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A note on the parabolic identification problem with involution and Dirichlet condition

A space source of identification problem for parabolic equation with involution and Dirichlet condition is studied. The well-posedness theorem on the differential equation of the source identification parabolic problem is established. The stable difference scheme for the approximate solution of this problem is presented. Furthermore, stability estimates for the difference scheme of the source identification parabolic problem are presented. Numerical results are given.

Keywords: well-posedness, elliptic equations, positivity, coercive stability, source identification, exact estimates, boundary value problem.

Introduction

The theory and applications of source identification problems for partial differential equations have been studied by many authors (see, e.g., [1–9] and the references given therein). Numerous source identification problems for hyperbolic-parabolic equations and their applications have been investigated too (see, e.g., [10–13] and the references given therein). In the last decade, partial differential equations with involutions were investigated in [14–18]. However, source identification problems for parabolic equations with involution have not been well-investigated.

The present paper is devoted to study a space source of identification problem for parabolic equation with involution and Dirichlet condition. The well-posedness theorem on the differential equation of the source identification parabolic problem is proved. The stable difference schemes for the approximate solution of this problem are constructed. Furthermore, stability estimates for the difference schemes of the source identification parabolic problem are established. Numerical results are provided.

Well-posedness of differential problem

We consider the space source identification problem

$$\begin{cases} u_t(t,x) - \left(a(x)u_x\left(t,x\right)\right)_x - \beta\left(a(-x)u_x\left(t,-x\right)\right)_x + \delta u(t,x) \\ = p(x) + f(t,x),\ -l < x < l,\ 0 < t < T, \\ u(t,-l) = u(t,l) = 0,\ 0 \le t \le T, \\ u(0,x) = \varphi(x), u(T,x) = \psi(x),\ -l \le x \le l \end{cases} \tag{1}$$

for the one dimensional parabolic differential equation with involution. Problem (1) has a unique solution $(u(t,x), p(x))$ for the smooth functions $f(t,x)$ $(t \in (0,T) \times (-l,l))$, $a \ge a\left(x\right) = a\left(-x\right) \ge \delta > 0$, $\delta - a\left|\beta\right| \ge 0$ $\ (x \in (-l,l))$, and $\varphi(x),\ \psi(x),\ x \in [-l,l]$.

In the present paper $C_0^\alpha([0,T],H)$ $(0<\alpha<1)$ stands for Banach spaces of all abstract continuous functions $\varphi(t)$ defined on $[0,T]$ with values in H satisfying a Hölder condition with weight t^α for which the following norm is finite

$$\|\varphi\|_{C_0^\alpha([0,T],H)} = \|\varphi\|_{C([0,T],H)} + \sup_{0\le t<t+\tau\le T} \frac{(t+\tau)^\alpha \|\varphi(t+\tau)-\varphi(t)\|_H}{\tau^\alpha}.$$

Here, $C([0,T],H)$ stands for the Banach space of all abstract continuous functions $\varphi(t)$ defined on $[0,T]$ with values in H equipped with the norm

$$\|\varphi\|_{C([0,T],H)} = \max_{0\le t\le T} \|\varphi(t)\|_H .$$

Theorem 1. Suppose that $\varphi,\psi\in W_2^2[-l,l]$. Let $f(t,x)$ be continuously differentiable in t on $[0,T]\times[-l,l]$ function. Then the solutions of the identification problem (1) satisfy the stability estimates

$$\|u\|_{C([0,T],L_2[-l,l])} + \left\|(A^x)^{-1}p\right\|_{L_2[-l,l]}$$
$$\le M_1(\delta,\sigma,\beta,l)\left[\|\varphi\|_{L_2[-l,l]} + \|\psi\|_{L_2[-l,l]} + \|f\|_{C([0,T],L_2[-l,l])}\right], \tag{2}$$
$$\|u\|_{C^{(1)}([0,T],L_2[-l,l])} + \|u\|_{C\left([0,T],W_2^2[-l,l]\right)} + \|p\|_{L_2[-l,l]}$$
$$\le M_2(\delta,\sigma,\beta,l)\left[\|\varphi\|_{W_2^2[-l,l]} + \|\psi\|_{W_2^2[-l,l]} + \|f\|_{C^{(1)}([0,T],L_2[0,l])}\right]. \tag{3}$$

Here $M_1(\delta,\sigma,\beta,l)$ and $M_2(\delta,\sigma,\beta,l)$ do not depend on $\varphi(x),\psi(x)$ and $f(t,x)$. The Sobolev space $W_2^2[-l,l]$ is defined as the set of all functions $u(x)$ defined on $[0,l]$ such that $u(x)$ and the second order derivative function $u''(x)$ are all locally integrable in $L_2[-l,l]$, equipped the norm

$$\|u\|_{W_2^2[-l,l]} = \left(\int_{-l}^{l} |u(x)|^2\,dx\right)^{\frac12} + \left(\int_{-l}^{l} |u''(x)|^2\,dx\right)^{\frac12}.$$

Proof. Problem (1) can be written in abstract form

$$\begin{cases} \frac{du(t)}{dt} + Au(t) = p + f(t), 0<t<T, \\ u(0)=\varphi, u(T)=\psi \end{cases} \tag{4}$$

in a Hilbert space $H=L_2[-l,l]$ with self-adjoint positive definite operator $A=A^x$ defined by the formula

$$A^x u(x) = -(a(x)u_x(x))_x - \beta(a(-x)u_x(-x))_x + \delta u(x) \tag{5}$$

with the domain $D(A^x)=\left\{u\in W_2^2[-l,l] : u(-l)=u(l)=0\right\}$ [14]. The proof of Theorem 1 is based on the symmetry properties of this space operator A and on the following stability results.

Theorem 2 [5]. Assume that $\varphi,\psi\in D(A)$ and $f(t)$ be continuously differentiable in t on $[0,T]$ function. Then, for the solution $\{u(t),p\}$ of the source identification problem (4) the following stability inequalities hold:

$$\|u\|_{C([0,T],H)} + \left\|A^{-1}p\right\|_H \le M\ \left[\|\varphi\|_H + \|\psi\|_H + f_{C([0,T],H)}\right] \tag{6}$$

$$\|u\|_{C^{(1)}([0,T],H)} + \|Au\|_{C([0,T],H)} + \|p\|_H \le M\left[\|A\varphi\|_H + \|A\psi\|_H + \|f\|_{C^{(1)}([0,T],H)}\right], \tag{7}$$

where M is independent of φ,ψ and $f(t)$.

Moreover, we have the following coercive stability results.

Theorem 3. Suppose that $\varphi, \psi \in W_2^2[-l,l]$ and $f(t,x) \in C_0^\alpha([0,T], L_2[-l,l])$. Then the solutions of the identification problem ((1) satisfy coercive stability estimates

$$\|u_t\|_{C_0^\alpha([0,T],L_2[-l,l])} + \|u\|_{C_0^\alpha([0,T],W_2^2[-l,l])} + \|p\|_{L_2[-l,l]}$$

$$\leq M(\delta,\sigma,\alpha,\beta,l)\left[\|\varphi\|_{W_2^2[-l,l]} + \|\psi\|_{W_2^2[-l,l]} + \|f\|_{C_0^\alpha([0,T],L_2[-l,l])}\right],$$

where $M(\delta,\sigma,\alpha,\beta,l)$ is independent of $\varphi(x), \psi(x)$ and $f(t,x)$.

The proof of Theorem 3 is based on the following abstract Theorem on coercive stability of the identification problem (4) in $C_0^\alpha([0,T],H)$ spaces and on self-adjointness and positive definite of the unbounded operator A defined by formula (5) in $L_2[-l,l]$ space.

Theorem 4. Assume that $\varphi, \psi \in D(A)$ and $f(t)$ and $f \in C_0^\alpha([0,T],H)$ $(0<\alpha<1)$. Then, for the solution $\{u(t),p\}$ of the source identification problem (4) the following coercive stability inequalities hold:

$$\|u'\|_{C_0^\alpha([0,T],H)} + \|Au\|_{C_0^\alpha([0,T],H)} + \|p\|_H$$

$$\leq M\left[\|A\varphi\|_H + \|A\psi\|_H + \frac{1}{\alpha(1-\alpha)}\|f\|_{C([0,T],H)}\right],$$

where M is independent of φ, ψ and $f(t)$.

Stability of difference schemes

Now, we study the stable difference schemes for the approximate solution of identification problem (1). The discretization of source identification problem (1) is carried out in two stages. In the first stage, we define the grid space

$$[-l,l]_h = \{x = x_n : x_n = nh,\ -M \leq n \leq M,\ Mh = l\}.$$

We introduce the Hilbert spaces $L_{2h} = L_2([-l,l]_h)$ and $W_{2h}^2 = W_2^2([-l,l]_h)$ of the grid functions $\varphi^h(x) = \{\varphi^r\}_{-M}^M$ defined on $[-l,l]_h$, equipped with the norms

$$\left\|\varphi^h\right\|_{L_{2h}} = \left(\sum_{x\in[-l,l]_h}\left|\varphi^h(x)\right|^2 h\right)^{1/2}$$

and

$$\left\|\varphi^h\right\|_{W_{2h}^2} = \left\|\varphi^h\right\|_{L_{2h}} + \left(\sum_{x\in[-l,l]_h}\left|\left(\varphi^h\right)_{x\overline{x},j}\right|^2 h\right)^{1/2},$$

respectively. To the differential operator A generated by problem (5), we assign the difference operator A_h^x by the formula

$$A_h^x\varphi^h(x) = \{-(a(x)\varphi_{\overline{x}}(x))_{x,r} - \beta\left(a(-x)\varphi_{\overline{x}}(-x)\right)_{x,r} + \delta\varphi_r\}_{-M+1}^{M-1}, \tag{8}$$

acting in the space of grid functions $\varphi^h(x) = \{\varphi_r\}_{-M}^M$ satisfying the conditions $\varphi_{-M} = \varphi_M = 0$.

It is well-known that A_h^x is a self-adjoint positive definite operator in L_{2h}. With the help of A_h^x, we reach the identification problem

$$\begin{cases} u_t^h(t,x) + A_h^x u^h(t,x) = p^h(x) + f^h(t,x),\ x \in [-l,l]_h,\ 0<t<T, \\ u^h(0,x) = \varphi^h(x), u^h(T,x) = \psi^h(x),\ x \in [-l,l]_h. \end{cases} \tag{9}$$

In the second stage, we replace identification problem (9) with a first order of accuracy difference scheme

$$\left\{\begin{array}{l} \frac{u_k^h(x)-u_{k-1}^h(x)}{\tau}+A_h^x u_k^h(x)=p^h(x)+f_k^h, f_k^h(x)=f(t_k,x), \\ t_k=k\tau, 1\le k\le N, N\tau=T, x\in[-l,l]_h\,, \\ u_0^h(x)=\xi^h(x), u_N^h(x)=\varphi^h(x),\ x\in[-l,l]_h\,. \end{array}\right. \tag{10}$$

Let $\alpha\in(0,1)$ is a given number and $C_\tau(H)$ and $C_\tau^\alpha(H)$ be Banach spaces of H-valued grid functions $w_\tau=\{w_k\}_{k=0}^N$ with the corresponding norms

$$\|w_\tau\|_{C_\tau(H)}=\max_{0\le k\le N}\|w_k\|_H\,,\ \|w_\tau\|_{C_\tau^\alpha(H)}=\sup_{1\le k<k+n\le N}(n\tau)^{-\alpha}(k\tau)^\alpha\|w_{k+n}-w_k\|_H+\|w_\tau\|_{C_\tau(H)}\,.$$

Theorem 5. For the solution $\left\{\left\{u_k^h(x)\right\}_0^N, p^h(x)\right\}$ of problem (10), the following stability estimates

$$\left\|\left\{u_k^h\right\}_0^N\right\|_{C_\tau(L_{2h})}+\left\|\left(A_h^x\right)^{-1}p^h\right\|_{L_{2h}}$$

$$\le M_3(\delta,\sigma,\beta,l)\left[\left\|\varphi^h\right\|_{L_{2h}}+\left\|\psi^h\right\|_{L_{2h}}+\left\|\left\{f_k^h\right\}_1^N\right\|_{C_\tau(L_{2h})}\right], \tag{11}$$

$$\left\|\left\{\frac{u_k^h-u_{k-1}^h}{\tau}\right\}_0^N\right\|_{C_\tau(L_{2h})}+\left\|\left\{u_k^h\right\}_0^N\right\|_{C_\tau(W_{2h}^2)}$$

$$\le M_4(\delta,\sigma,\beta,l)\left[\left\|\varphi^h\right\|_{W_{2h}^2}+\left\|\psi^h\right\|_{W_{2h}^2}+\left\|f_1^h\right\|_{L_{2h}}+\max_{2\le k\le N}\left\|\left\{\frac{1}{\tau}\left(f_k^h-f_{k-1}^h\right)\right\}_2^N\right\|_{L_{2h}}\right] \tag{12}$$

hold, where $M_3(\delta,\sigma,\beta,l)$ and $M_4(\delta,\sigma,\beta,l)$ do not depend on $\tau,h,f_k^h,\ 1\le k\le N,\ \varphi^h(x)$ and $\psi^h(x)$.

Proof. Difference scheme (10) can be written in the following abstract forms

$$\left\{\begin{array}{l} \frac{u_k-u_{k-1}}{\tau}+Au_k=p+f_k, 1\le k\le N, \\ u_0=\varphi, u_N=\psi \end{array}\right. \tag{13}$$

in a Hilbert space $H=L_{2h}$ with operator $A=A_h^x$ by formula (8). Here, $f_k=f_k^h(x)$ is a given abstract mesh function, $u_k=u_k^h(x)$ is unknown mesh function and $p=p^h(x)$ is the unknown mesh element of L_{2h}. Therefore, the proof of Theorem 5 is based on the self- adjointness and positive definiteness of the space difference operator A in L_{2h} [14] and on the following stability results.

Theorem 6. [5]. For the solution $\left\{\{u_k\}_0^N, p\right\}$ of the source identification difference problem (13), the following stability inequalities hold:

$$\left\|\{u_k\}_0^N\right\|_{C_\tau(H)}+\left\|A^{-1}p\right\|_H$$

$$\le M_5(\delta,\sigma,\beta,l)\left[\|\varphi\|_H+\|\psi\|_H+\left\|\{f_k\}_1^N\right\|_{C_\tau(H)}\right],$$

$$\left\|\left\{\frac{u_k-u_{k-1}}{\tau}\right\}_0^N\right\|_{C_\tau(H)}+\left\|\{Au_k\}_0^N\right\|_{C_\tau(H)}$$

$$\le M_5(\delta,\sigma,\beta,l)\left[\|A\varphi\|_H+\|A\psi\|_H+\|f_1\|_H+\max_{2\le k\le N}\left\|\left\{\frac{1}{\tau}(f_k-f_{k-1})\right\}_2^N\right\|_H\right],$$

where $M_5(\delta,\sigma,\beta,l)$ is independent of φ,ψ and $f(t)$.

Moreover, we have the following coercive stability results.

Theorem 7. The solutions of the identification difference problem (10) satisfies coercive stability estimate

$$\left\| \left\{ \frac{u_k^h - u_{k-1}^h}{\tau} \right\}_1^N \right\|_{C_\tau^\alpha(L_{2h})} + \left\| \left\{ u_k^h \right\}_0^N \right\|_{C_\tau^\alpha(W_{2h}^2)} \leq$$

$$\leq M_5 (\delta, \sigma, \beta, l) \left[\left\| \varphi^h \right\|_{W_{2h}^2} + \left\| \psi^h \right\|_{W_{2h}^2} + \left\| \left\{ f_k^h \right\}_1^N \right\|_{C_\tau^\alpha(L_{2h})} \right],$$

where $M_5 (\delta, \sigma, \beta, l)$ does not depends on $\tau, h, f_k^h,\ 1 \leq k \leq N,\ \ \varphi^h (x)$ and $\psi^h (x)$.

The proof of Theorem 7 is based on the self-adjointness and positive definiteness of the space difference operator A in L_{2h} [14] and on the following coercive stability results.

Theorem 8. For the solution $\left\{ \{u_k\}_0^N, p \right\}$ of the source identification difference problem (13) the following coercive stability inequality holds:

$$\left\| \left\{ \frac{u_k - u_{k-1}}{\tau} \right\}_1^N \right\|_{C_\tau^\alpha(H)} + \left\| \{u_k\}_0^N \right\|_{C_\tau^\alpha(H)} \leq M_6 (\delta, \sigma, \beta, l) \left[\|A\varphi\|_H + \|A\psi\|_H + \left\| \{f_k\}_1^N \right\|_{C_\tau^\alpha(H)} \right],$$

where $M_6 (\delta, \sigma, \beta, l)$ does not depends on $\tau, h, f_k,\ 1 \leq k \leq N,\ \ \varphi$ and ψ.

Numerical experiment

When the analytical methods do not work properly, the numerical methods for obtaining approximate solutions of partial differential equations play an important role in applied mathematics. We can say that there are many considerable works in the literature. In present section for the approximate solutions of a problem, we use the first order of accuracy difference scheme. We apply a procedure of modified Gauss elimination method to solve the problem. Finally, the error analysis of first order of accuracy difference scheme is given.

We consider the identification problem with the Dirichlet condition

$$\begin{cases} u_t (t, x) - u_{xx} (t, x) - \frac{1}{2} u_{x,x} (t, -x) + u(t, x) \\ = p (x) - \sin x + \cos t \sin x + \frac{3}{2} \sin t \sin x, \quad x \in (-\pi, \pi), t \in (0, \pi), \\ u (0, x) = 0, u (\pi, x) = 0, x \in [-\pi, \pi], \\ u (t, -\pi) = u (t, \pi) = 0, t \in [0, \pi] \end{cases} \tag{14}$$

for parabolic equation with involution. The exact solution pair of this problem is

$$(u (t, x), p (x)) = (\sin t \sin x, \sin x), -\pi \leq x \leq \pi, 0 \leq t \leq \pi.$$

Here and in future, we denote the set $[0, \pi]_\tau \times [-\pi, \pi]_h$ of all grid points

$$[0, \pi]_\tau \times [-\pi, \pi]_h = \{(t_k, x_n) : t_k = k\tau, 0 \leq k \leq N,$$

$$N\tau = \pi, x_n = nh, -M \leq n \leq M, Mh = \pi\}.$$

For the numerical solution of SIP (14), we present the first order of accuracy difference scheme in t

$$\begin{cases} \tau^{-1} \left(u_n^k - u_n^{k-1} \right) - h^{-2} \left(u_{n+1}^k - 2u_n^k + u_{n-1}^k \right) \\ -\frac{1}{2} h^{-2} \left(u_{-n+1}^k - 2u_{-n}^k + u_{-n-1}^k \right) + u_n^k = p_n - \sin x_n \\ + \cos t_k \sin x_n + \frac{3}{2} \sin t_k \sin x_n, 1 \leq k \leq N, -M + 1 \leq n \leq M - 1, \\ u_n^0 = 0, u_n^N = 0, -M \leq n \leq M, \\ u_{-M}^k = u_M^k = 0, 0 \leq k \leq N. \end{cases} \tag{15}$$

In the first step, we obtain $\left\{\left\{\omega_n^k\right\}_0^N\right\}_{n=-M}^M$ as solution of nonlocal BVP

$$\begin{cases} \tau^{-1}\left(\omega_n^k-\omega_n^{k-1}\right)-h^{-2}\left(\omega_{n+1}^k-2\omega_n^k+\omega_{n-1}^k\right) \\ -\frac{1}{2}h^{-2}\left(\omega_{-n+1}^k-2\omega_{-n}^k+\omega_{-n-1}^k\right)+\omega_n^k \\ =-\sin x_n+\cos t_k\sin x_n+\frac{3}{2}\sin t_k\sin x_n, 1\leq k\leq N, -M+1\leq n\leq M-1, \\ \omega_n^0-\omega_n^N=0, -M\leq n\leq M, \\ \omega_{-M}^k=\omega_M^k=0, 0\leq k\leq N. \end{cases} \tag{16}$$

Here and in future, ω_n^k denotes the numerical approximation of $\omega(t,x)$ at (t_k,x_n). For obtaining the solution of difference scheme (16), we rewrite it in the matrix form

$$\begin{cases} A\omega_{n+1}+B\omega_n+A\omega_{n-1}+C\omega_{-n+1}+D\omega_{-n}+C\omega_{-n-1}=f_n, \\ A\omega_{-n+1}+B\omega_{-n}+A\omega_{-n-1}+C\omega_{n+1}+D\omega_n+C\omega_{n-1}=f_{-n}, \end{cases}, \tag{17}$$

$$1\leq n\leq M-1, \begin{pmatrix} \omega_M \\ \omega_{-M} \end{pmatrix}=\begin{pmatrix} \overrightarrow{0} \\ \overrightarrow{0} \end{pmatrix},$$

where $\overrightarrow{0}$, w_s for $s=n,\ n\pm1$, and f_n are $(N+1)\times1$ column matrices, and $(N+1)\times(N+1)$ square matrices A,B,C,D are defined as follows:

$$A=\begin{bmatrix} 0 & 0 & 0 & . & 0 & 0 \\ 0 & -h^{-2} & 0 & . & 0 & 0 \\ 0 & 0 & -h^{-2} & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & -h^{-2} & 0 \\ 0 & 0 & 0 & . & 0 & -h^{-2} \end{bmatrix},$$

$$B=\begin{bmatrix} 1 & 0 & 0 & . & 0 & -1 \\ -\tau^{-1} & \tau^{-1}+2h^{-2}+1 & 0 & . & 0 & 0 \\ 0 & -\tau^{-1} & \tau^{-1}+2h^{-2}+1 & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & \tau^{-1}+2h^{-2}+1 & 0 \\ 0 & 0 & 0 & . & -\tau^{-1} & \tau^{-1}+2h^{-2}+1 \end{bmatrix},$$

$$C=\frac{1}{2}A,\ D=-A.$$

Grouping the above expression (17) as

$$\begin{cases} A\omega_{n+1}+C\omega_{-n-1}+B\omega_n+D\omega_{-n}+A\omega_{n-1}+C\omega_{-n+1}=f_n, \\ C\omega_{n+1}+A\omega_{-n-1}+D\omega_n+B\omega_{-n}+C\omega_{n-1}+A\omega_{-n+1}=f_{-n} \end{cases},$$

and defining $z_n=\begin{pmatrix} w_n \\ w_{-n} \end{pmatrix}$ and $\phi_n=\begin{pmatrix} f_n \\ f_{-n} \end{pmatrix}$, the system can be written as

$$\begin{pmatrix} A & C \\ C & A \end{pmatrix} z_{n+1}+\begin{pmatrix} B & D \\ D & B \end{pmatrix} z_n+\begin{pmatrix} A & C \\ C & A \end{pmatrix} z_{n-1}=\phi_n,\ 1\leq n\leq M-1,\ z_M=\begin{pmatrix} \overrightarrow{0} \\ \overrightarrow{0} \end{pmatrix}. \tag{18}$$

For solving the system (18), we use the Gauss elimination method. Thus, let's define

$$z_n=\alpha_{n+1}z_{n+1}+\beta_{n+1}, n=M-1,...,1, z_M=\begin{pmatrix} \overrightarrow{0} \\ \overrightarrow{0} \end{pmatrix}. \tag{19}$$

where α_n $(1 \le n \le M)$ are $(2N+2)\times(2N+2)$ square matrices and β_n $(1 \le n \le M)$ are $(2N+2)\times 1$ column vectors, calculated as,

$$\begin{cases} \alpha_{n+1} = -\left(P\alpha_n + Q\right)^{-1} P, \\ \beta_{n+1} = \left(P\alpha_n + Q\right)^{-1}\left(R\phi_n - P\beta_n\right), \\ n = 1, ..., M-1, \end{cases} \tag{20}$$

where $P = \begin{pmatrix} A & C \\ C & A \end{pmatrix}$ and $Q = \begin{pmatrix} B & D \\ D & B \end{pmatrix}$ and R is $(2N+2)\times(2N+2)$ identity matrix.

First, we evaluate α_n and β_n $(1 \le n \le M)$. Since,

$$\phi_0 = \begin{pmatrix} f_0 \\ f_0 \end{pmatrix} = \begin{pmatrix} A\omega_1 + C\omega_{-1} \\ C\omega_1 + A\omega_{-1} \end{pmatrix} + \begin{pmatrix} B\omega_0 + D\omega_0 \\ D\omega_0 + B\omega_0 \end{pmatrix} + \begin{pmatrix} A\omega_{-1} + C\omega_1 \\ C\omega_{-1} + A\omega_1 \end{pmatrix},$$

we get

$$z_0 = \begin{pmatrix} \omega_0 \\ \omega_0 \end{pmatrix} = \begin{pmatrix} B & D \\ D & B \end{pmatrix}^{-1} \left\{ -\begin{pmatrix} A+C & A+C \\ A+C & A+C \end{pmatrix} z_1 + \phi_0 \right\}$$

and

$$\alpha_1 = -\begin{pmatrix} B & D \\ D & B \end{pmatrix}^{-1} \begin{pmatrix} A+C & A+C \\ A+C & A+C \end{pmatrix},$$

$$\beta_1 = \begin{pmatrix} B & D \\ D & B \end{pmatrix}^{-1} \phi_0.$$

Using the iteration (20), we obtain all α_n and β_n $(1 \le n \le M)$ values. Second, using the formula (19), we obtain z_n and the equality $z_n = \begin{pmatrix} w_n \\ w_{-n} \end{pmatrix}$ gives the values of ω_n.

In the second step, using [5, Equation 8], we get

$$p_n = \frac{\omega_{n+1}^N - 2\omega_n^N + \omega_{n-1}^N}{h^2} + \frac{1}{2}\frac{\omega_{-n+1}^N - 2\omega_{-n}^N + \omega_{-n-1}^N}{h^2} - \omega_n^N$$

for $-M+1 \le n \le M-1$.

In the last step, using formula (see, [5])

$$u_n^k = \omega_n^k - \omega_n^N, n = -M, -M+1, ..., M, k = 0, ..., N, \tag{21}$$

we obtain $\left\{ \left\{ u_n^k \right\}_{k=0}^N \right\}_{n=-M}^M$.

Here, we compute the error between the exact solution and numerical solution by

$$\begin{cases} \|E_u\|_\infty = \max\limits_{0\le k\le N, -M\le n\le M} \left| u\left(t_k, x_n\right) - u_n^k \right|, \\ \|E_p\|_\infty = \max\limits_{-M<n<M} \left| p\left(x_n\right) - p_n \right|, \end{cases} \tag{22}$$

where $u\left(t,x\right)$, $p(x)$ represent the exact solution, u_n^k represent the numerical solutions at $\left(t_k, x_n\right)$ and p_n represent the numerical solutions at x_n. The numerical results are given in the Table 1.

Table 1.

Errors	$\|E_p\|_\infty$	$\|E_u\|_\infty$
$N=20, M=20$	0.1117	0.0195
$N=40, M=40$	0.0557	0.0101
$N=80, M=80$	0.0278	0.0052
$N=160, M=160$	0.0139	0.0026

Conclusion

In this paper, we considered a space source of identification problem for parabolic equation with involution and Dirichlet condition. The theoretical considerations that prove well-posedness theorem on the differential equation of the source identification parabolic problem and stability estimates for the difference schemes of the source identification parabolic problem were given. To support the theoretical results by a numerical experiment, we constructed a stable difference scheme for the approximate solution of the problem. Obtained results given in Table 1 also support the theoretical results.

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А. Ашыралыев, А.С. Ердоган, А. Сарсенби

Дирихле шартымен және инволюциясымен сәйкестендірілген параболалық теңдеу туралы ескерту

Дирихле шартымен және инволюциясымен сәйкестендірілген параболалық теңдеу үшін кеңістіктік есептері зерттелген. Параболалық дифференциалдық теңдеу үшін дереккөзді сәйкестендіру есебінің корректілігі теоремасы құрылған. Осы есептің жуық шешімі үшін орнықты айырымдық схемасы көрсетілген. Сонымен қатар, дереккөзді сәйкестендіру параболалық теңдеуінің орнықтылық айырымдық схемасының бағамы берілген. Сандық нәтижелер келтірілген.

Кілт сөздер: корректілігі, эллипстік теңдеу, оң таңбалы, коэрцитивті орнықтылық, дереккөзді сәйкестендіру, дәл бағамы, шеттік есеп.

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Замечание о параболической проблеме идентификации с инволюцией и условием Дирихле

Исследована пространственная задача идентификации источника для параболического уравнения с инволюцией и условием Дирихле. Установлена теорема корректности задачи идентификации источника для параболического дифференциального уравнения. Представлена устойчивая разностная схема для приближенного решения этой задачи. Кроме того, даны оценки устойчивости разностной схемы параболической задачи идентификации источника. Приведены численные результаты.

Ключевые слова: корректность, эллиптические уравнения, положительность, коэрцитивная устойчивость, идентификация источника, точные оценки, краевая задача.

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