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Method of functional parametrization for solving a semi-periodic initial problem for fourth-order partial differential equations

A semi-periodic initial boundary-value problem for a fourth-order system of partial differential equations is considered. Using the method of functional parametrization, an additional parameter is carried out and the studied problem is reduced to the equivalent semi-periodic problem for a system of integro-differential equations of hyperbolic type second order with functional parameters and integral relations. An interrelation between the semi-periodic problem for the system of integro-differential equations of hyperbolic type and a family of Cauchy problems for a system of ordinary differential equations is established. Algorithms for finding of solutions to an equivalent problem are constructed and their convergence is proved. Sufficient conditions of a unique solvability to the semi-periodic initial boundary value problem for the fourth-order system of partial differential equations are obtained.

Keywords: semi-periodic initial boundary-value problem, fourth-order system of partial differential equations, the method of functional parametrization, semi-periodic problem, system of integro-differential equations of hyperbolic type second order, family of Cauchy problems, algorithm, unique solvability.

Introduction

In the present paper, on the domain $\Omega = [0,T] \times [0,\omega]$ we consider the following semi-periodic initial boundary value problem for a fourth order system of partial differential equations

$$\frac{\partial^4 u}{\partial t^3 \partial x} = A_1(t,x)\frac{\partial^3 u}{\partial t^2 \partial x} + A_2(t,x)\frac{\partial^3 u}{\partial t^3} + A_3(t,x)\frac{\partial^2 u}{\partial t^2} + A_4(t,x)\frac{\partial^2 u}{\partial t \partial x} +$$

$$+A_5(t,x)\frac{\partial u}{\partial t} + A_6(t,x)\frac{\partial u}{\partial x} + A_7(t,x)u + f(t,x), \tag{1}$$

$$u(0,x) = \varphi_1(x), \qquad x \in [0,\omega], \tag{2}$$

$$\frac{\partial u(t,x)}{\partial t}|_{t=0} = \varphi_2(x), \qquad x \in [0,\omega], \tag{3}$$

$$\frac{\partial^2 u(t,x)}{\partial t^2}|_{t=0} = \frac{\partial^2 u(t,x)}{\partial t^2}|_{t=T}, \qquad x \in [0,\omega], \tag{4}$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \tag{5}$$

where $u(t,x) = col(u_1(t,x),...,u_n(t,x))$ is unknown function, the $n \times n$ matrices $A_i(t,x), (i = \overline{1,7})$, and n vector–function $f(t,x)$ are continuous on Ω; n vector–function $\psi(t)$ are continuously three times differentiable on $[0,T]$; the n vector–functions $\varphi_1(x)$ and $\varphi_2(x)$ are continuously differentiable on $[0,\omega]$.

Let $C(\Omega, \mathbb{R}^n)$ be a space of continuous on Ω vector functions $u(t,x)$ with the norm
$||u||_0 = \max\limits_{(t,x)\in\Omega} ||u(t,x)||$, $||u(t,x)|| = \max\limits_{i=\overline{1,n}} |u_i(t,x)|$.

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A function $u(t,x) \in C(\Omega, \mathbb{R}^n)$ having partial derivatives

$$\frac{\partial u(t,x)}{\partial t} \in C(\Omega, \mathbb{R}^n), \frac{\partial u(t,x)}{\partial x} \in C(\Omega, \mathbb{R}^n), \frac{\partial^2 u(t,x)}{\partial t \partial x} \in C(\Omega, \mathbb{R}^n), \frac{\partial^2 u(t,x)}{\partial t^2} \in C(\Omega, \mathbb{R}^n),$$
$$\frac{\partial^3 u(t,x)}{\partial t^2 \partial x} \in C(\Omega, \mathbb{R}^n), \frac{\partial^3 u(t,x)}{\partial t^3} \in C(\Omega, \mathbb{R}^n), \frac{\partial^4 u(t,x)}{\partial t^3 \partial x} \in C(\Omega, \mathbb{R}^n),$$

is called a classical solution to problem (1)–(5) if it satisfies system (1) for all $(t,x) \in \Omega$, and the initial and the boundary conditions (2)–(5).

Mathematical modeling of various processes in physics, ecology, chemistry, biology and others are leaded to initial - boundary value problems for a higher-order partial differential equations with variable coefficients and boundary functions [1, 2]. Despite the presence of numerous works, general statements of initial-boundary value problems for the higher-order system of partial differential equations remain poorly studied up to now. Therefore, the problems of solvability of initial-boundary value problems for the fourth-order system of partial differential equations are important in applied problems [1-8]. Some classes of initial-boundary value problems for systems of fourth-order hyperbolic equations are studied in [4-8].

Aim of the paper is to study issues for an existence and uniqueness of classical solutions to the semi-periodic initial boundary value problem for the fourth-order system of partial differential equations (1)–(5). We will establish coefficient criteria for its unique solvability and construct algorithms for finding its approximate solutions. For reaching this goal, we use method of functional parametrization [9-19] for solving the problem (1)–(5).

First, we introduce a new unknown function $w(t,x) = \dfrac{\partial^2 u(t,x)}{\partial t^2}$ and rewrite problem (1)–(5) in the following from

$$\frac{\partial^2 w}{\partial t \partial x} = A_1(t,x)\frac{\partial w}{\partial x} + A_2(t,x)\frac{\partial w}{\partial t} + A_3(t,x)w + f(t,x) +$$
$$+ A_4(t,x)\frac{\partial^2 u}{\partial t \partial x} + A_5(t,x)\frac{\partial u}{\partial t} + A_6(t,x)\frac{\partial u}{\partial x} + A_7(t,x)u, \tag{6}$$
$$w(0,x) = w(T,x), \qquad x \in [0,\omega], \tag{7}$$
$$w(t,0) = \ddot{\psi}(t), \qquad t \in [0,T], \tag{8}$$
$$\frac{\partial u}{\partial t} = \varphi_2(x) + \int_0^t w(\tau,x)\,d\tau, \quad u(t,x) = \varphi_1(x) + t \cdot \varphi_2(x) + \int_0^t \int_0^\tau w(\tau_1,x)\,d\tau_1 d\tau, \tag{9}$$
$$\frac{\partial^2 u}{\partial t \partial x} = \dot{\varphi}_2(x) + \int_0^t \frac{\partial w(\tau,x)}{\partial x}\,d\tau, \quad \frac{\partial u}{\partial x} = \dot{\varphi}_1(x) + t \cdot \dot{\varphi}_2(x) + \int_0^t \int_0^\tau \frac{\partial w(\tau_1,x)}{\partial x}\,d\tau_1 d\tau. \tag{10}$$

A solution of problem (6)–(10) is a function $w(t,x) \in C(\Omega, \mathbb{R}^n)$ having partial derivatives $\frac{\partial w(t,x)}{\partial t} \in$ $\in C(\Omega, \mathbb{R}^n)$, $\frac{\partial w(t,x)}{\partial x} \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial^2 w(t,x)}{\partial t \partial x} \in C(\Omega, \mathbb{R}^n)$, where the function $u(t,x)$ and its partial derivatives $\frac{\partial u(t,x)}{\partial t}$, $\frac{\partial u(t,x)}{\partial x}$ and $\frac{\partial^2 u(t,x)}{\partial t \partial x}$ are determined from integral relations (9), (10).

The method of functional parametrization is based on the introduction of additional parameters as the value of the desired solution on the line $t = 0$ of the domain Ω. The semi-periodic boundary-value problem for system of hyperbolic equations with integral conditions (6)–(10) is reduced to an equivalent semi-periodic problem for the system of integro-differential equations of hyperbolic type with functional parameter depending on x. The properties of solution and its partial derivatives pass into the properties of functional parameter. Using this method, we obtained coefficient conditions for the unique solvability of semi-periodic initial boundary value problem for the fourth-order system of partial differential equations (1)–(5).

Different types of initial-boundary value problems for some classes of fourth-order system of partial differential equations are studied in [20-22] by introducing additional new functions.

Scheme of the method functional parametrization without partitioning of the domain

We denote by $\lambda(x) = w(0,x)$ and in problem (6) – (10) make the change $\tilde{w}(t,x) = w(t,x) - \lambda(x)$. Then, the integral relations (9) and (10) have the following form

$$\frac{\partial u(t,x)}{\partial t} = \varphi_2(x) + t\cdot\lambda(x) + \int_0^t \tilde{w}(\tau,x)\,d\tau, \tag{11}$$

$$u(t,x) = \varphi_1(x) + t\cdot\varphi_2(x) + \frac{t^2}{2}\cdot\lambda(x) + \int_0^t\int_0^\tau \tilde{w}(\tau_1,x)\,d\tau_1 d\tau, \tag{12}$$

$$\frac{\partial^2 u(t,x)}{\partial t\partial x} = \dot{\varphi}_2(x) + t\cdot\dot{\lambda}(x) + \int_0^t \frac{\partial\tilde{w}(\tau,x)}{\partial x}\,d\tau, \tag{13}$$

$$\frac{\partial u(t,x)}{\partial x} = \dot{\varphi}_1(x) + t\cdot\dot{\varphi}_2(x) + \frac{t^2}{2}\cdot\dot{\lambda}(x) + \int_0^t\int_0^\tau \frac{\partial\tilde{w}(\tau_1,x)}{\partial x}\,d\tau_1 d\tau. \tag{14}$$

Further, in system (6) instead of functions $\frac{\partial u(t,x)}{\partial t}$, $u(t,x)$ $\frac{\partial^2 u(t,x)}{\partial t\partial x}$ and $\frac{\partial u(t,x)}{\partial x}$ we substitute their representations from (11)-(14), respectively. We get the following equivalent nonlocal problem for system of integro-differential equations of hyperbolic type with an unknown function $\lambda(x)$:

$$\frac{\partial^2\tilde{w}}{\partial t\partial x} = A_1(t,x)\frac{\partial\tilde{w}}{\partial x} + A_2(t,x)\frac{\partial\tilde{w}}{\partial t} + A_3(t,x)\tilde{w} + A_4(t,x)\int_0^t \frac{\partial\tilde{w}(\tau,x)}{\partial x}\,d\tau +$$

$$+A_5(t,x)\int_0^t \tilde{w}(\tau,x)\,d\tau + A_6(t,x)\int_0^t\int_0^\tau \frac{\partial\tilde{w}(\tau_1,x)}{\partial x}\,d\tau_1 d\tau + A_7(t,x)\int_0^t\int_0^\tau \tilde{w}(\tau_1,x)\,d\tau_1 d\tau +$$

$$+\Big[A_1(t,x) + A_4(t,x)t + A_6(t,x)\frac{t^2}{2}\Big]\dot{\lambda}(x) + \Big[A_3(t,x) + A_5(t,x)t + A_7(t,x)\frac{t^2}{2}\Big]\lambda(x) +$$

$$+f(t,x) + g_1(t,x) + g_2(t,x), \tag{15}$$

$$\tilde{w}(0,x) = 0, \qquad x\in[0,\omega], \tag{16}$$

$$\tilde{w}(t,0) = \ddot{\psi}(t) - \ddot{\psi}(0), \qquad t\in[0,T], \tag{17}$$

$$\tilde{w}(T,x) = 0, \qquad x\in[0,\omega], \tag{18}$$

where $g_1(t,x) = A_4(t,x)\dot{\varphi}_2(x) + A_5(t,x)\varphi_2(x)$,
$g_2(t,x) = A_6(t,x)[\dot{\varphi}_1(x) + t\cdot\dot{\varphi}_2(x)] + A_7(t,x)[\varphi_1(x) + t\cdot\varphi_2(x)]$.

The compatibility condition is valid:

$$\lambda(0) = \ddot{\psi}(0). \tag{19}$$

Problems (6)–(10) and (15)–(18) are equivalent in the sense that if the function $w(t,x)$ is a solution of problem (6)–(10), then the pair $\{\lambda(x) = w(0,x), \tilde{w}(t,x) = w(t,x) - w(0,x)\}$ will be a solution of problem (15)–(18), and vice versa, if a pair $\{\lambda(x), \tilde{w}(t,x)\}$ is a solution to problem (15)–(18), then the function $\{\lambda(x) + \tilde{w}(t,x)\}$ will be the solution to problem (6)–(10).

For fixed $\lambda(x)$, $\dot{\lambda}(x)$ the function $\tilde{w}(t,x)$ is a solution to the Goursat problem on Ω with conditions (16), (17). From (16), (17) we obtain $\frac{\partial\tilde{w}(0,x)}{\partial x} = 0$, $\frac{\partial\tilde{w}(t,0)}{\partial t} = \dddot{\psi}(t)$ and reduce the Goursat problem to an equivalent system of three integral equations

$$\frac{\partial\tilde{w}(t,x)}{\partial x} = \int_0^t \Big[A_1(\tau,x)\frac{\partial\tilde{w}(\tau,x)}{\partial x} + A_2(\tau,x)\frac{\partial\tilde{w}(\tau,x)}{\partial\tau} + A_3(\tau,x)\tilde{w}(\tau,x)\Big]d\tau +$$

$$+\int_0^t \Big[A_4(\tau,x)\int_0^\tau \frac{\partial\tilde{w}(\tau_1,x)}{\partial x}\,d\tau_1 + A_5(\tau,x)\int_0^\tau \tilde{w}(\tau_1,x)\,d\tau_1\Big]d\tau +$$

$$+\int_0^t \Big[A_6(\tau,x)\int_0^\tau\int_0^{\tau_1}\frac{\partial\tilde{w}(\tau_2,x)}{\partial x}\,d\tau_2 d\tau_1 + A_7(\tau,x)\int_0^\tau\int_0^{\tau_1}\tilde{w}(\tau_2,x)\,d\tau_2 d\tau_1\Big]d\tau+$$

$$+\int_0^t\Big[A_1(\tau,x)+A_4(\tau,x)\tau+A_6(\tau,x)\frac{\tau^2}{2}\Big]d\tau\dot{\lambda}(x)+\int_0^t\Big[A_3(\tau,x)+A_5(\tau,x)\tau+A_7(\tau,x)\frac{\tau^2}{2}\Big]d\tau\lambda(x)+$$

$$+\int_0^t[f(\tau,x)+g_1(\tau,x)+g_2(\tau,x)]d\tau, \qquad (20)$$

$$\frac{\partial\tilde{w}(t,x)}{\partial t}=\dddot{\psi}(t)+\int_0^x\Big[A_1(t,\xi)\frac{\partial\tilde{w}(t,\xi)}{\partial\xi}+A_2(t,\xi)\frac{\partial\tilde{w}(t,\xi)}{\partial t}+A_3(t,\xi)\tilde{w}(t,\xi)\Big]d\xi+$$

$$+\int_0^x\Big[A_4(t,\xi)\int_0^t\frac{\partial\tilde{w}(\tau,\xi)}{\partial\xi}\,d\tau+A_5(t,\xi)\int_0^t\tilde{w}(\tau,\xi)\,d\tau\Big]d\xi+$$

$$+\int_0^x\Big[A_6(t,\xi)\int_0^t\int_0^\tau\frac{\partial\tilde{w}(\tau_1,\xi)}{\partial\xi}\,d\tau_1 d\tau+A_7(t,\xi)\int_0^t\int_0^\tau\tilde{w}(\tau_1,\xi)\,d\tau_1 d\tau\Big]d\xi+$$

$$+\int_0^x\Big[A_1(t,\xi)+A_4(t,\xi)t+A_6(t,\xi)\frac{t^2}{2}\Big]\dot{\lambda}(\xi)d\xi+\int_0^x\Big[A_3(t,\xi)+A_5(t,\xi)t+A_7(t,\xi)\frac{t^2}{2}\Big]\lambda(\xi)d\xi+$$

$$+\int_0^x[f(t,\xi)+g_1(t,\xi)+g_2(t,\xi)]d\xi, \qquad (21)$$

$$\tilde{w}(t,x)=\dddot{\psi}(t)-\ddot{\psi}(0)+\int_0^x\frac{\partial\tilde{w}(\tau,\xi)}{\partial\xi}d\xi. \qquad (22)$$

Instead of $\frac{\partial\tilde{w}(\tau,x)}{\partial x}$, $\frac{\partial\tilde{w}(\tau_1,x)}{\partial x}$, $\frac{\partial\tilde{w}(\tau_2,x)}{\partial x}$ we substitute the corresponding right-hand side of (20) and, repeating this procedure $m(m=1,2,3,...)$ times, we obtain

$$\frac{\partial\tilde{w}}{\partial x}=D_m(t,x)\cdot\dot{\lambda}(x)+E_m(t,x)\cdot\lambda(x)+G_m\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)+H_m\Big(t,x,\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big)+F_m(t,x), \qquad (23)$$

where

$$D_m(t,x)=D_m^{(1)}(t,x)+D_m^{(2)}(t,x)+D_m^{(3)}(t,x),$$

$$E_m(t,x)=E_m^{(1)}(t,x)+E_m^{(2)}(t,x)+E_m^{(3)}(t,x),$$

$$G_m\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)=G_m^{(1)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)+G_m^{(2)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)+G_m^{(3)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big),$$

$$H_m\Big(t,x,\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big)=H_m^{(1)}\Big(t,x,\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big)+H_m^{(2)}(t,x,\tilde{w})+H_m^{(3)}(t,x,\tilde{w}),$$

$$F_m(t,x)=F_m^{(1)}(t,x)+F_m^{(2)}(t,x)+F_m^{(3)}(t,x),$$

$$D_m^{(1)}(t,x)=\int_0^t A_1(\tau_1,x)d\tau_1+\int_0^t A_1(\tau_1,x)\int_0^{\tau_1}A_1(\tau_2,x)d\tau_2 d\tau_1+$$

$$+...+\int_0^t A_1(\tau_1,x)...\int_0^{\tau_{m-1}}A_1(\tau_m,x)d\tau_m...d\tau_1,$$

$$E_m^{(1)}(t,x)=\int_0^t A_3(\tau_1,x)d\tau_1+...+\int_0^t A_1(\tau_1,x)...\int_0^{\tau_{m-2}}A_1(\tau_{m-1},x)\int_0^{\tau_{m-1}}A_3(\tau_m,x)d\tau_m...d\tau_1,$$

$$G_m^{(1)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)=\int_0^t A_1(\tau_1,x)...\int_0^{\tau_{m-2}}A_1(\tau_{m-1},x)\int_0^{\tau_{m-1}}A_1(t_m,x)\frac{\partial\tilde{w}(\tau_m,x)}{\partial x}d\tau_m...d\tau_1,$$

$$H_m^{(1)}\Big(t,x,\tilde{w},\frac{\partial\tilde{w}}{\partial t}\Big)=\int_0^t[A_2(\tau_1,x)\frac{\partial\tilde{w}}{\partial\tau_1}+A_3(\tau_1,x)\tilde{w}]d\tau_1+...+$$

$$+\int_0^t A_1(\tau_1,x)...\int_0^{\tau_{m-2}} A_1(\tau_{m-1},x)\int_0^{\tau_{m-1}}[A_2(\tau_m,x)\frac{\partial\tilde{w}}{\partial\tau_m}+A_3(\tau_m,x)\tilde{w}]d\tau_m...d\tau_1,$$

$$F_m^{(1)}(t,x)=\int_0^t f(\tau_1,x)d\tau_1+...+\int_0^t A_1(\tau_1,x)...\int_0^{\tau_{m-2}} A_1(\tau_{m-1},x)\int_0^{\tau_{m-1}} f(\tau_m,x)d\tau_m...d\tau_1,$$

$$D_m^{(2)}(t,x)=\int_0^t A_4(\tau_1,x)\cdot\tau_1 d\tau_1+\int_0^t A_4(\tau_1,x)\int_0^{\tau_1}\int_0^{\tau_2} A_4(\tau_3,x)\cdot\tau_3 d\tau_3 d\tau_2 d\tau_1+$$

$$+...+\int_0^t A_4(\tau_1,x)\int_0^{\tau_1}\int_0^{\tau_2} A_4(\tau_3,x)...\int_0^{\tau_{2m-3}}\int_0^{\tau_{2m-2}} A_4(\tau_{2m-1},x)\tau_{2m-1}d\tau_{2m-1}d\tau_{2m-2}...d\tau_1,$$

$$E_m^{(2)}(t,x)=\int_0^t A_5(\tau_1,x)\tau_1 d\tau_1+...+$$

$$+\int_0^t A_4(\tau_1,x)...\int_0^{\tau_{2m-5}}\int_0^{\tau_{2m-4}} A_4(\tau_{2m-3},x)\int_0^{\tau_{2m-3}}\int_0^{\tau_{2m-2}} A_5(\tau_{2m-1},x)\tau_{2m-1}d\tau_{2m-1}...d\tau_1,$$

$$G_m^{(2)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)=\int_0^t A_4(\tau_1,x)...\int_0^{\tau_{2m-3}}\int_0^{\tau_{2m-2}} A_4(\tau_{2m-1},x)\int_0^{\tau_{2m-1}}\frac{\partial\tilde{w}(\tau_{2m},x)}{\partial x}d\tau_{2m}...d\tau_1,$$

$$H_m^{(2)}(t,x,\tilde{w})=\int_0^t A_5(\tau_1,x)\int_0^{\tau_1}\tilde{w}(\tau_2,x)d\tau_2 d\tau_1+...+\int_0^t A_4(\tau_1,x)...$$

$$...\int_0^{\tau_{2m-5}}\int_0^{\tau_{2m-4}} A_4(\tau_{2m-3},x)\int_0^{\tau_{2m-3}}\int_0^{\tau_{2m-2}} A_5(\tau_{2m-1},x)\int_0^{2m-1}\tilde{w}(\tau_{2m},x)d\tau_{2m}...d\tau_1,$$

$$F_m^{(2)}(t,x)=\int_0^t g_1(\tau_1,x)d\tau_1+...+$$

$$+\int_0^t A_4(\tau_1,x)...\int_0^{\tau_{2m-5}}\int_0^{\tau_{2m-4}} A_4(\tau_{2m-3},x)\int_0^{\tau_{2m-3}}\int_0^{\tau_{2m-2}} g_1(\tau_{2m-1},x)d\tau_{2m-1}...d\tau_1,$$

$$D_m^{(3)}(t,x)=\int_0^t A_6(\tau_1,x)\cdot\frac{\tau_1^2}{2}d\tau_1+$$

$$+\int_0^t A_6(\tau_1,x)\int_0^{\tau_1}\int_0^{\tau_2}\int_0^{\tau_3} A_6(\tau_4,x)\cdot\frac{\tau_4^2}{2}d\tau_4 d\tau_3 d\tau_2 d\tau_1+...+\int_0^t A_6(\tau_1,x)...$$

$$...\int_0^{\tau_{3m-8}}\int_0^{\tau_{3m-7}}\int_0^{\tau_{3m-6}} A_6(\tau_{3m-5},x)\int_0^{\tau_{3m-5}}\int_0^{\tau_{3m-4}}\int_0^{\tau_{3m-3}} A_6(\tau_{3m-2},x)\frac{\tau_{3m-2}^2}{2}d\tau_{3m-2}...d\tau_1,$$

$$E_m^{(3)}(t,x)=\int_0^t A_7(\tau_1,x)\frac{\tau_1^2}{2}d\tau_1+...+\int_0^t A_6(\tau_1,x)...$$

$$...\int_0^{\tau_{3m-8}}\int_0^{\tau_{3m-7}}\int_0^{\tau_{3m-6}} A_6(\tau_{3m-5},x)\int_0^{\tau_{3m-5}}\int_0^{\tau_{3m-4}}\int_0^{\tau_{3m-3}} A_7(\tau_{3m-2},x)\frac{\tau_{3m-2}^2}{2}d\tau_{3m-2}...d\tau_1,$$

$$G_m^{(3)}\Big(t,x,\frac{\partial\tilde{w}}{\partial x}\Big)=$$

$$=\int_0^t A_6(\tau_1,x)...\int_0^{\tau_{3m-5}}\int_0^{\tau_{3m-4}}\int_0^{\tau_{3m-3}} A_6(\tau_{3m-2},x)\int_0^{\tau_{3m-2}}\int_0^{\tau_{3m-1}}\frac{\partial\tilde{w}(\tau_{3m},x)}{\partial x}d\tau_{3m}...d\tau_1,$$

$$H_m^{(3)}(t,x,\tilde{w})=\int_0^t A_7(\tau_1,x)\int_0^{\tau_1}\int_0^{\tau_2}\tilde{w}(\tau_3,x)d\tau_3 d\tau_2 d\tau_1+...+\int_0^t A_6(\tau_1,x)...\int_0^{\tau_{3m-8}}\int_0^{\tau_{3m-7}}$$

$$\int_0^{\tau_{3m-6}} A_6(\tau_{3m-5},x)\int_0^{\tau_{3m-5}}\int_0^{\tau_{3m-4}}\int_0^{\tau_{3m-3}} A_7(\tau_{3m-2},x)\int_0^{\tau_{3m-2}}\int_0^{\tau_{3m-1}}\tilde{w}(\tau_{3m},x)d\tau_{3m}...d\tau_1,$$

$$F_m^{(3)}(t,x) = \int_0^t g_2(\tau_1,x)d\tau_1 + ... +$$

$$+\int_0^t A_6(\tau_1,x)...\int_0^{\tau_{3m-8}}\int_0^{\tau_{3m-7}}\int_0^{\tau_{3m-6}} A_6(\tau_{3m-5},x)\int_0^{\tau_{3m-5}}\int_0^{\tau_{3m-4}}\int_0^{\tau_{3m-3}} g_2(\tau_{3m-2},x)d\tau_{3m-2}...d\tau_1.$$

Assumptions regarding the data of problem (6)–(10) allow us to differentiate relation (18) with respect to x:

$$\frac{\partial \tilde{w}(T,x)}{\partial x} = 0. \qquad (24)$$

Relation (24) will be equivalent to relation (18) if the compatibility condition (19) is satisfied.

From the right-hand side of (23), finding the value of $\tilde{w}(t,x)$ for $t = T$ and substituting it in (24), we obtain a system of n ordinary first-order differential equations that are not resolved with respect to the derivatives:

$$D_m(T,x)\cdot\dot{\lambda}(x) = -E_m(T,x)\cdot\lambda(x) - G_m\Big(T,x,\frac{\partial\tilde{w}}{\partial x}\Big) - H_m\Big(T,x,\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big) - F_m(T,x). \qquad (25)$$

For fixed $\frac{\partial\tilde{w}}{\partial x}$, $\frac{\partial\tilde{w}}{\partial t}$, $\tilde{w}$ system of differential equations (25) with initial condition (19) is the Cauchy problem with respect to $\lambda(x)$ for all $x\in[0,\omega]$. We solve the Cauchy problem (25), (19) using the fundamental matrix.

Let the matrix $D_m(T,x)$ be invertible for all $x\in[0,\omega]$ and $\Phi(x)$ the fundamental matrix to system of differential equations

$$\frac{d\lambda(x)}{dx} = -[D_m(T,x)]^{-1}E_m(T,x)\cdot\lambda(x). \qquad (26)$$

We re-write system (25) in the following form

$$\dot{\lambda}(x) = -[D_m(T,x)]^{-1}E_m(T,x)\cdot\lambda(x) - \tilde{F}\Big(T,x,\frac{\partial\tilde{w}}{\partial x},\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big), \qquad (27)$$

where

$$\tilde{F}\Big(T,x,\frac{\partial\tilde{w}}{\partial x},\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big) = -[D_m(T,x)]^{-1}\Big\{G_m\Big(T,x,\frac{\partial\tilde{w}}{\partial x}\Big) + H_m\Big(T,x,\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big) + F_m(T,x)\Big\}.$$

A solution to the Cauchy problem (27), (19) is written as

$$\lambda(x) = \Phi(x)\ddot{\psi}(0) + \Phi(x)\int_0^x \Phi^{-1}(\xi)\tilde{F}\Big(T,\xi,\frac{\partial\tilde{w}}{\partial\xi},\frac{\partial\tilde{w}}{\partial t},\tilde{w}\Big)d\xi, \qquad x\in[0,\omega].$$

Thus, the invertibility of the matrix $D_m(T,x)$ for all $x\in[0,\omega]$ allows us to find a solution to the original problem (1)–(5) by using the fundamental matrix of a system of ordinary differential equations (26) and constructing solutions to the Goursat problem (15)–(17).

Note that a similar technique was applied to the semi-periodic boundary value problem for systems of quasi-linear and semi-linear hyperbolic equations of second-order in [23-24]. These problems were reduced to an equivalent problems, consisting of a family of periodic boundary-value problems for quasi-linear and semi-linear ordinary differential equations, respectively, and functional relations. To solve a families of periodic boundary-value problems for ordinary differential the parametrization method were used. Algorithms for finding periodic boundary-value problem's solution for systems of the quasi-linear and semi-linear system of hyperbolic equations are offered. To construct the algorithms were used a solutions to families of Cauchy problems for systems of ordinary differential equations and systems of functional equations with respect to the introduced parameters. This approach allowed to establish sufficient conditions for the existence of an solution to considered problems.

Algorithm for finding solution to problem (6)–(10)

As well-known, the fundamental matrix can be constructed for a narrow class of differential equations. Therefore, we propose an algorithm for finding an approximate solution to problem (6)–(10) without using the fundamental matrix.

So, the method of functional parametrization divides the process of finding unknown functions into two stages:

1) finding the introduced functional parameter $\lambda(x)$ $(\dot{\lambda}(x))$ from system (25) with condition (19).

2) finding unknown functions $\frac{\partial\tilde{w}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}$, $\tilde{w}(t,x)$ from the system of integral equations (20)–(22).

If the functions $\dot{\lambda}(x)$, $\lambda(x)$ are known, then we will find the functions $\frac{\partial\tilde{w}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}$, $\tilde{w}(t,x)$ to solve the system of integral equations (20)–(22), and the function $\lambda(x)+\tilde{w}(t,x)$ will be the solution to problem (6)–(10). If the functions $\frac{\partial\tilde{w}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}$, $\tilde{w}(t,x)$ are known, then solving system of differential equations (25) with condition (19), we find $\dot{\lambda}(x)$, $\lambda(x)$ and again determining the sum of the functions $\lambda(x)+\tilde{w}(t,x)$ we find a solution to problem (6)–(10).

Here unknown are both the functions $\dot{\lambda}(x)$, $\lambda(x)$ and the functions $\frac{\partial\tilde{w}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}$, $\tilde{w}(t,x)$. Therefore, we use an iterative method and the solution to system of integral equations (20)–(22) and the Cauchy problem (25), (19) is found as the limits of the sequences $\{\dot{\lambda}(x),\lambda(x),\frac{\partial\tilde{w}(t,x)}{\partial x},\frac{\partial\tilde{w}(t,x)}{\partial t},\tilde{w}(t,x)\}$, determined by the following algorithm:

Step 0. Assuming on the right-hand side of (25) $\lambda(x)=\ddot{\psi}(0)$, $\frac{\partial\tilde{w}(t,x)}{\partial x}=0$, $\frac{\partial\tilde{w}(t,x)}{\partial t}=\dddot{\psi}(t)$, $\tilde{w}(t,x)=\dddot{\psi}(t)-\ddot{\psi}(0)$, and taking into account the invertibility of the matrix $D_m(T,x)$ for all $x\in[0,\omega]$, we find $\dot{\lambda}^{(0)}(x)$ from equation (25). Using conditions (19) we find the function $\lambda^{(0)}(x)$: $\lambda^{(0)}(x)=\ddot{\psi}(0)+\int\limits_0^x\dot{\lambda}^{(0)}(\xi)d\xi$, $x\in[0,\omega]$. From the system of integral equations (20)–(22), where $\lambda(x)=\lambda^{(0)}(x),\dot{\lambda}(x)=\dot{\lambda}^{(0)}(x)$, we define the functions $\frac{\partial\tilde{w}^{(0)}(t,x)}{\partial x},\frac{\partial\tilde{w}^{(0)}(t,x)}{\partial t},\tilde{w}^{(0)}(t,x)$ for all $(t,x)\in\Omega$.

Step 1. From equation (25), where on the right-hand side of $\lambda(x)=\lambda^{(0)}(x)$, $\frac{\partial\tilde{w}(t,x)}{\partial x}=\frac{\partial\tilde{w}^{(0)}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}=\frac{\partial\tilde{w}^{(0)}(t,x)}{\partial t}$, $\tilde{w}(t,x)=\tilde{w}^{(0)}(t,x)$, by virtue of the invertibility of $D_m(T,x)$ for all $x\in[0,\omega]$, we find $\dot{\lambda}^{(1)}(x)$.

Using conditions (19) again, we find the function $\lambda^{(1)}(x)=\ddot{\psi}(0)+\int_0^x\dot{\lambda}^{(1)}(\xi)d\xi$, $x\in[0,\omega]$. From the system of integral equations (20)–(22), where $\lambda(x)=\lambda^{(1)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(1)}(x)$, we define the functions $\frac{\partial\tilde{w}^{(1)}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}^{(1)}(t,x)}{\partial t}$, $\tilde{w}^{(1)}(t,x)$ for all $(t,x)\in\Omega$.

And so on.

Step k. From equation (25), where on the right-hand side of $\lambda(x)=\lambda^{(k-1)}(x)$, $\frac{\partial\tilde{w}(t,x)}{\partial x}=\frac{\partial\tilde{w}^{(k-1)}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}(t,x)}{\partial t}=\frac{\partial\tilde{w}^{(k-1)}(t,x)}{\partial t}$, $\tilde{w}(t,x)=\tilde{w}^{(k-1)}(t,x)$, by virtue of the reversibility of $D_m(T,x)$ for all $x\in[0,\omega]$ we find $\dot{\lambda}^{(k)}(x)$.

Using conditions (19), we find the function $\lambda^{(k)}(x)=\ddot{\psi}(0)+\int_0^x\dot{\lambda}^{(k)}(\xi)d\xi$, $x\in[0,\omega]$. From the system of integral equations (20)–(22), where $\lambda(x)=\lambda^{(k)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(k)}(x)$, we define the functions $\frac{\partial\tilde{w}^{(k)}(t,x)}{\partial x}$, $\frac{\partial\tilde{w}^{(k)}(t,x)}{\partial t}$, $\tilde{w}^{(k)}(t,x)$ for all $(t,x)\in\Omega$.

Here $k=1,2,3,\ldots$.

The following statement gives conditions for the convergence of the proposed algorithm and the unique solvability of problem (6)–(10) in terms of the initial data.

Theorem 1. Suppose that for some $m, m=1,2,3,...$, the $n\times n$ -matrix $D_m(T,x)$ is invertible for all $x\in[0,\omega]$ and the following inequalities hold:

a) $||[D_m(T,x)]^{-1}||\le\gamma_m(T,x)$, and $\gamma_m(T,x)$ is a positive continuous function for all $x\in[0,\omega]$;

b) $q_m(T,x) = \gamma_m(T,x) \cdot \left\{ e^{\alpha(x)T} - 1 - \alpha(x)T - ... - \frac{1}{m!}[\alpha(x)T]^m \right\} \leq \chi < 1,$
where $\alpha(x) = \max\limits_{t\in[0,T]} (||A_1(t,x)||, ||A_4(t,x)||, ||A_6(t,x)||)$*,* χ *is constant.*

Then there is a unique solution $w^*(t,x)$ *to problem (6)–(10), determining by equality*

$$w^*(t,x) = \lambda^*(x) + \tilde{w}^*(t,x)$$

with

$$\frac{\partial u^*(t,x)}{\partial t} = \varphi_2(x) + t \cdot \lambda^*(x) + \int_0^t \tilde{w}^*(\tau,x)\, d\tau,$$

$$u^*(t,x) = \varphi_1(x) + t \cdot \varphi_2(x) + \frac{t^2}{2} \cdot \lambda^*(x) + \int_0^t \int_0^\tau \tilde{w}^*(\tau_1,x)\, d\tau_1 d\tau,$$

$$\frac{\partial^2 u^*(t,x)}{\partial t \partial x} = \dot{\varphi}_2(x) + t \cdot \dot{\lambda}^*(x) + \int_0^t \frac{\partial \tilde{w}^*(\tau,x)}{\partial x}\, d\tau,$$

$$\frac{\partial u^*(t,x)}{\partial x} = \dot{\varphi}_1(x) + t \cdot \dot{\varphi}_2(x) + \frac{t^2}{2} \cdot \dot{\lambda}^*(x) + \int_0^t \int_0^\tau \frac{\partial \tilde{w}^*(\tau_1,x)}{\partial x}\, d\tau_1 d\tau,$$

where $\lambda^*(x) = \lim\limits_{k\to\infty} \lambda^{(k)}(x)$, $\dot{\lambda}^*(x) = \lim\limits_{k\to\infty} \dot{\lambda}^{(k)}(x)$ *for all* $x \in [0,\omega]$,

$\tilde{w}^*(t,x) = \lim\limits_{k\to\infty} \tilde{w}^{(k)}(t,x)$, $\frac{\partial \tilde{w}^*(t,x)}{\partial x} = \lim\limits_{k\to\infty} \frac{\partial \tilde{w}^{(k)}(\tau,x)}{\partial x}$ *for all* $(t,x) \in \Omega$.

Proof of the Theorem 1 is provided according to proposed algorithm above.

Therefore, from the equivalence of problems (6)–(10) and (1)–(5) it follows

Theorem 2. Suppose that for some $m, m = 1,2,3,...$*, the* $n \times n$ *-matrix* $D_m(T,x)$ *is invertible for all* $x \in [0,\omega]$ *and the inequalities a), b) of Theorem 1 are fulfilled.*

Then there is a unique classical solution $u^*(t,x)$ *to problem (1)–(5), defining from the following integral representation*

$$u^*(t,x) = \varphi_1(x) + t \cdot \varphi_2(x) + \int_0^t \int_0^\tau w^*(\tau_1,x)\, d\tau_1 d\tau, \qquad (t,x) \in \Omega.$$

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Төртінші ретті дербес туындылы дифференциалдық теңдеулер үшін жартылайпериодты бастапқы есепті шешудің функционалдық параметрлеу әдісі

Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін жартылайпериодты бастапқы шеттік есеп қарастырылды. Функционалдық параметрлеу әдісі көмегімен авторлар қосымша параметрін енгізіп, зерттеліп отырған есеп екінші ретті гиперболалық тектес интегралдық-дифференциалдық теңдеулер жүйесі үшін функционалдық параметрлері мен интегралдық қатынастары бар пара-пар жартылайпериодты есепке келтірді. Гиперболалық тектес интегралдық-дифференциалдық теңдеулер жүйесі үшін жартылайпериодты есеп пен жай дифференциалдық теңдеулер жүйесі үшін Коши есептері әулетінің өзара байланысы тағайындалған. Пара-пар есептің шешімін табу алгоритмдері құрылған және олардың жинақтылығы дәлелденген. Төртінші ретті дербес туындылы дифференциалдық теңдеулер үшін жартылайпериодты бастапқы шеттік есептің бірмәнді шешілімділігінің жеткілікті шарттары алынған.

Кілт сөздер: жартылайпериодты бастапқы шеттік есеп, төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі, функционалдық параметрлеу әдісі, жартылайпериодты есеп, екінші ретті гиперболалық тектес интегралдық-дифференциалдық теңдеулер жүйесі, Коши есептерінің әулеті, алгоритм, бірмәнді шешілімділік.

А.Т. Асанова, Ж.С. Токмурзин

Метод функциональной параметризации решения полупериодической начальной задачи для дифференциальных уравнений в частных производных четвертого порядка

Рассмотрена полупериодическая начальная краевая задача для системы дифференциальных уравнений в частных производных четвертого порядка. Авторами с помощью метода функциональной параметризации введен дополнительный параметр, и исследуемая задача сведена к эквивалентной полупериодической задаче для системы интегро-дифференциальных уравнений гиперболического типа второго порядка с функциональными параметрами и интегральными соотношениями. Установлена взаимосвязь полупериодической задачи для системы интегро-дифференциальных уравнений гиперболического типа и семейства задач Коши для системы обыкновенных дифференциальных уравнений. Построены алгоритмы нахождения решений эквивалентной задачи и доказана их сходимость. Получены достаточные условия однозначной разрешимости полупериодической начальной краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка.

Ключевые слова: полупериодическая начальная краевая задача, система дифференциальных уравнений в частных производных четвертого порядка, метод функциональной параметризации, полупериодическая задача, система интегро-дифференциальных уравнений гиперболического типа второго порядка, семейство задач Коши, алгоритм, однозначная разрешимость.

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