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Determining stress intensity factor in bending reinforced Concrete beams

There has been analytically solved the problem of determining the stress state in the cross section of bending reinforced concrete beams with a crack in a linear formulation. For this, the beam is cut along the crack line and the equilibrium condition of the cut-off part of the beam determines the height of the compression zone and the tensile stress at the crack tip. The remaining parameters of the stress state are expressed in terms of these values. The value of the bending moment is determined, above which there takes place increasing the initial length of the crack. For this case, the length of the operational crack is determined. The solution is valid for beams of arbitrary section shape. Determining the stress intensity factor (SIF) is based on the assumptions that the longitudinal forces at the tip of the crack are equal with and without taking into account stress concentration. The size of the stress concentration zone is determined from the condition that the local stress is equal to the nominal stress. On this basis a formula has been obtained for determining the stress intensity factor in rectangular beams. The paper analyzes the dependence of the stress intensity factor on the crack length, the moment and other geometric and operational factors. The obtained results allow estimating the bearing capacity of beams with a crack, as well as their crack resistance by a stress intensity factor.

Keywords: reinforced concrete, beam, bending, compressed zone, crack, reinforcement, stress state, bearing capacity, stress intensity factor, crack resistance.

A very common type of defects in reinforced concrete structures are cracks. They appear both at the manufacturing stage and at the operation stage [1–3]. The appearance of cracks in bending elements does not mean exhaustion of its bearing capacity. It leads to increasing the efforts in the sections with a crack, which reduces the strength of the element. Due to the opening of the crack width corrosion of reinforcement increases that reduces the durability of structures. The calculation of the stress state of reinforced concrete structures with cracks is the focus of many books and articles [1–9]. The crack opening width is one of the criteria for the ultimate state of reinforced concrete structures with cracks. Its definition is dealt with a lot of works [10–14]. The issues of crack formation are considered in works [15–17].

The above problems are dealt with in a series of experimental studies [8, 18, 19]. A lot of studies have been carried out by numerical methods [6, 7, 11, 20–22].

Under certain conditions unstable crack development is possible, which is estimated through the parameters of fracture mechanics [1, 23]. The stress intensity factor (SIF) is the main parameter of linear fracture mechanics. Some works dealing with determining the SIF in reinforced concrete beams have appeared only recently [24, 25]. It should be noted that these calculations were carried out in a linear formulation. However, the relationship between stress and strain in concrete is not linear. In this paper an approximate analytical method is proposed for determining the SIF in reinforced concrete beams.

We will first define the nominal stress in a beam with a crack of the known length l in a linear formulation. Let's consider an I-section with the vertical axis of symmetry (Fig. 1, *a*). We will introduce the notations:

A_{ct}, A_c is the area of the shelves overhang in the tension and compression zones;

h_t, h_c is the thickness of the shelves in the tension and compression zones;

a, a' is the thickness of the protective layer of concrete in the tension and compression zones;

A_s, A'_s is the area of reinforcement in the tension and compression zones;

N_a, N'_a are internal forces in the reinforcement in the tension and compression zones.

Let's cut the beam by the section passing through the crack and show the curve of the stress distribution in it (Fig. 1, *b*).

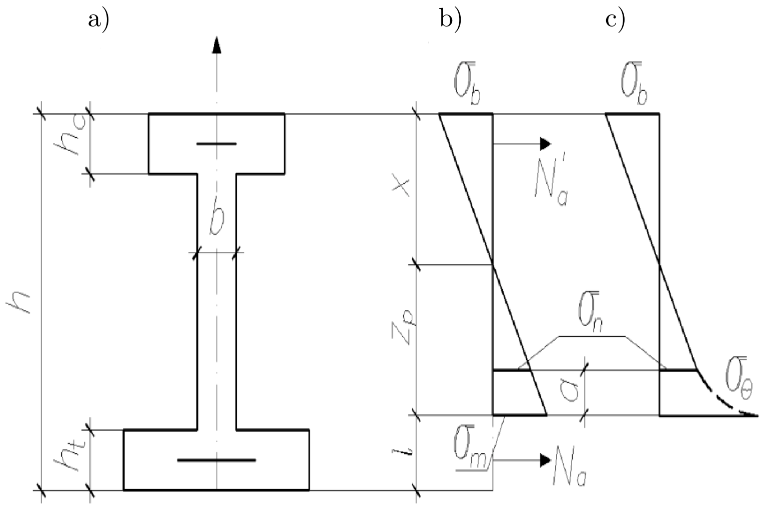

Figure 1. Towards determining stresses and SIF

The main unknown tasks will be tensile stress at the tip of the crack σ_m and the height of the compressed zone x. Given the linearity of the stress profile, we will determine the maximum compressive stress in concrete

$$\sigma_b = \sigma_m E/z_p, \tag{1}$$

where z_p is the height of the tensile zone.

In the zone of the crack the adhesion of the reinforcement to the concrete is broken, and the tensile forces are mainly perceived by the reinforcement. In sections between cracks tensile forces are also perceived by reinforcement and concrete. The deformations and stresses in the reinforcement and concrete, as well as the height of the compressed zone between the cracks vary. This non-uniformity in the calculations is taken into account by introducing special coefficients that are equal to the ratios of the average values in the section between the cracks and the values in the section with a crack [2]

$$\psi_s = \varepsilon_{sm}/\varepsilon_s, \quad \psi_b = \varepsilon_{bm}/\varepsilon_b.$$

Taking this into account, we will determine the deformations at the reinforcement level as:

$$\varepsilon_s = \varepsilon_m \frac{\psi_b}{\psi_s} \cdot \frac{h-a-x}{z_p} = \varepsilon_m \psi_{bs}(h-a-x)/z_p, \quad \varepsilon_s' = \varepsilon_m \psi_{bs}(x-a')/z_p.$$

The stress in the reinforcement will be

$$\sigma_s = E_s\varepsilon_s = \alpha\sigma_m\psi_{bs}(h-a-x)/z_p \quad \sigma_s' = \sigma_{sp}' - E_s\varepsilon_s' = \sigma_{sp}' - \alpha\sigma_m\psi_{bs}(x-a')/z_p, \tag{2}$$

where α is the ratio of elasticity modulus of the reinforcement and concrete; σ_{sp}' is pre-stress in the reinforcement of the compression zone.

The condition of longitudinal forces equilibrium in the section

$$\sigma_m b z_p/2 + \sigma_s A_s - \sigma_b b x/2 + \sigma_s' A_s' - \sigma_b A_c(x - h_c/2)/x = 0.$$

The second equilibrium equation in the form of the sum of all the forces moments relative to the zero line has the form

$$\sigma_b b x^2/3 + \sigma_m b z_p^2/3 + \sigma_s A_s(h-a-x) + \sigma_b A_c(x-h_c/2)^2/x - \sigma_s' A_s'(x-a') = M_{bn},$$

where the external moment M_{bn} is equal to the bending moment in the section with a crack M for the reinforcement without pre-stress. For the pre-stressed reinforcement of the tension zone the σ s stress occurs after the moment of external forces M exceeds the moment of pre-stress force. Then in the equation the external moment will be

$$M_{bn} = M - \sigma_{sp} A_s(h-a-x).$$

The equilibrium equations are common for all reinforced concrete elements with and without pre-stress, with different cross-sectional shapes: I-shaped, T-shaped, rectangular beams. For a T-section A_c or A_{ct} is equal to zero. For a rectangular cross section, the area of both overhangs is zero. In order to obtain explicit calculation ratios, we will perform a further solution for the most common case of a rectangular cross section with reinforcement in the tension zone. In this case the equilibrium equations will take the form:

$$\sigma_m b z_p/2 + \sigma_s A_s - \sigma_b b x/2 = 0;$$

$$\sigma_b b x^2/3 + \sigma_m b z_p^2/3 + \sigma_s A_s(h - a - x) = M_{bn}.$$

Let's introduce the following dimensionless parameters:

$$\xi = x/h, \quad z = l/h, \quad \lambda = z_p/h = 1 - z - \xi, \quad \overline{h} = (h - a)/h, \quad \mu = A_s/bh.$$

Taking into account the expressions for σ_b and σ_s, the equilibrium equations will transform into the form:

$$\lambda^2 - \xi^2 + \alpha\mu\psi_{bs}(\overline{h}_0 - \xi) = 0;$$

$$\sigma_m\left[\xi^3/\lambda + \lambda^2 + 3\alpha\mu\psi_{bs}(\overline{h} - \xi)^2/\lambda\right] = 3M_{bn}/bh^2.$$

From the first equation we will determine the height of the compression zone, and then that of the tension zone

$$\xi = \frac{(1-z)^2 + \alpha\mu\psi_{bs}\overline{h}}{2(1-z) + \alpha\mu\psi_{bs}}, \quad \lambda = 1 - z - \xi. \tag{3}$$

From the second equation of equilibrium we will determine the nominal stress at the crack tip

$$\sigma_m = LM_{bn}/W, \tag{4}$$

where

$$W = bh^2/6, \quad L = 0.5\lambda/\left[\xi^3 + \lambda^3 + 3\alpha\mu\psi_{bs}(\overline{h} - \xi)^2\right].$$

The $\lambda(\xi)$, L parameters do not depend on the section dimensions and the acting loads. Therefore, it is expedient to build in advance the graph of these parameters dependence on the crack length. Figure 2 shows such a graph for

$$\overline{h} = 0.907, \quad \nu = \alpha\mu\psi_{bs} = (200/24)\cdot 0.015(0.6/0.5) = 0.15.$$

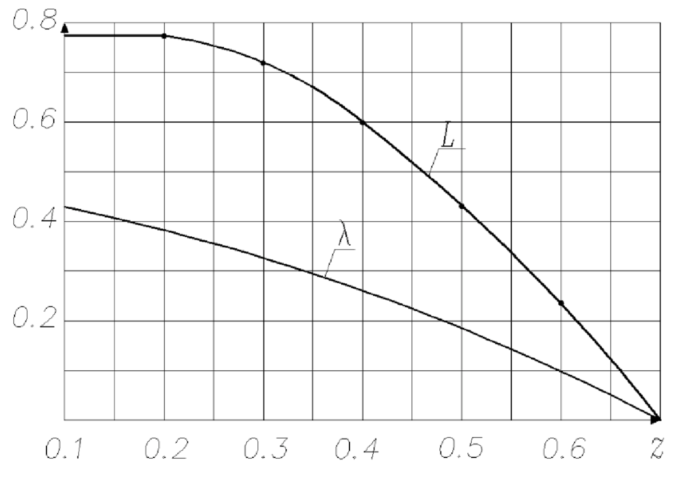

Figure 2. Stress state parameters dependence on the crack length

This solution is valid as long as the nominal stress is less than or equal to the concrete tensile strength R_{bt}. Otherwise, the initial crack length will increase. The value of the moment at which this happens is equal to

$$M_m = R_{bt}W/L.$$

Using the graph in Figure 2 and this formula, we can find the moment value for crack growth M_m. The second line of Table 1 shows the values of this moment in kNm for the beam section (15×30) cm, where the destructive moment $M_p = 53.6kNm$.

If the nominal stress at the crack tip σ_m exceeds the tensile strength of the concrete $R_{bt}(M_{bn} > M_m)$, then the crack length will increase. To determine a new crack length in (4), we assume $L = R_{bt}W/M$. Then, from the system of nonlinear equations (3) and (4) we also determine ξ and z. To solve this system, we can use the graphs in Figure 2.

Let's determine the SIF at the crack tip K_I. The tip of the crack is a strong stress concentrator. According to the linear theory of elasticity, the stress there is determined by the formula

$$\sigma_\theta = K_I/\sqrt{2\pi r}, \quad (5)$$

where r is the distance from the crack tip to the point considered.

Determining the SIF is based on the equality of the longitudinal forces at the crack tip with and without stress concentration. Local stresses are determined by formula (5), and nominal stresses by formulas (3) and (4).

Let us determine the length of the stress concentration zone "a" from the condition of equality of the nominal and local stresses at the end of this zone (Fig. 1, c). The rated stresses at the distance «a» from the crack tip

$$\sigma_n = \sigma_m(1 - a/z_p) = \sigma_m(1 - t).$$

Equating the local stress to this value, we will obtain

$$a = K_I^2/2\pi\sigma_n^2.$$

The longitudinal forces per unit width of the beam in the zone of concentration are

$$I_1 = \int_0^a \sigma_\theta dr = K_I\sqrt{2a/\pi} = K_I^2/\pi\sigma_n.$$

The longitudinal forces in the same zone without considering concentration are equal

$$I_2 = \frac{\sigma_m + \sigma_n}{2}a = \frac{\sigma_m + \sigma_n}{2}\frac{K_I^2}{2\pi\sigma_n^2}.$$

Equating these forces, we will obtain

$$\sigma_m = 3\sigma_n.$$

From here it follows that $t = 2/3$.

Now we will determine the SIF

$$K_I = \sqrt{2\pi a}\sigma_n = 0.683\sigma_m\sqrt{z_p}.$$

Substituting here (4), we will finally obtain

$$K_I = 0.683L\sqrt{\lambda h}M_{bn}/W. \quad (6)$$

If the nominal stress at the crack tip σm exceeds the tensile strength of the concrete $R_{bt}(M_{bn} > M_m)$, then it is necessary to find a new operational crack length, and the SIF is calculated using the formula

$$K_I = 0.683\sqrt{\lambda h}R_{bt}.$$

Let's analyze the SIF changing dependent on the crack length. In third line of Table 1 there is shown the SIF changing ($N/m^{3/2}$) in the beam of the (15×30) cm section at the acting moment $M = 4.5kNm$ ($M < M_m$).

Table 1

SIF change depending on the crack length

Z	0.1	0.2	0.3	0.4	0.5	0.6	0.7
M_m	4.61	4.63	5.0	6.0	8.28	15.5	$> M_p$
K_I	0.378	0.347	0.293	0.217	0.131	0.051	0

We see that with increasing the crack length, the SIF decreases to zero. This is explained by the fact that in bending there necessarily exists a minimum zone of compressive stresses. With increasing the length of the crack, this reduces the length of the zone of tensile stresses to zero. At $z_p = 0$ tensile forces are perceived only by the reinforcement, the crack does not develop. In this case, the SIF becomes equal to zero and the beam is destroyed due to the achievement of the limit of the yield strength in the reinforcement or crushing the concrete of the compression zone.

If, with a known crack length and a given moment, the SIF will be smaller than the maximum SIF value $_{IC}$ for this material, then there is a steady development of the crack. If $K_I \geq K_{IC}$, the crack is unstable and a rapid crack growth is possible with a slight increase of the moment. The growth of the crack length continues until the SIF drops to the $_{IC}$ value. This SIF value corresponds to a certain value of the crack length, which is determined from expression (6). This value is easier to find graphically by plotting the $_I$ dependence on z using the graphs in Figure 2.

The assessment of the bearing capacity of the beam is made by the magnitude of the stress in the reinforcement (2) and the maximum compressive stress in the concrete (1). The parameters σ_m, x, z_p, found through the initial ($M < M_m$) or operational ($M > M_m$) crack length are substituted into these formulas.

Let's analyze the SIF changing dependent on the crack length at $M = 6kHm$. From Table 1 we conclude that in this case $M > M_m$ for $z_0 = 0.1 \div 0.3$, $M = M_m$ at $z_0 = 0.4$ and $M < M_m$ at $z_0 = 0.5$ and 0.6. At $z_0 \leq 0.4$

$$L = 1.6 \cdot 10^3 \cdot 2.25 \cdot 10^{-3}/6 = 0.6.$$

From the graph in Figure 2 we will find $\lambda = 0,233$ and then by the formula for the SIF we will obtain

$$K_I = 0.683 \cdot 1.6\sqrt{0.233 \cdot 0.3} = 0.29MN/m^{3/2}.$$

At $z_0 = 0.5$

$$\sigma = M/W = 6 \cdot 10^3/2.25 \cdot 10^{-3} = 2.667MPa.$$

By the graph in Figure 2 we will find $L = 0.435$, $\lambda = 0.164$. Then by formula (6) we will obtain $K_I = 0.175MN/m^{3/2}$. Similarly we determine the SIF at $z_0 = 0.6 : K_I = 0.069MN/m^{3/2}$.

From here we see that at $M > M_m$ the SIF does not depend on the length of the initial crack. It depends only on the length of the operational crack that depends on the acting moment.

Let's analyze the SIF changing with the external moment changing. To do this, we will calculate the SIF at $z_0 = 0.2$ ($M_m = 4.63kNm$). The results are shown in Table 2, where the moment is in kNm, the SIF is in $MN/m^{3/2}$.

Table 2

SIF changing dependent on the moment

M	2	4	4.63	6	8	12	16	20
K_I	0.154	0.308	0.357	0.29	0.255	0.203	0.178	0.159

The Table shows that if $M < M_m$, then the SIF increases in direct proportion to the moment, reaching its maximum at $M = M_m$. At $M > M_m$ the SIF decreases due to increasing the length of the operational crack.

The obtained results allow estimating the bearing capacity of beams with a crack, as well as their crack resistance by the stress intensity factor.

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Иілетін темірбетонды арқалықтардағы кернеудің қарқындылық коэффициентін анықтау

Есептің аналитикалық шешімі жарығы бар иілетін темірбетонды арқалықтардың көлденең қимасындағы кернеуді анықтайды. Ол үшін арқалық жарық бойымен кесіледі және арқалықтың бөлінген бөлігінің тепе-теңдік жағдайынан сығылған аймақтық биіктігі мен жарықтың ұшындағы созылатын кернеу анықталды. Кернеудің қалған параметрлері осы өлшемдер арқылы көрсетілді. Жарықтың бастапқы ұзындығының ұлғаюына байланысты иілу моменті анықталды. Ол үшін жарықтың пайдалану ұзындығы анықталған. Бұл қиманың еркін формасындағы арқалықтар үшін әділ шешім. Кернеудің қарқындылық коэффициентін анықтау кернеудің шоғырлануын елемей және жарық ұшындағы бойлық күштердің теңдігіне негізделген. Кернеудің шоғырлану аймағының өлшемі жергілікті кернеудің номиналдық кернеудегі теңдік шарттарынан анықталды. Осы негізде арқалықтардың тікбұрышты қимасындағы кернеудің қарқындылық коэффициентін анықтайтын формула алынған. Мақалада кернеудің қарқындылық коэффициенті жарықтың ұзындығына, моментіне, басқа да геометриялық және пайдалану факторларына тәуелділігі талданған. Алынған нәтижелер жарығы бар арқалықтардың көтергіш қабілетін, сондай-ақ кернеудің қарқындылық коэффициенті бойынша олардың жарыққа төзімділігін бағалауға мүмкіндік береді.

Кілт сөздер: темірбетон, арқалық, иілу, сығылған аймақ, жарық, арматура, кернеулі жағдай, салмақ көтеру қабілеті, кернеудің қарқындылық коэффициенті, жарыққа төзімділік.

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Определение коэффициента интенсивности напряжений в изгибаемых железобетонных балках

Аналитически решена задача об определении напряженного состояния в сечении изгибаемых железобетонных балок с трещиной в линейной постановке. Для этого балка разрезается по линии трещины и из условий равновесия отсеченной части балки определяются высота сжатой зоны и растягивающее напряжение у вершины трещины. Остальные параметры напряженного состояния выражаются через эти величины. Определено значение изгибающего момента, при превышении которого происходит увеличение первоначальной длины трещины. Для этого случая определена длина эксплуатационной трещины. Решение справедливо для балок произвольной формы сечения. Определение коэффициента интенсивности напряжений основано на предположении о равенстве продольных сил у вершины трещины с учетом и без учета концентрации напряжений. Размер зоны концентрации напряжений определяется из условия равенства местного напряжения номинальному напряжению. На этой основе получена формула для определения коэффициента интенсивности напряжений в балках прямоугольного сечения. В работе проанализирована зависимость коэффициента интенсивности напряжений от длины трещины, момента и других геометрических и эксплуатационных факторов. Полученные результаты позволяют оценить несущую способность балок с трещиной, а также их трещиностойкость по коэффициенту интенсивности напряжения.

Ключевые слова: железобетон, балка, изгиб, сжатая зона, трещина, арматура, напряженное состояние, несущая способность, коэффициент интенсивности напряжений, трещиностойкость.

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