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Mapes of secondary sources in the problem of ERT probing 2D medium: numerical method and analytical solutions

The paper considers a mathematical model of electrical tomography above the media with local inclusions. Numerical solutions of a system of integral equations for a medium with local inclusion are compared against a numerical implementation of the analytical solution of the problem for a case of a sphere in homogeneous space. The parameters of local inclusion and the depth of heterogeneity are varied. Maps of secondary sources in the ERT (Electrical Resistivity Tomography) probing problem are constructed: for local inclusion in the form of the ellipsoid, an ellipsoid in a homogeneous space (analytical solution of the problem) and for two-layer half-spaces as well. Numerical results are presented, and maps of secondary sources in the cases where the immersed heterogeneity is an insulator and a conductor are computed.

Keywords: map of secondary sources; analytical solution of the problem with immersed heterogeneity; ellipsoid in a homogeneous space; the integral equation method.

Introduction

Modeling the problems of electrical exploration is very relevant nowadays. The solution of direct and inverse problems of electrical tomography are the main subject of many works ([1–13] and references therein). In solving the problems of electrical tomography [14, 15], the finite element method is used; the novelty of our work is the application of the method of integral equations [16–22] to the solution of the problem specified below.

When modeling the electric field in complex media, it is important to take into account the geometric parameters of the desired objects (shape, number of elements, depth and dimensions) [23, 24]. The listed geometric parameters strongly affect the amplitude and shape of the electric field anomalies [23]. Theoretical calculations by analytical formulas and numerical algorithms should be performed for models that are found in the practice of geophysical research.

In addition to the size and the depth, the resistivity of the heterogeneity and the peculiarity of the medium such as angles of incidence of flat boundaries and the orientation of the buried object have a great influence on the results of the work [24]. In our work, we consider a model containing heterogeneity in the form of an ellipsoid located in a homogeneous half-space and full space (analytical solution) and a two-layer horizontally layered medium.

The numerical results are obtained for two types of modeling:

1. Tests of the numeral solutions have been performed using the method of integral equations against the analytical solutions by A. I. Zaborovsky [23].

2. The distribution of secondary sources on the earth's surface and the internal contact boundary are shown.

The study of the electrical field for such kind of media is important for isolating and tracing local objects, their depth and surface shape. Approximate solutions are known for a sphere, and for compressed and elongated ellipsoids in a homogeneous medium [23, 24].

A special case of an ellipsoid is a sphere in a homogeneous medium. Due to the complexity of calculations by explicit formulas, for mass calculations of field parameters in the software, approximate solutions of A.I. Zaborovsky are implemented [23, 25].

The electrical potential inside the medium with a sphere near the surface of a half-space is determined using the exact formulas [23]. Depending on the location of the source and receiving electrodes along the measuring profile, which does not necessarily passes above the center of the inclusion, there are four possible analytical formulas for the potential of the electrical field:

$$U_1 = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + \int_{n=0}^{\infty}\frac{npc^n}{d^{n+1}}P_n(\cos\theta)\right], d > a > r;$$

$$U_2 = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + \int_{n=0}^{\infty}\frac{npd^n}{c^{n+1}}P_n(\cos\theta)\right], c > a > d; \tag{1}$$

$$U_3 = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + \int_{n=0}^{\infty}\frac{(-1)pa^{2n+1}}{(cd)^{n+1}}P_n(\cos\theta)\right], a > r, d > a;$$

$$U_4 = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + \int_{n=0}^{\infty}\frac{(-1)p(n+1)(cd)^n}{c^{2n+1}}P_n(\cos\theta)\right], a > c, d < a;$$

where r is a distance from the source to the receiver; a is a radius of the half sphere; d, c are the distances between the supply and receiving electrodes and the center of the hemisphere, respectively; θ is the angle between directions c and d; $P_n(\cos\theta)$ is the Legendre polynomial of the first kind of the order n from cos θ and

$$\theta = \begin{cases} 0 \\ \pi \end{cases}, \quad P_n(\cos\theta) = \begin{cases} 1^n \\ (-1)^n \end{cases}.$$

Using these formulas (1), we can numerically solve the problem with submerged heterogeneity in the form of a ball and compare with the solution of the problem with immersed inhomogeneity obtained by the method of integral equations [16] - [22].

The second part of our testing was the construction of distributions of secondary field sources. Secondary sources of the anomalous field appear as a result of the excitation of the Earth's surface by a source electrode. Maps of secondary sources determine abnormal electric fields.

The authors constructed a map of secondary sources for a two-layer half-space, and for an immersed inhomogeneity by the method of integral equations and for the analytical solution for a ball in a homogeneous space according to A.I. Zaborovsky to show how secondary sources are distributed over the surface.

Numerical solutions

As mentioned above, the algorithm was tested in two ways: comparing the solution obtained by the method of integral equations with the analytical solution by A.I. Zaborovsky. For the best of our knowledge there is no analytical solutions of the problem for the heterogeneity placed inside a homogeneous half-space. On the other hand, we implement the method of integral equations for half-space, whereas the Zaborovsky solution is obtained for full-space; therefore, to make a reasonable comparison, the inclusion should be placed in such depth where the boundary of the half space does not significantly impact on the electric field.

Comparison with the analytical solution has been carried out for different parameters of the immersed heterogeneity, its size and depth. In the computations by the method of integral equations, when we placed the ball to the lower depths than $z = 1.5\ r$, the influence of the inhomogeneity on the anomalies of the resistivity curves became very small, so we have to reduce the depth of the inclusion. But in the analytical solution of A.I. Zaborovsky, the higher we lift up the ball from this depth, the more the difference in models appears, namely, the reflection from the boundary of upper half-space influences on the electric field.

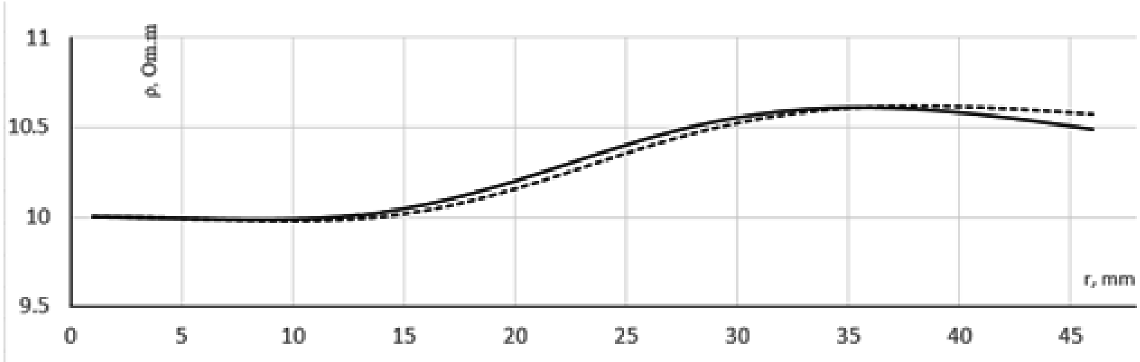

Figure 1. Comparison of a solution by the method of integral equations with an analytical solution by A.I. Zaborovsky, (-) - a solution by the method of integral equations, (- -) - an analytical solution by A.I. Zaborovsky

With the parameters of the sphere $a=1\ r$, $z = 1.5\ r$, it turned out that similar results are obtained. With these data, a comparison result about of 5% is obtained.

To construct maps of secondary sources for each case, media models are considered for inclusions that are an insulator or a conductor.

In Figure 2, the upper layer is flat, the parameters of local inclusion in taken the form of an ellipsoid at a depth of $z = 0.5\ r$ with parameters $ax = 0.21r$, $by = 0.2128\ r$, $cz = 0.21\ r$ with layer resistivities $\rho 1 = 10\ Ohm{\cdot}m$, $\rho 2 = 100\ Ohm{\cdot}m$. Although Matlab's state-of-the-art math package makes it possible to construct triangulations, for our purposes these triangulations turn to be unacceptable. This is due to the fact that the thickening of the grid should occur in the vicinity of the measuring line, and the source and measuring electrodes should be located at the vertices of the triangles, in nodes with the same geometry of triangulation. Therefore, we have had to construct our own algorithm of the triangulation. An example of a grid constructed for a case with spherical local inclusion is shown in Figure 2.

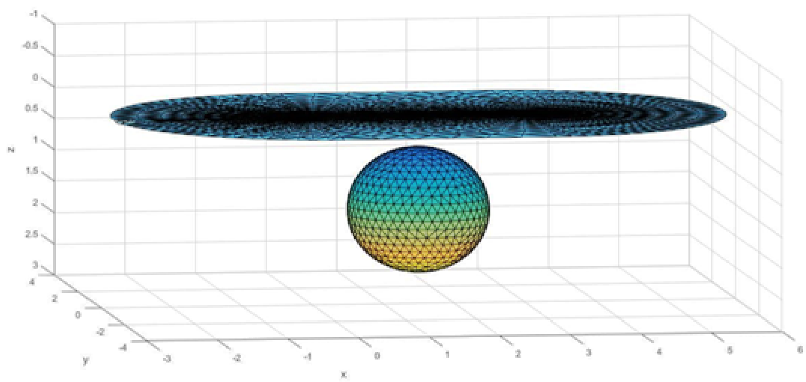

Figure 2. Triangulation for spherical inclusion under the flat surface

Figure 3 shows the secondary sources for resistivities of the surrounding medium $\rho 1 = 10\ Ohm{\cdot}m$ and an inclusion with resistivity $\rho 2 = 100\ Ohm{\cdot}m$.

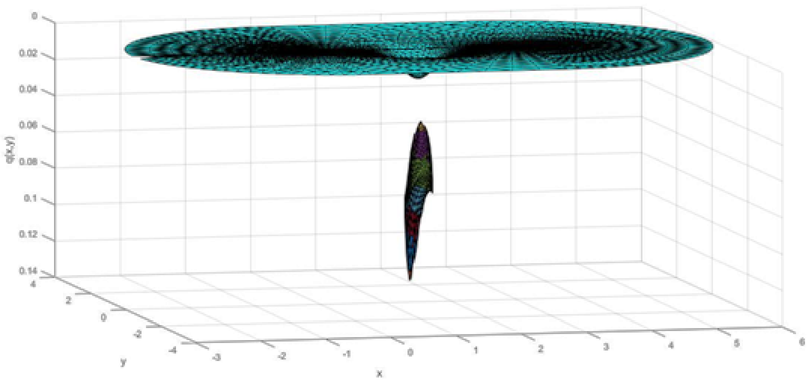

Figure 3. Map of secondary sources at $\rho 1$ = *10 Ohm·m* and $\rho 2$ = *100 Ohm·m*

Figure 4 shows a map of secondary sources for the case of resistivity of the surrounding medium $\rho 1$ = *100 Ohm·m* and there is an immersed conductor with resistivity $\rho 2$ = *10 Ohm·m*.

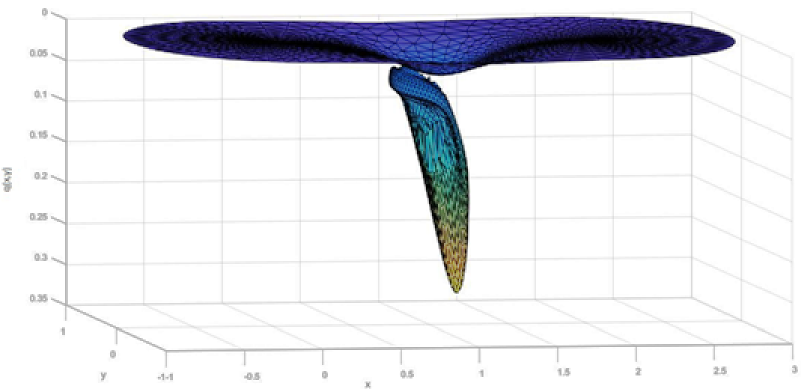

Figure 4. Map of secondary sources for $\rho 1$ = *100 Ohm·m and* $\rho 2$ = *10 Ohm·m*

Figure 5 shows a ball in homogeneous space, the parameters of the ball *a=1 r*. The triangulation constructed for this case is shown in Figure 5.

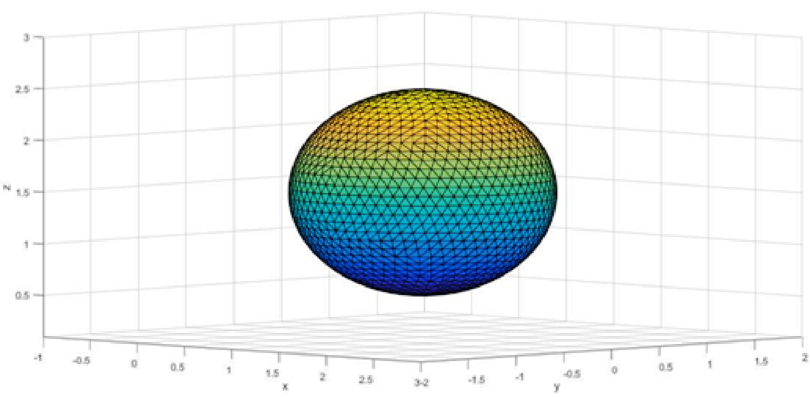

Figure 5. Triangulation constructed for a ball in a homogeneous space

Figure 6 shows a map of secondary sources projected on the plane (Oxy) for a ball in a homogeneous space, for resistivities $\rho 1$ = *10 Ohm·m,* $\rho 2$ = *100 Ohm·m*.

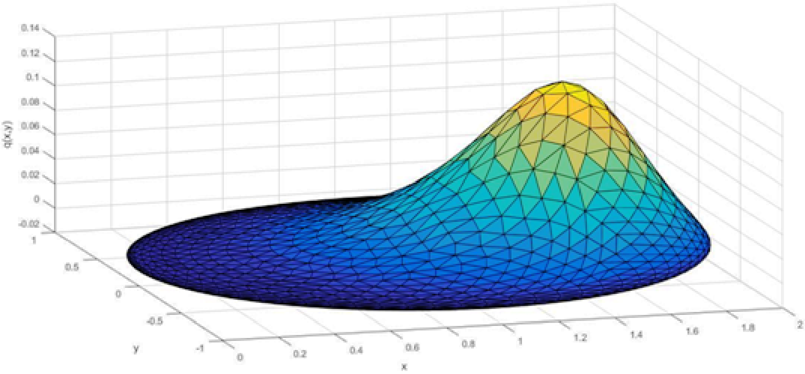

Figure 6. Map of secondary sources at *ρ1 = 10 Ohm·m and ρ2 = 100 Ohm·m*

Figure 7 shows a map of secondary sources projected on the plane (Oxy) for the resistivity of the containing medium *ρ1 = 100 Ohm·m* and the resistivity of the ball *ρ2 = 10 Ohm·m*.

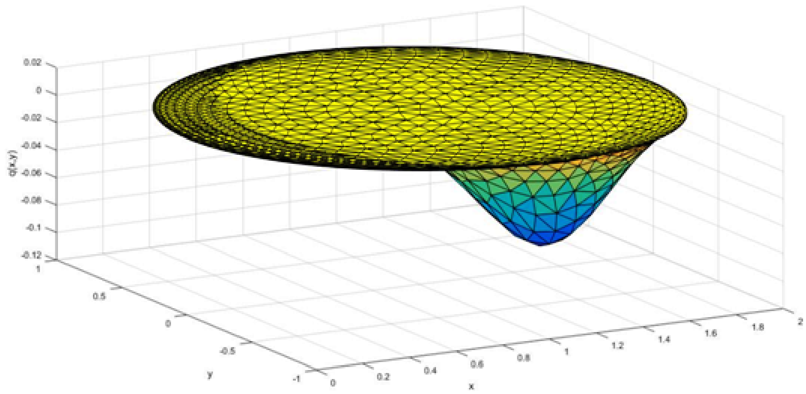

Figure 7. Map of secondary sources at *ρ1 = 100 Ohm·m* and *ρ2 = 10 Ohm·m*

Additional computations are performed for a two-layer medium in a half-space; both layers are supposed to be plane. The triangulation constructed for this case is shown in the Figure 8.

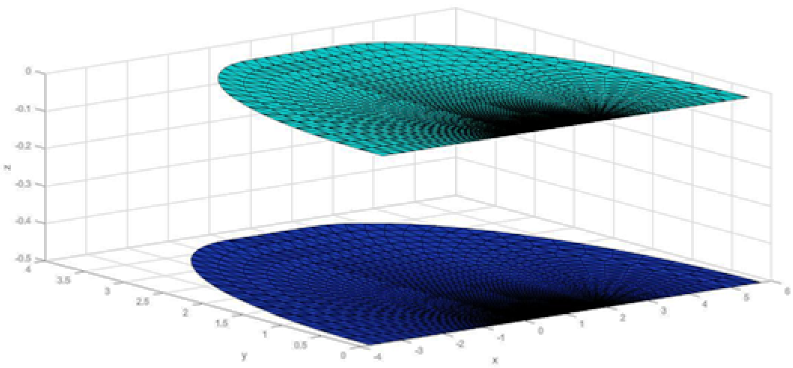

Figure 8. Triangulation built for a two-layer environment

Figure 9 shows a map of secondary sources for the case when the second insulating layer has a resistivity *ρ2 = 100 Ohm·m* and is contacting with the layer of *ρ1 = 10 Ohm·m*.

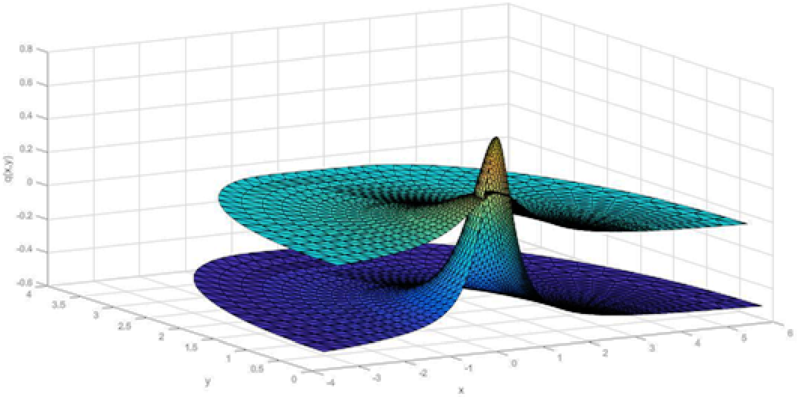

Figure 9. Map of secondary sources at *ρ1 = 10 Ohm·m and ρ2 = 100 Ohm·m*

Figure 10 shows a map of secondary sources for the resistivity of the upper layer *ρ1 = 100 Ohm·m*, and the lower layer with *ρ2 = 10 Ohm·m*.

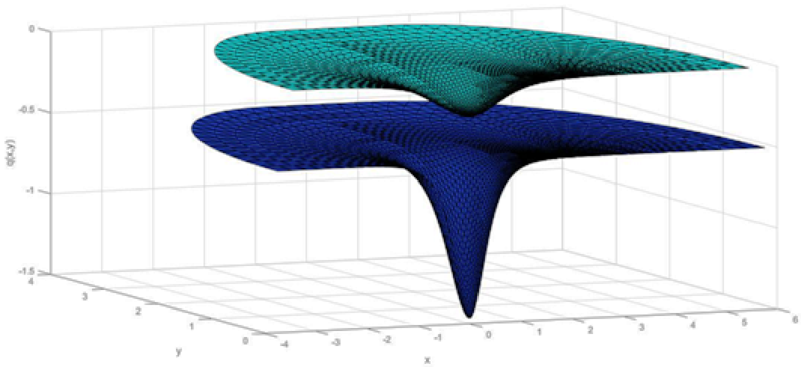

Figure 10. Map of secondary sources with *ρ1 = 100 Ohm·m and ρ2 = 10 Ohm·m*

Conclusion

Numerical solutions obtained by the method of integral equations are compared with analytical solutions of A.I. Zaborovsky. It turns out that even the models are different (half space and full space) the difference in apparent resistivity curves for the depth of inclusion $a = 1.5r$ are above 5%. Maps of secondary sources of the electric field for the following cases are constructed:

– buried inclusion by the method of integral equations,

– a sphere in homogeneous space according to A.I. Zaborovsky,

– a case of two-layer medium.

Calculations by the method of integral equations have shown that the distribution of secondary sources on the surface of inhomogeneities that determine the structure of an anomalous electric field is close to the solutions known in the theory of geophysics.

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2D орталарының ERT зондтау есебіндегі қайталама зарядтардың карталары: сандық әдіс және аналитикалық шешімдер

Мақалада локальді қосылуы бар аналитикалық шешім үшін электрлік томографияның математикалық моделі қарастырылған. Интегралдық теңдеулер жүйесіне негізделген эллипсоид түрінде қосылуы бар математикалық модель мен біртектес ортадағы шар аналитикалық шешімнің сандық есептелуі салыстырылды. Екі жағдай үшін локальді қосылудың параметрлерімен тереңдігін өзгертіп, оптималды тереңдік анықталды. Сонымен қатар, ERT барлау есебінде, эллипсойд және біртектес ортадағы шар түрінде локальді қосылуы бар және екі қабатты орта жағдайлары үшін қосалқы зарядтар картасы құрылған. Аналитикалық шешіммен салыстырудың сандық нәтижелерімен аталып кеткен үш жағдайдың өткізгіш пен изолятор орналасқан қосалқы зарядтар картасы келтірілген.

Кілт сөздер: қосалқы зарядтар картасы, қондырылған біртектесі бар аналитикалық шешім, біртектес ортадағы шар, интегралдық теңдеулер әдісі.

Д.С. Ракишева, И.Н. Модин, Б.Г. Муканова

Карты вторичных источников в задаче ERT зондирования 2D сред: численный метод и аналитические решения

В статье рассмотрена математическая модель электрической томографии для аналитического решения с локальным включением. Проведено сравнение с математической моделью, основанной на системе интегральных уравнений с локальным включением в виде эллипсоида, с численным решением аналитического решения задачи с шаром в однородном пространстве. Варьировались параметры локального включения и глубина залегания для определения оптимальной глубины для обоих случаев. Построены карты вторичных источников в задаче ERT зондирования: для локального включения в виде эллипсоида, для эллипсоида в однородном пространстве (аналитическое решение задачи) и двухслойных полупространств. Приведены численные результаты для сравнения с аналитическим решением и карты вторичных источников в случаях, когда погружен изолятор или проводник.

Ключевые слова: карта вторичных источников, аналитическое решение задачи с погруженной неоднородностью, эллипсоид в однородном пространстве, метод интегральных уравнений.

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