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# On lattice of existential formulas for fragment of Jonsson set

This paper its content associated with the study of model-theoretic properties of Jonsson theories and their semantic models. In particular, considering Jonsson sets concerning of their definable circuit described fragments and their relationship to their centers. In the article considered Jonsson analogue of the question A.D.Taymanov of the existence of a sequence of Boolean algebras isomorphic to the corresponding algebras Lindenbaum-Tarski considered a complete theory. When Jonsson theory instead of Boolean algebras are considered lattices of existential formulas. Is studied the connection of companions of fragment of Jonsson set with respect to the conservation of properties of stability.

*Key words:* Jonsson set, lattice of existential formulas, fragment of Jonsson set, Boolean algebra, centre of fragment.

In the study of incomplete inductive theories, a special place among them is occupied Jonsson theories. In this paper we study the analogue of the question A.D.Taymanov for a fragment Jonsson set, which is a subset of the semantic model of some fixed Jonsson theory.

We distinguish two directions in the development of model theory. In the famous book [1] they are called Western and Eastern model theory, since one of the founders of model theory A.Tarsky lived on the west coast of the United States with 1940, and the other founder of A.Robinson — on the east. Western model theory developed in the traditions of Skolem and Tarski. It is largely motivated by problems in number theory, analysis and set theory, and in it uses all the formulas of first order logic.

Eastern model theory developed in the tradition of Maltsev and Robinson. She was motivated by problems in abstract algebra, where formulas theories usually have at most two blocks of quantifiers. It emphasizes the set of quantifier-free formulas and existential formulas. In contrast to the Western model theory, which studies the complete theory, Eastern model theory, in general, it has to deal with incomplete theories. Class of incomplete theories is wide enough so that you can to confine of inductive theories ($\forall\exists$-axiomatizable). In terms of the completeness of considered the theory the maximum demand is usually — $\forall\exists$-completeness. All of these conditions are satisfied Jonsson theory. Thus, we can conclude that the study of Jonsson theories relates essentially to the problems of «eastern» model theory.

In the study of the properties of models of complete first-order theories useful are informations about Boolean algebras (algebras Lindenbaum-Tarski) $F_n(T)$, $n \in \omega$, of the theory $T$ [1]. In connection with these Boolean algebras $F_n(T)$, $n \in \omega$, well-known question of Academician A.D. Taimanov (can be found in [2, 3]):

(*) What properties should have Boolean algebras $B_n, n \in \omega$, that there be a complete theory $T$, so that $B_n$ was isomorphic $F_n(T)$, $n \in \omega$?

In [3] Professor T.G.Mustafin were given answers to particular cases of this issue. He obtained the following results:

*Theorem 1* [3]. For any Boolean algebra $B$ exists there a complete theory $T$ that:

a) $B \cong F_1(T)$;

b) if $B$ is the final, then $T$ is categorical in countable power;

c) if the Stone space of the algebra $B$ is countable, then $T$ is totally transcendental.

*Theorem 2* [3]. In order to final Boolean algebras $B_1$, $B_2$ exist such categorical in countable capacity the theory $T$ that $F_1(T) \cong B_1$, $F_2(T) \cong B_2$ it is necessary and sufficient that the number of atoms $B_2$ had more of square of the number of atoms $B_1$.

Therefore, we say that the question (*) has a positive solution for the theory $T$, if there exists a sequence of Boolean algebras $B_n$, $n \in \omega$, that $B_n$ isomorphic $F_n(T)$, $n \in \omega$.

In this article we consider the above question in the study of incomplete theories, namely, in the classroom Jonsson theories.

*Definition 1.* Theory $T$ called Jonsson if:

1) the theory $T$ has infinite models;
2) the theory $T$ is inductive;
3) the theory $T$ has the joint embedding property (JEP);
4) the theory $T$ has the property of amalgam (AP).

Theory is inductive if it is stable under union of chains. Known the following theorem:

*Theorem 3. (Chang-Los-Sushko)* The theory $T$ is stable under union of chains, if and only if it is $\forall\exists$-axiomatizable, ie is equivalent to the set of $\forall\exists$-sentences**.**

The theory $T$ has the joint embedding property, if for any models $U$, $B$ of theory $T$ exists the model $M$ of theory $T$ and isomorphic investments $f: U \to M$, $g: B \to M$.

The theory $T$ has the property of amalgam, if for any models $U$, $B_1, B_2$ of theory $T$ and isomorphic investments $f_1: U \to B_1$, $f_2: U \to B_2$ there are $M \models T$ isomorphic embeddings $g_1: B_1 \to M$, $g_2: B_2 \to M$ such that $g_1 \circ f_1 = g_2 \circ f_2$.

The following theories are examples of Jonsson theories:

1) groups;
2) Abelian groups;
3) Boolean algebras;
4) linear orders;
5) fields of characteristic *p* (*p* — a prime number or a zero);
6) ordered fields.

Consider the theory $T$ of countable first-order language *L*.

The next result is the main in the description of the model-theoretic properties of perfect Jonsson theories.

*Theorem 4.* Let $T$ is Jonsson theory. Then the following conditions are equivalent:

1) $T$ is perfect;
2) $T^*$ is a model companion of *T*;

If Jonsson theory $T$ is perfect, the

3) $ModT^* = E_T$;
4) $T^* = T^f$,

where $E_T$ is class $T$ — existentially closed models of $T$, $T^0$ is shell of Kaiser (maximum $\forall\exists$-theory one-model together with $T$), $T^f = Th(F_T)$, where $F_T$ is the class of generic models of $T$ (in the sense of the final forcing of Robinson), $T^M$ is model companion of Jonsson theory $T$.

Known following facts and theorems on the relationship Jonsson theories and their companions [4].

*Definition 2.* Let $T$ is Jonsson theory. Companion of Jonsson theory $T$ called a such theory $T^\#$ of the same signature that satisfies the following conditions:

1) $\left(T^\#\right)_\forall = T_\forall$;
2) for any Jonsson theory $T'$ if $T_\forall = T'_\forall$, then $T^\# = (T')^\#$.
3) $T_{\forall\exists} \subseteq T^\#$.

Natural interpretations of a companion $T^\#$ are $T^*$, $T^0$, $T^f$, $T^M$, $T^e$, where there $T^0$-companion is a shell of Kaiser; $T^*$-companion is the center; $T^M$-companion is a model companion; $T^f$-companion is the final forcing companion in terms of Robinson; $T^e$-companion is an elementary theory of the class of existentially closed models of the theory $T$.

*Fact 1.* For any Jonsson theory $T$ the following conditions are equivalent:

1) $T$ is perfect;
2) $T^*$ is model complete.

*Fact 2.* For any complete for $\exists$-sentences of Jonsson theory $T$ the following conditions are equivalent:

1) $T^*$ is model complete;
2) for each $n < \omega$ $E_n(T)$ — Boolean algebra, where $E_n(T)$ is lattice of $\exists$-formulas with $n$ free variables.

*Theorem 5* [4]. Let $T$ is Jonsson theory. Then the following conditions are equivalent:

1) $T$ is perfect;

2) $T*$ is $\forall\exists$-axiomatizable.

*Theorem 6.* [4] Let $T$ is Jonsson theory. Then the following conditions are equivalent:

1) $T$ is perfect;

2) $T* = T^0$.

*Theorem 7.* [4] Let $T$ is Jonsson theory. Then the following conditions are equivalent:

1) $T$ is perfect;

2) $T$ has a model companion.

The following lemma is a result of applying of the above theorems.

*Lemma 1.* [4] If $T^\#$ is a companion of Jonsson theory $T$ and $T^M$ is model companion of $T$, then $T^\# = T^M$.

*Lemma 2.* [4] Let $T_1$ and $T_2$ is Jonsson theory. Then the following conditions are equivalent:

1) $T_1$ and $T_2$ mutually compatible model;

2) $T_1^\# = T_2^\#$.

It is well known that working with Jónsson theories, in some cases we are able to limit yourself to existential formulas and existentially closed models consider Jonsson theory. In this case, instead algebras of the Lindenbaum-Tarski $F_n(T)$, $n \in \omega$, should be considered lattices of existential formulas $E_n(T)$, $n \in \omega$. Thus, the above question of A.D.Taymanov can be to formulate as follows:

(**) What properties should have lattices $E_n$, $n \in \omega$, that existed Jonsson theory $T$, such that $E_n$ was isomorphic $E_n(T)$, $n \in \omega$?

Similarly, the question (**) can be solved positively for Jonsson theory $T$, if there is a sequence of lattices $E_n$, $n \in \omega$, that $E_n$ is isomorphic $E_n(T)$, $n \in \omega$.

In connection with these questions (*), (**) following results were obtained in [5]:

*Theorem 8.* Let $T$ is a perfect, full for existential sentences, Jonsson theory. Then the following conditions are equivalent:

1) a positive solution to the question (**) on the theory *T*;

2) a positive solution to the question (*) on the theory $T^*$, where $T^*$ is the center of the theory *T*;

3) a positive solution to the question (**) on the #-companion of the theory *T*, $\# \in \{*, 0, m, f, e\}$, where 0 — companion is shell of Kaiser, * — companion is the center, $m$ — companion is a model companion, $f$ — companion is the final forcing in the sense Robinson, $e$ — companion is elementary theory of the class of all existentially closed models of the theory *T*.

If a set of universal consequences of Jonsson theory is also an Jonsson theory (which in general is not always the case), then we get the following result.

*Theorem 9* [5]. Let $T$ is the perfect, full for existential sentences Jonsson theory and the theory $T_\forall$ is Jonsson, where $T_\forall$ is the set of universal sentences withdrawn from *T*. Then the following conditions are equivalent:

1) a positive solution to the question (**) regarding the theory $T_\forall$, where $T_\forall$ is the set of sentences withdrawn from *T*;

2) a positive solution to the question (*) on the theory $T^*$, where $T^*$ is the center of the theory $T$.

In general, in the study of structures of lattices of existential formulas fixed Jonsson theory were obtained [6–14] many results in which the model-theoretic properties of language first-order of the center under consideration Jonsson theory were moved on itself Jonsson theory.

In this article we consider the more general situation and move on to the fragments of Jonsson set of fixed Jonsson theory. Notion Jonsson set and its fragments was introduced in [15].

Let $T$ is Jonsson perfect theory complete for the existential sentences in the language *L*, and its a semantic model is *C*.

We say that the set $X - \Sigma$-definable if it is definable some existential formula.

The set *X* is called Jonsson in the theory *T* if it satisfies the following properties:

1) *X* is $\Sigma$-definable subset of *C*;

2) dcl(*X*) is a carrier some existentially closed submodel of *C*.

To study the behavior of the elements of alveolus in the case of Jonsson sets, we can always consider $\forall\exists$ -investigation true in the above closings of Jonsson set. In view of the above, in this case, considered set sentences will Jonsson theory.

Resulting Jonsson theory in this case it will be called the Jonsson fragment of corresponding Jonsson set. It is clear that we can carry out research of Jonsson fragments with respect to an initial theory, which is a new formulation of problem of the research Jonsson theories.

This article discusses the fragments Jonsson sets which are subsets of a semantic model of some Jonsson theory of countable language first-order. A series of results that establish a connection between the properties of the fragment and the theory Jonsson, central replenishment of the Jonsson theory and the properties of the lattice of classes equivalence of existential formulas regarding this considered fragment.

In connection with the above results on the introduced concepts, we obtained results relating the concepts of [16] with Jonsson theories and fragments Jonsson subsets according to their semantic models.

Note that in the case of Jonsson set and its fragment we can obtain corresponding results similar to the above [5].

Let given a certain Jonsson theory *T* and *X* is a Jonsson subset of its semantic model. *M* is existentially closed model where dcl(*X*) = *M*.

Consider $Th_{\forall\exists}(M) = T_M$ $Th_{\forall\exists}(M) = T_M$.

*Theorem 10.* Let $T_M$ is a perfect, full for existential sentences, Jonsson theory. Then the following conditions are equivalent:

1) a positive solution to the question (**) on the theory $T_M$ ;

2) a positive solution to the question (*) on the theory $T_M^*$, where $T_M^*$ is the center of the theory $T_M$ ;

3) a positive solution to the question (**) on the # — companion of the theory $T_M$ , $\# \in \{*, 0, m, f, e\}$, where 0 — companion is shell of Kaiser; * — companion is the center, $m$ — companion is a model companion; $f$ — companion is the final forcing in the sense Robinson; $e$ — companion is elementary theory of the class of all existentially closed models of the theory $T_M$ .

*Proof.* This follows from the application of the above facts and theorems.

If a set of universal consequences of Jonsson theory also is a Jonsson theory (which in general is not always the case), then we get the following result.

*Theorem 11.* Let $T_M$ is the perfect, full for existential sentences Jonsson theory and the theory $(T_M)_\forall$ is Jonsson, where $(T_M)_\forall$ is the set of universal sentences withdrawn from $T_M$ . Then the following conditions are equivalent:

1) a positive solution to the question (**) regarding the theory $(T_M)_\forall$, where $T_M$ is the set of sentences withdrawn from $T_M$ ;

2) a positive solution to the question (*) on the theory $T_M$ , where $T_M$ $T^*$ is the center of the theory $T_M$ .

*Proof.* It follows from Theorem 10.

Well-known fact (*) that for a complete theory it is true that its companion retains the concept of stability [1] in the sense S.Shelah.

When we are dealing with Jonsson theories, we need to work within the framework of other stability. Note that in general, the Jonsson theory is not complete. To date, device of the model theory developed mainly for complete theories, and so the study of the general model-theoretic properties (eg, stability) of Jónsson theories of interesting in the class of incomplete inductive theories. In particular in class of Jonsson theories. What we are seeing interest in this generalization the above concepts? Firstly existential types is part of all types — ie it is not necessary to consider all types (for perfect Jónsson theories it is), and secondly most of the algebraic examples to allow the elimination of such formulas (if it is not, you can do this up enrichment of signature), in the third, as the class of models central replenishment (and in perfect and imperfect cases), we consider the class of existentially closed models of Jonsson theory. And their behavior adequately describe the existential types in the definition of these models. Such considerations, and suggest, abovementioned synthesis of stability in class Jónsson theories.

Consider stability properties of Jónsson theories. This will be discussed Jonsson analogues of the classical definition of stability and some of its generalizations.

Let us give the following definitions.

Let $T$ is Jonsson theory, $S^J(X)$ is the set of all existential complete $n$-types over *X*, joint with *T*, for each final *n*.

*Definition 3.* We say that Jonsson theory $T$ is $J-\lambda$-stable if for any $T$-existentially closed model *A*, for all subsets $X$ of set *A*, $|X|\leq\lambda \Rightarrow |S^J(X)|\leq\lambda$.

In the case of fragment Jonsson set, which is a subset of the semantic model for the fixed-perfect complete for existential sentences of Jonsson theory, we have the following result generalizes the result (*).

*Lemma 3.* Let $T$ as above, then it is true that $T_M$ is Jonsson theory a perfect, complete for existential sentences.

*Theorem 12.* Let $T_M$ is a perfect, complete for existential sentences Jonsson theory, $\lambda\geq\omega$, # — a companion of theory $T_M$, $T_M^*$ is the center of the theory $T_M$. Then #-companion of theory $T_M$ save *J*-λ-stable of fragment $T_M$, namely, the following are equivalent:

1) $T_M$ is $J-\lambda$-stable;
2) $T_M^*$ is $\lambda$- stable, where $T_M^*$ is centre of theory $T_M$;
3) # — companion = $T_M^*$.

*Proof.* 1⇒2: Theory $T_M\subset T_M^* \Rightarrow E_n(T_M)\subset E_n(T_M^*)$, where $E_n(T_M)$, $E_n(T_M^*)$ it is appropriate lattice of existential formulas. Theory $T_M$ is complete for existential sentences, thus, $E_n(T_M)=E_n(T_M^*)$. $T_M$ perfect, thus, $T_M^*$ model complete ⇔ $\forall n<\omega$, $\forall\varphi\in F_n(T_M^*)$ $\exists\theta\in E_n(T_M^*)$: $T_M^*$ |- $\varphi\leftrightarrow\theta$.

Let $T_M$ *J*-λ- stable, then to determine for each $A\in E_T$ we have that for each $X\subset A$, $|X|\leq\lambda\Rightarrow|S^J(X)|\leq\lambda$.

Assume that $T_M^*$ not λ- stable. Then there $A\in E_T=\mathrm{Mod}\,T_M^*$, so that there $X\subset A$, such that $|X|<\lambda$, $\exists n<\omega$, $\Rightarrow|S^J(X)|>\lambda$. For each formula $\varphi\in p$, где $p\in S_n(X)$, we will replace φ on θ, where θ satisfies $T_M^*$ |- $\varphi\leftrightarrow\theta$ and $\theta\in E_n(T_M^*)$. Let $p'$ will be *p* after replacing. Then $p'\in S^J(X)$ and $|S^J(X)|>\lambda$. This contradicts *J*-λ- stable of theory $T_M$.

2⇒1 trivially.

1⇒3 it is due to the perfectness $T_M$.

3⇒1 should the completeness shell of Kaiser.

All necessary definitions, but uncertain directly can be found in [4].

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## Йонсондық жиын фрагментінің экзистенциалды формулаларының торы туралы

Мақалада йонсондық теориялардың модельді-теориялық қасиеттері мен олардың семантикалық модельдерін зерттеумен байланысты дербес жағдайда йонсондық жиындарды олардың анықталатын тұйықталуына қатысты қарастыра отырып, фрагменттер мен олардың центрлерімен байланысы көрсетілді. Авторлар толық теорияның Линденбаум-Тарский алгебрасына сәйкес изоморфты Буль алгебралы тізбектің бар болуы туралы А.Д.Тайманов сұрағының йонсондық баламасын қарастырған. Йонсондық теориялар жағдайында Буль алгебраларының орнына экзистенциалды формулалардың торлары зерттелді. Стабильділік қасиеттерін сақтауға қатысты йонсондық жиынның фрагментінің компаньондарының байланысы айқындалды.

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## О решётке экзистенциальных формул фрагмента йонсоновского множества

Статья по своему содержанию связана с изучением теоретико-модельных свойств йонсоновских теорий и их семантических моделей. В частности, рассматривая йонсоновские множества относительно их определимого замыкания, описаны фрагменты и их связи со своими центрами. Авторами рассмотрен йонсоновский аналог вопроса А.Д.Тайманова о существовании последовательности булевых алгебр изоморфных соответствующим алгебрам Линденбаума-Тарского рассматриваемой полной теории. В случае йонсоновской теории вместо булевых алгебр изучены решётки экзистенциальных формул, а также связь компаньонов фрагмента йоносоновского множества относительно сохранения свойства стабильности.

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