially loaded equation, was studied in [3], [4]. Boundary value problems for hyperbolic equations with a load along one of the characteristic curves are studied in [5], [6]. For $\lambda = 0$, the formulated problem is fully investigated in [7]. Here important special cases

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of the same problem were investigated, namely, the problem of complete oscillation excitation and damping. The boundary control problem for equation (1) for $\lambda = 0$ with various nonlocal, including integral form conditions, were studied in [8–12].

A number of specific formulations for distributed control problems, with oscillatory nature of the movement, are described in detail in the monograph [13], and moreover various solution methods are proposed. For example, the damping fluid flow pulsation problem in automated long main pipelines design and pipeline irrigation systems [14], [15]. As earliest works devoted to the study of boundary control problems (1) at $\lambda = 0$ the woks [16–18] are worth mentioning.

There are following main results in the work:

1. Necessary and sufficient conditions are established for the functions $\varphi_0(x)$, $\psi_0(x)$, $\varphi_1(x)$, $\psi_1(x)$, which ensure the existence of the desired boundary values.

$$\psi_0(l) + \varphi_0'(l) - \psi_1(0) - \varphi_1'(0) = 0, \tag{5}$$

$$\psi_0(0) - \varphi_0'(0) - \psi_1(l) + \varphi_1'(l) = 0, \tag{6}$$

$$\lambda\left[\psi_0(l) + \varphi_0'(l) + \psi_1(0) + \varphi_1'(0)\right] = 0, \tag{7}$$

$$\varphi_0''(l) + \psi_0'(l) - \varphi_1''(0) - \varphi_1'(0) = 0, \tag{8}$$

$$\varphi_0''(0) - \psi_0'(0) - \varphi_1''(l) + \varphi_1'(l) = 0, \tag{9}$$

$$\varphi_0(0) - \varphi_1(l) + [\varphi_0(l) - \varphi_1(0)]\, e^{\frac{\lambda\, l^2}{8}} +$$
$$+\frac{1}{2}\int_0^l e^{\frac{\lambda\, t^2}{8}}\,[\psi_0(t) - \psi_1(l-t)]\,dt - \frac{\lambda}{4}\int_0^l t\, e^{\frac{\lambda\, t^2}{8}}\,[\varphi_0(t) - \varphi_1(l-t)]\,dt = 0. \tag{10}$$

2. Under conditions (5)–(10), an explicit analytical form of the sought boundary controls is found

$$\mu(t) = \varphi_1(l-t) + F_1(t,\,\lambda) - \frac{\lambda}{4}\,t\,F_2(t,\,\lambda), \tag{11}$$

$$\nu(t) = \varphi_0(l-t) + F_1(l-t,\,\lambda) - \frac{\lambda}{4}\,t\,F_2(l-t,\,\lambda), \tag{12}$$

where

$$F_1(t,\,\lambda) = \frac{1}{2}\,e^{-\frac{\lambda\, t^2}{8}}\,[\varphi_0(0) - \varphi_1(l)] + \frac{1}{2}\,[\varphi_0(t) - \varphi_1(l-t)] -$$
$$-\frac{\lambda}{8}\int_0^t \xi\, e^{\frac{\lambda\left(t^2-\xi^2\right)}{8}}\,[\varphi_0(\xi) - \varphi_1(l-\xi)]\,d\xi - \frac{1}{8}\int_0^t e^{\frac{\lambda\left(t^2-\xi^2\right)}{8}}\,[\psi_0(\xi) - \psi_1(l-\xi)]\,d\xi,$$

$$F_2(t,\,\lambda) = \sqrt{\frac{2\,\pi}{\lambda}}\,\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)[\varphi_0(0) - \varphi_1(l)] +$$
$$+\frac{1}{2}\int_0^t [\varphi_0(\xi) - \varphi_1(l-\xi)]\,d\xi -$$
$$-\sqrt{\frac{\lambda\,\pi}{8}}\int_0^t \left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right) - \Phi_0\left(\frac{\sqrt{\lambda}}{2}\,\xi\right)\right]\xi\, e^{\frac{\lambda \xi^2}{8}}\,[\varphi_0(\xi) - \varphi_1(l-\xi)]\,d\xi +$$

$$+\sqrt{\frac{2\,\pi}{\lambda}}\int_0^t\left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)-\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,\xi\right)\right]e^{\frac{\lambda\xi^2}{8}}\,[\psi_0(\xi)-\psi_1(l-\xi)]\,d\xi,$$

where $\Phi_0(x)=\frac{1}{\sqrt{2\,\pi}}\int_0^x e^{-\frac{\xi^2}{2}}\,d\xi$ is the distribution function for normalizing and centering processes.

1 Main part

1.1 An analogue of the d'Alembert formula

Using the characteristic variables $\xi=x-t$, $\eta=x+t$ equation (1) is written out as

$$v_{\xi\eta}=\frac{\lambda}{4}\,v(0,\,\eta),\tag{13}$$

where $v(\xi,\,\eta)=u\Big(\frac{\xi+\eta}{2},\,\frac{\eta-\xi}{2}\Big)$.

Any solution to equation (13) is a solution to the following loaded integral equation.

$$v(\xi,\,\eta)-\frac{\lambda}{4}\,\xi\int_a^\eta v(0,\,t)\,dt=f(\xi)+g(\eta),\tag{14}$$

where $f(\xi)$ and $g(\eta)$ are arbitrary twice continuously differentiable functions. Since for $\xi=0$ by (14) it follows that $v(0,\,\eta)=f(0)+g(\eta)$, then solution (14) takes the form

$$v(\xi,\,\eta)=f(\xi)+g(\eta)+\frac{\lambda}{4}\,\xi\int_a^\eta[f(0)+g(t)]\,dt.$$

Replacing $P(\eta)=\int_a^\eta[f(0)+g(t)]\,dt$ in the last formula and then renaming $P(\eta)$ by $g(\eta)$, we obtain

$$v(\xi,\,\eta)=f(\xi)-f(0)+g'(\eta)+\frac{\lambda}{4}\,\xi\,g(\eta).$$

Or when using the old coordinates, get

$$u(x,\,t)=f(x-t)-f(0)+g'(x+t)+\frac{\lambda}{4}\,(x-t)\,g(x+t).\tag{15}$$

We call formula (15) an analogue of the d'Alembert formula for equation (1).

Formula (15) below is more convenient for further use. It should be noted that in (15) the functions $f(t)$ and $g'(t)$ are twice continuously differentiable.

1.2 Searching algorithm for $\mu(t)$ and $\nu(t)$

Before searching for boundary controls, let us remember that when $\lambda=0$ the minimum time t required for the desired control is uniquely equal to l. As noted in [12], this time is determined by the characteristics of the initial equation that simulates the oscillation process. From a mathematical point of view, problem (2), (4) for equation (1) with $\lambda=0$ in the rectangle $(0,\,l)\times(0,\,T)$ has a unique solution if and only if $T=l$. If $T<l$, the problem is overdetermined, $T>l$, the problem is underdetermined. In our case when $\lambda\neq0$ the equation commands the condition $T=l$. This is due to the fact that for any arbitrary point $(x,\,t)\in(0,\,l)\times(0,\,T)$, belonging to the point $\big(\frac{x+t}{2},\,\frac{x+t}{2}\big)\in(0,\,l)\times(0,\,T)$ is possible if and only if $T=l$. Therefore, further we assume that $T=l$.

We introduce the following equation for the functions $f(x)$ and $g(x)$ in formula (15).

$$f(x)=\begin{cases}f_0(x), & x\in[0,\,l]\,,\\ f_1(x), & x\in[-l,\,0]\,,\end{cases}\tag{16}$$

$$g(x)=\begin{cases} g_0(x), & x\in[0,\,l]\,,\\ g_1(x), & x\in[l,\,2\,l]\,. \end{cases} \tag{17}$$

We call the function $u(x,\,t)\in C^1\left(\overline{\Omega}\right)\cap C^2\left(\Omega\right)$ a regular solution to equation (1) where $\Omega=(0,\,l)\times(0,\,l)$.

Letting (15) satisfy (3) and taking into account (16), (17), we obtain

$$\mu(t)=f_1(-t)-f_0(0)+g_0'(t)-\frac{\lambda}{4}\,t\,g_0(t),\quad t\in[0,\,l]\,, \tag{18}$$

$$\nu(t)=f_0(l-t)-f_0(0)+g_0'(l+t)+\frac{\lambda}{4}\,(l-t)\,g_0(l+t),\quad t\in[0,\,l]\,. \tag{19}$$

Letting (15) satisfy (2) and (4) for $T=l$ we obtain

$$f_0(x)-f_0(0)+g_0'(x)+\frac{\lambda}{4}\,x\,g_0(x)=\varphi_0(x), \tag{20}$$

$$-f_0'(x)+g_0''(x)-\frac{\lambda}{4}g_0(x)+\frac{\lambda}{4}\,x\,g_0'(x)=\psi_0(x), \tag{21}$$

$$f_1(x-l)-f_0(0)+g_1'(x+l)+\frac{\lambda}{4}\,(x-l)\,g_1(x+l)=\varphi_1(x), \tag{22}$$

$$-f_1'(x-l)+g_1''(x+l)-\frac{\lambda}{4}\,g_1(x+l)+\frac{\lambda}{4}\,(x-l)\,g_1'(x+l)=\psi_1(x), \tag{23}$$

where $x\in[0,\,l]$.

Differentiating (20), (22), subtracting and adding term by term with (21), (23), respectively, obtain

$$\begin{gathered} f_0'(x)+\frac{\lambda}{4}\,g_0(x)=\frac{1}{2}\,\varphi_0'(x)-\frac{1}{2}\,\psi_0(x),\\ g_0''(x)+\frac{\lambda}{4}\,x\,g_0'(x)=\frac{1}{2}\,\varphi_0'(x)+\frac{1}{2}\,\psi_0(x), \end{gathered} \tag{24}$$

$$\begin{gathered} f_1'(x-l)+\frac{\lambda}{4}\,g_1(x+l)=\frac{1}{2}\,\varphi_1'(x)-\frac{1}{2}\,\psi_1(x),\\ g_1''(x+l)+\frac{\lambda}{4}\,(x-l)\,g_1'(x+l)=\frac{1}{2}\,\varphi_1'(x)+\frac{1}{2}\,\psi_1(x). \end{gathered} \tag{25}$$

Employing the second equation of (24) we obtain

$$\left[g_0'(x)\,e^{\frac{\lambda\,x^2}{8}}\right]'=\frac{1}{2}\,e^{\frac{\lambda\,x^2}{8}}\left[\varphi_0'(x)+\psi_0(x)\right],$$

$$g_0'(x)=g_0'(0)\,e^{-\frac{\lambda\,x^2}{8}}+\frac{1}{2}\,e^{-\frac{\lambda\,x^2}{8}}\int_0^x e^{\frac{\lambda\,t^2}{8}}\,\varphi_0'(t)\,dt+\frac{1}{2}\,e^{-\frac{\lambda\,x^2}{8}}\int_0^x e^{\frac{\lambda\,t^2}{8}}\,\psi_0(t)\,dt.$$

Using integration by parts for the first integral and taking into account that $g_0'(0)=\varphi_0(0)$, obtain

$$g_0'(x)=\frac{1}{2}\,\varphi_0(0)\,e^{-\frac{\lambda\,x^2}{8}}+\frac{1}{2}\,\varphi_0(x)-\frac{\lambda}{8}\,e^{-\frac{\lambda\,x^2}{8}}\int_0^x t\,e^{\frac{\lambda\,t^2}{8}}\,\varphi_0(t)\,dt+\frac{1}{2}\,e^{-\frac{\lambda\,x^2}{8}}\int_0^x e^{\frac{\lambda\,t^2}{8}}\,\psi_0(t)\,dt. \tag{26}$$

Hence

$$g_0(x)=g_0(0)+\frac{1}{2}\,\varphi_0(0)\int_0^x t\,e^{-\frac{\lambda\,t^2}{8}}\,dt+\frac{1}{2}\int_0^x\varphi_0(t)\,dt-$$

$$-\frac{\lambda}{8}\int_0^x e^{-\frac{\lambda\xi^2}{8}}\int_0^\xi t\,e^{\frac{\lambda t^2}{8}}\,\varphi_0(t)\,dt\,d\xi+\frac{1}{2}\int_0^x e^{-\frac{\lambda\xi^2}{8}}\int_0^\xi e^{\frac{\lambda t^2}{8}}\,\psi_0(t)\,dt\,d\xi.$$

Changing the order of integration in the last two integrals, get

$$g_0(x)=g_0(0)+\sqrt{\frac{2\pi}{\lambda}}\,\varphi_0(0)\,\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)+\frac{1}{2}\int_0^x\varphi_0(t)\,dt-$$

$$-\sqrt{\frac{\lambda\pi}{8}}\int_0^x\left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)-\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)\right]t\,e^{\frac{\lambda t^2}{8}}\,\varphi_0(t)\,dt+$$

$$+\sqrt{\frac{2\pi}{\lambda}}\int_0^x\left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)-\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)\right]e^{\frac{\lambda t^2}{8}}\,\psi_0(t)\,dt. \qquad (27)$$

By (20) we have

$$f_0(x)=\varphi_0(x)-g_0'(x)-\frac{\lambda}{4}\,x\,g_0(x)+f_0(0),$$

$$f_0(l-t)=\varphi_0(l-t)-g_0'(l-t)-\frac{\lambda}{4}\,(l-t)\,g_0(l-t)+f_0(0). \qquad (28)$$

Employing the second relation of (25) we obtain

$$\left(g_1'(x+l)\,e^{\frac{\lambda(x-l)^2}{8}}\right)'=\frac{1}{2}\,e^{\frac{\lambda}{8}(x-l)^2}\,\varphi_1'(x)+\frac{1}{2}\,e^{\frac{\lambda}{8}(x-l)^2}\,\psi_1(x),$$

$$g_1'(x+l)=g_1'(2\,l)\,e^{\frac{\lambda}{8}(x-l)^2}+\frac{1}{2}\,e^{\frac{\lambda}{8}(x-l)^2}\int_l^x e^{\frac{\lambda}{8}(t-l)^2}\,\varphi_1'(t)\,dt+\frac{1}{2}\,e^{-\frac{\lambda}{8}(x-l)}\int_l^x e^{\frac{\lambda}{8}(x-l)^2}\,\psi_1(t)\,dt.$$

Taking into account $g_1'(2\,l)=\varphi_1(l)$, integrating by parts the first integral, substituting x by $l-x$ and then substituting in both integrals $t=l-t$ get

$$g_1'(2\,l-x)=\frac{1}{2}\,\varphi_1(l)\,e^{-\frac{\lambda x^2}{8}}+\frac{1}{2}\,\varphi_1(l-x)+\frac{\lambda}{8}\,e^{-\frac{\lambda x^2}{8}}\int_0^x t\,e^{-\frac{\lambda t^2}{8}}\,\varphi_1(l-t)\,dt+$$

$$+\frac{1}{2}\,e^{-\frac{\lambda x^2}{8}}\int_0^x t\,e^{-\frac{\lambda t^2}{8}}\,\psi_1(l-t)\,dt. \qquad (29)$$

Hence

$$g_1(2\,l-x)=g_1(2\,l)+\frac{1}{2}\,\varphi_1(l)\int_0^x e^{-\frac{\lambda t^2}{8}}\,dt+\frac{1}{2}\int_0^x\varphi_1(l-t)\,dt-$$

$$-\frac{\lambda}{8}\int_0^x e^{-\frac{\lambda\xi^2}{8}}\int_0^\xi t\,e^{\frac{\lambda t^2}{8}}\,\varphi_1(l-t)\,dt\,d\xi+\frac{1}{2}\int_0^x e^{-\frac{\lambda\xi^2}{8}}\int_0^\xi e^{\frac{\lambda t^2}{8}}\,\psi_1(l-t)\,dt\,d\xi.$$

Changing the integration order in the last two integrals we get

$$g_1(2\,l-x)=g_1(2\,l)+\sqrt{\frac{2\pi}{\lambda}}\,\varphi_1(l)\,\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)+\frac{1}{2}\int_0^x\varphi_1(l-t)\,dt-$$

$$-\sqrt{\frac{\lambda\pi}{8}}\int_0^x\left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)-\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)\right]t\,e^{\frac{\lambda t^2}{8}}\,\varphi_1(l-t)\,dt+$$

$$+\sqrt{\frac{2\pi}{\lambda}}\int_0^x\left[\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,x\right)-\Phi_0\left(\frac{\sqrt{\lambda}}{2}\,t\right)\right]e^{-\frac{\lambda t^2}{8}}\,\psi_1(l-t)\,dt. \qquad (30)$$

By (22) we have

$$f_1(x-l) = \varphi_1(x) + f_0(0) - g_1'(x+l) - \frac{\lambda}{4}\,(x-l)\,g_1(x+l)$$

or

$$f_1(-t) = \varphi_1(l-t) + f_0(0) - g_1'(2\,l-t) + \frac{\lambda}{4}\,t\,g_1(2\,l-t). \tag{31}$$

Substituting $f_1(-t)$ by (31) into (18), and $f_0(l-t)$ by (28) into (19), we obtain

$$\mu(t) = \varphi_1(l-t) + g_0'(t) - g_1'(2\,l-t) - \frac{\lambda}{4}\,t\left[g_0(t) - g_1(2\,l-t)\right], \tag{32}$$

$$\nu(t) = \varphi_0(l-t) - \left[g_0'(l-t) - g_1'(l+t)\right] - \frac{\lambda}{4}\,(l-t)\left[g_0(l-t) - g_1(l+t)\right]. \tag{33}$$

Substituting $g_0'(t)$, $g_0(t)$, $g_1'(2\,l-t)$, $g_1(2\,l-t)$ calculated by (26), (27), (29), (30) into formulas (32), (33), and taking into account, due to the assumed continuity of $f'(x)$, that

$$\frac{\lambda}{2}\left[g_0(0) - g_1(2\,l)\right] = \varphi_0' - \psi_0(0) - \varphi_1'(l) + \psi_0(l)$$

obtain the sought result (11), (12).

In order for the solution to be regular, sufficient conditions must be satisfied that ensure the existence of a boundary control. It is therefore only natural these conditions provide the functions $f(x)$ and $g(x)$ with required smoothness determined by rule (16), (17) respectively, and by the initial and boundary conditions

$$f_0(0) = f_1(0), \quad f_0'(0) = f_1'(0), \quad f_0''(0) = f_1''(0),$$

$$g_0(l) = g_1(l), \quad g_0'(l) = g_1'(l), \quad g_0''(l) = g_1''(l), \quad g_0'''(l) = g_1'''(l),$$

$$\mu(0) = \varphi_0(0), \quad \mu(l) = \varphi_1(0), \quad \nu(0) = \varphi_0(l), \quad \nu(l) = \varphi_1(l),$$

$$\mu'(0) = \psi_0(0), \quad \mu'(l) = \psi_1(0), \quad \nu'(0) = \psi_0(l), \quad \nu'(l) = \psi_1(l).$$

Satisfying the expression for the function $f_0(x)$, $f_1(x)$, $g_0(x)$, $g_1(x)$, $\mu(x)$, $\nu(x)$ represented by formulas (28), (31), (27), (30), (32), (33) according to the conditions above and after some simple transformations obtain (5)–(10). It should be noted that for $\lambda = 0$ the obtained results in this work coincide with the results obtained in [7].

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Гиперболалық теңдеу үшін оның сипаттамаларының бірі бойынша жүктелген шекаралық бақылау есебі

Мақалада регулярлы шешім тұрғысынан бір өлшемді жолдың тербеліс теңдеуі үшін сипаттамаларының бірі бойынша жүктелген шекаралық бақылау есебінің бірегей шешімі зерттелген. Шешу әдісі осы теңдеу үшін құрылған Даламбер формуласының аналогына негізделген. Ұзындығы бірдей кесінділер бойынша бастапқы және соңғы Коши деректері берілгенде бұл теңдеудің шешімінің анықталу облысы квадрат болатыны атап өтілген. Квадраттың қабырғасы берілген кесіндінің ұзындығына тең. Шекаралық бақылаудың өзі Даламбер формуласының аналогының құрамдас бөліктері бойынша анықталған, олар өз кезегінде Кошидің бастапқы және соңғы деректері бойынша бірегей түрде анықталады. Ізделінді шекаралық бақылауларға арналған соңғы формулаларда нормаланған және орталықтандырылған үлестірімдердің таралу функциясы қатысатынын атап өткен жөн, бұл жалпы айтқанда гиперболалық типті теңдеулермен басталатын бастапқы және шекаралық есептер үшін тән емес.

Кілт сөздер: гиперболалық теңдеу, таралған тербелмелі жүйе, газ немесе сұйық ағындарының пульсациясын бәсеңдету есебі, жүктелген теңдеу, бастапқы шарттар, шекаралық шарттар, Даламбер формуласының аналогы, шекаралық бақылау, қалыпты үлестірім, үлестірім функциясы.

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Задача граничного управления для нагруженного вдоль одной из своих характеристик гиперболического уравнения

В статье исследована однозначная разрешимость задачи граничного управления для нагруженного вдоль одной из своих характеристик одномерного уравнения колебания струны в терминах регулярного решения. Метод решения основан на аналоге формулы Даламбера, построенного для данного уравнения. Отмечено, что областью определения решения данного уравнения, когда начальные и финальные данные Коши задаются на отрезках одинаковой длины, является квадрат. Сторона квадрата равна длине данного отрезка. Сами граничные управления определены через компоненты аналога формулы Даламбера, которые, в свою очередь, однозначно определяются через начальные и финальные данные Коши. Следует отметить, что в окончательных формулах для искомых граничных управлений участвует функция распределения нормированного и центрированного распределения, что, вообще говоря, не характерно для начальных и граничных задач инициированных уравнениями гиперболического типа.

Ключевые слова: гиперболическое уравнение, распределённая колебательная система, задача гашения пульсации потоков газа или жидкости, нагруженное уравнение, начальные условия, граничные условия, аналог формулы Даламбера, граничные управления, нормальное распределение, функция распределения.

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