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Chains of existentially closed models of positive (n_1, n_2)-Jonsson theories

In this article are considered model-theoretical properties of chains of positive (n_1, n_2)-Jonsson theories. Herewith considered theories is perfect in the sense of the existence of appropriate model companion. The main obtained results are as follows: introduced new concepts n_2-elimination of quantifier for positive theory, (n_1, n_2)-Jonsson theory, n_1-Jonsson chain; indicated a feature of perfect (n_1, n_2)-positive Jonsson theory.

Keywords: (n_1, n_2)-Jonsson theory, positive (n_1, n_2)-Jonsson theory, n_2-elimination of quantifier, n_1-Jonsson chain.

This article is devoted to the study of positive properties of (n_1, n_2)-Jonsson theories. At the study of positive Jonsson theories in the framework of [1], were considered various aspects of positivity which appear at translating of properties of Jonsson theories into the language of positivity [2].

In this paper, we continue to study (n_1, n_2)-Jonsson theories with the condition of positive axiomatizability, and instead of morphisms we consider immersions, special cases of homomorphisms [1]. The indices n_1, n_2 respectively determine n_1-model completeness, n_2-elimination of quantifiers. These indices are related to the concepts of chains of theories under consideration.

Many classical examples from algebra satisfy the natural axioms that define Jonsson theories. On Jonsson theories, more detailed information can be extracted in the monograph [3] and in the works [4–8]. We consider groups, Abelian groups, Boolean algebras, fields of fixed characteristic, rings, modules, polygons. All of these types of algebras are examples of such algebras whose theories are Jonsson. Abelian groups and Boolean algebras are good because their invariants are known, that is, some constants that allow us to describe these types of algebras up to elementary equivalence. In Jonsson theories, the concepts of elementary equivalence are replaced by cosemanticness. In the works [9, 10] are presented results that describe Abelian groups and modules up to cosemanticness. In the case of Boolean algebras the work [11] is known were defined generalized Jonsson theories (α-Jonsson) were defined and a description of α-Jonson theories of Boolean algebras was developed on the basis of the apparatus developed in the framework of the definitions of this article. Regardless of the study of Jonsson theories, in [12] the authors considered various generalizations of the concepts of model completeness and elimination of quantifiers. Also, within the framework of these generalizations, n-chains were considered in [12] which naturally appear as a consequence of the n-inductance of the theories under consideration. In fact, all of the above concepts definitely are related to the central concept of Robinson model theory, namely, the concept of a model companion. Well known the achievements of A. Robinson and his followers related to the application of theorems on the properties of model companions of the considered theories for classical algebraic objects, such as fields of fixed characteristic, Boolean algebras, Abelian groups, rings, modules, etc. Unfortunately, not all inductive theories have a model companion. It is well known from [13] that a theory has a model companion iff the class of its existentially closed models is elementary. In particular, the Jonsson theories that have a model companion are reasonably well arranged in the sense that their semantic invariant is a saturated model. Also in [14], was considered a generalization of the concept of a model companion, but somewhat from a different perspective. Therefore, in view of the foregoing, it would be interesting to redefine some of the concepts from works [11, 12] and [14], in the framework of the study of positive Johnson theory, where all the above properties of companions of Jonsson theories will be considered in the framework of positivity, which will be defined in this work.

Now we want to define the notion of (n_1, n_2)-positive Jonsson's theories.

Let L be a first-order language. At is a set of atomic formulas of the language. $B^+(At)$ is a closed set with respect to the positive Boolean combinations (conjunction and disjunction) of all atomic formulas and their subformulas and substitution variables. $Q_n(B^+(At))$ is a set of formulas in prenix normal form obtained by the use of quantifiers (∀ and ∃) to $B^+(At)$, where n is a number of quantifier changes in prenix. We call the formula positive if it belongs to $Q_n(B^+(At))$. The theory is positively axiomatizable if its axioms are positive.

Following [15] we define the Δ-morphisms between structures.

Let M and N, the structure of language, $\Delta \in Q_n(B^+(At))$. The mapping $h : M \to N$ is called Δ-homomorphism (in symbols $h : M \underset{\Delta}{\longrightarrow} N$) if for any $\varphi(\overline{x}) \in \Delta$, $\forall \overline{a} \in M$ from the fact that $M \models \varphi(\overline{a})$, it follows that $N \models \varphi(h(\overline{a}))$.

Following [2], the model M is said to begin in N and we say that M continues to N, with $h(M)$ is a continuation of M. If the map h is injective, we say that h immersion from M into N (symbolically h : $M \underset{\Delta}{\longleftrightarrow} N$). In the following we will use the terms Δ-continuation and Δ-immersion. In frames of this definition (Δ-homomorphism), it is easy to see that an isomorphic embedding, and an elementary embedding are Δ-immersions, when $\Delta = B(At)$ and $\Delta = L$, respectively.

Definition 1 [15]. If C - class of L-structures, then we say that an element M of C Δ-positive existential closed in C if every Δ-homomorphism from M to any element of C is a Δ-immersion. The class of all existential positive Δ-closed models will be denoted by $(E_C^{\Delta})^+$; if $C = ModT$ for some theory T, then by E_T, $(E_T^{\Delta})^+$ we mean, respectively, the class of existential closed and Δ-positive existential of the closed models of theory T.

Definition 2 [15]. We say that theory T admits a $\Delta - JEP$, if for any two $A, B \in ModT$ exists $C \in ModT$ and Δ-homomorphisms $h_1 : A \underset{\Delta}{\longrightarrow} C$, $h_2 : B \underset{\Delta}{\longrightarrow} C$.

Definition 3 [15]. We say that theory T admits a of $\Delta - AP$, if for any $A, B, C \in ModT$ such that $h_1 : A \underset{\Delta}{\longrightarrow} C$, $g_1 : A \underset{\Delta}{\longrightarrow} B$, where h_1, g_1 – Δ-homomorphisms, there exists a $D \in ModT$ and $h_2 : C \underset{\Delta}{\longrightarrow} D$, $g_2 : B \underset{\Delta}{\longrightarrow} D$, where h_2, g_2 – Δ-homomorphisms such that $h_2 \circ h_1 = g_2 \circ g_1$.

Definition 4 [15]. The theory T is called Δ-positive Jonsson's $(\Delta - PJ)$-theory if it satisfies the following conditions:

1) T has an infinite model;
2) T positive ∀∃-axiomatizable;
3) T admits $\Delta - JEP$;
4) T admits $\Delta - AP$.

When $\Delta = B(At)$ we obtain the usual Jonsson's theory, the only difference that it has only positive ∀∃-axiom.

In future, by (n_1, n_2)-positive Jonsson theory we understand some Δ-positive Jonsson theory $(\Delta - PJ)$ which n_1-positive model complete and n_2-positive elimination of quantifiers (E.Q.). Positive model completeness is model completeness of theory without using a symbol ¬. Positive E.Q. is a redaction to positive quantifiers-free formulas.

We give the necessary definitions of concepts which we will use.

A concept of model completeness introduced by A. Robinson is played large role in the study of model companions of various types of classical algebras.

Definition 5 [12]. A theory T is model complete if for any $B, D \in ModT$ and B is a submodel of D, then $B \prec D$.

Definition 6 [13]. A theory T is called model companion of T if:

1) T and T^* are mutually model consistent;
2) T^* is model complete.

The following fact is well known, connecting the theory with model companion with respect to E.Q.

Lemma 1 [13]. 1) Let T^* be model companion of theory T, where T is universal theory. In this case T^* is model completion of T iff T^* admit quantifier elimination.

2) Let T^* be model companion of theory T. In this case T^* is model completion of T iff T^* has amalgam property.

In the framework of concept of n-embedding, following work [11], we have n-Jonsson theory that differs from concept of generalized α-Jonsson theory only a semantic model [16]. Continuing to work within the framework of n-Jonsson theory we can fully define the concept n-model completeness, n-model companion and correspondingly on this base to prove the theorem which is a criterion of perfectness of generalized α-Jonsson theory [from 11] in following view.

Proposition 1 [11]. Let T be arbitrary n-Jonsson theory, then the following conditions are equivalent:

1) T is perfect;
2) T^* is n-model companion of T.

In work [12] are given following definitions generalizing a concepts of model completeness and E.Q.

Definition 7 [12]. $B \subseteq_n D$ means that $B \models \psi(\bar{b})$ if $D \models \psi(\bar{b})$ for all $\bar{b} \in B$ and all Σ_n (or Π_n) formulas ψ.

Definition 8 [12]. 1. A theory T is n-model complete if for all models B, D of T, $B \subseteq_n D$ implies $B \prec D$.

2. T is nearly n-model complete if any formula is equivalent (mod T) to Boolean combination of Σ_{n+1} (or Π_{n+1}) formulas.

3. A chain $\{B_n\}_{n\in\omega}$ under $\subseteq_n$ is called a n-chain.

Definition 9. We will call the theory n_2-E.Q. if any formula of this theory is presented in the view of Boolean combination of formulas from $Q_{n_2}(B(At))$.

Combining these concepts in the framework of study of Jonsson theories we can distinguish the following class of Jonsson theories:

Definition 10. Theory T is called (n_1, n_2)-Jonsson theory if it is n_1-model complete and n_2-E.Q.

In work [12] was determined the next kind of chain of models for some theory and statements with respect to this concept as 1-model completeness.

Definition 11 [12]. A chain $\{B_k\}_{k\in\omega}$ is eventually elementary if for all $\bar{b} \in \cup_{k\in\omega} B_k$ and all formulas $\psi(\overline{x})$ exists some $k_0 \in \omega$ such that either $B_k \models \psi(\bar{b})$ for every $k \geq k_0$ or $B_k \models \neg\psi(\bar{b})$ for every $k \geq k_0$.

Proposition 2 [12]. A theory T is 1-model complete iff for any formulas $\psi(\overline{x})$ exists a formula $\varphi(\overline{x})$ which is a $\forall_2 \cap \exists_2$-formulas such that $T \models \forall\overline{x}[\psi \leftrightarrow \varphi]$.

If the chain $\{B_k\}_{k\in\omega}$ is not eventually elementary, then there are $\psi(\overline{x})$ and $\bar{b}$ such that $B_k \models \psi(\bar{b})$ and $B_m \models \neg\psi(\bar{b})$ both hold for arbitrarily large $k, m \in \omega$. Thus, such a chain can be refined to be an alternating chain for $\psi(\overline{x})$ as in the following definition.

Definition 12 [12]. A chain $\{B_k\}_{k\in\omega}$ is an alternating chain for $\psi(\overline{x})$ if there is some $\bar{b} \in B_0$ such that $B_{2k} \models \psi(\bar{b})$ and $B_{2k+1} \models \neg\psi(\bar{b})$ for all $k \in \omega$.

Proposition 3. A theory T is 1-E.Q. iff for any formulas $\psi(\overline{x})$ exists a formula $\varphi(\overline{x})$ which is Boolean combination of $\forall_1 \cap \exists_1$-formulas such that $T \models \forall\overline{x}[\psi \leftrightarrow \varphi]$.

Recall the basic definitions and statements from work [11], in which are determined all basic concepts related with α-Jonsson theories. In our case, $\alpha = n_1$.

Definition 13 [11]. Let $\Gamma \subset L$. Then

1) map $f : A \to B$ is called a Γ-embedding if for any $\overline{a} \in A$ and $\varphi(\overline{x}) \in \Gamma$ and $A \models \varphi(\overline{a})$ follows $B \models \varphi(f(\overline{a}))$;

2) if $B \subseteq \mid A \mid$, then $Th_\Gamma(A, B)$ denote a set of all Γ-sentence of language L_B which are true in A;

3) if $A \subseteq B$, then notation $A \subseteq_\Gamma B$ denote that $Th_\Gamma(A, |A|) \subseteq Th_\Gamma(B, |B|)$;

4) sequence of models A_i, $i < \beta$, is called a Γ-chain if $A_i \subseteq_\Gamma A_j$ at $i < j < \beta$.

Lemma 2 [11]. map $f : A \to B$ is a Π_α-embedding iff it is a $\Sigma_{\alpha+1}$-embedding.

Definition 14 [11]. Theory T preserved with respect to the union of Π_α-chains (or α-inductive) if an union of any Π_α-chain of model of T again is a model of T.

Definition 15 [11]. A model $M \models T$ is called $\Sigma_{\alpha+1}$-saturated model if for any subset $E \subseteq \mid M \mid$ less power than M, for any model $B \models T$ such that $M \subseteq_{\Pi_\alpha} B$, and any element $b \in B$ exists element $m \in M$ satisfying an inclusion $Th_{\Sigma_{\alpha+1}}(M, E \cup m) \supseteq Th_{\Sigma_{\alpha+1}}(B, E \cup b)$.

Theorem 1 [11]. Let T be α-Jonsson theory, $M \models T$. Then the following conditions are equivalent:

1) M is $T - \alpha$-universal $T - \alpha$-homogeneous model;

2) M is $\Sigma_{\alpha+1}$-saturated model.

Proposition 4 [11]. For α-Jonsson theory T the following conditions are equivalent:

1) T is perfect;

2) T^* is α-model completion (i.e. $D(T^*) - \alpha$-model completion) of T.

Definition 16 [11]. A model $A \models T$ is called $\Sigma_{\alpha+1}$-closed if for any model $B \models T$ and any formula $\varphi(\overline{x}) \in \Sigma_{\alpha+1}$ with constants from A is performed $A \models \exists\overline{x}\varphi(\overline{x})$ provided that $A \subseteq_{\Pi_\alpha} B$ and $B \models \exists\overline{x}\varphi(\overline{x})$.

In future, a set $\Sigma_{\alpha+1}$-closed models of theory T denote by $\Sigma_{n_1}(T)$.

Definition 17. A chain $\{B_k\}_{k\in\omega}$ is called n_1-Jonsson if it will consist only from models $\Sigma_{n_1}(T)$.

The following facts are well known.

Lemma 3 [17]. Let A_β, $\beta < \alpha$, be some Σ_n-chain of models and $A = \cup_{\beta<\alpha} A_\beta$. Then

1) a model A is a Σ_n-extension of every model A_β;

2) any Π_{n+1}-sentence which is true in all models A_β, is true in A.

Theorem 2 [17]. The following statements are equivalent (where $n > 0$):

1) sentence φ equivalent to both some Σ_{n+1} and some Π_{n+1};

2) sentence φ equivalent to some Boolean combination Σ_n.

The result on the stability of theory is well known with respect to arbitrary chain of her models. Such theory must be $\forall\exists$-axiomatizable. The following definition is considered a generalization of concept of induction, i.e. stability with respect to a chain of models.

Definition 18 [17]. A theory T is stable with respect to a union Π_α-chains (or α-inductive) if a union of any Π_α-chain of models T is again a model of T.

The following result is known in the connection with the above definition.

Proposition 5 [17]. The following conditions are equivalent:

1) $T \in \Pi_{\alpha+2}C_\Delta$;

2) a theory T is α-inductive.

A class of models of her center in the connection with perfectness α-Jonsson theory coincides with a class of her $\Sigma_{\alpha+1}(T)$-models.

Lemma 4 [17]. Let T be perfect α-Jonsson theory, T^* be her center, $A \models T$. Then $A \in \Sigma_{\alpha+1}(T) \Leftrightarrow A \models T^*$.

In future, we will work in the case when the theory n_1-model complete and so that to use the result (Theorem 2) in the definition of $\Delta - PJ$-theory we suppose that Δ is contained in the $Q_{n_1}(B^+(At))$, where $Q_{n_1} = \Sigma_{n_1} \cap \Pi_{n_1}$.

The following theorem is positive generalization of theorem (Theorems 2.6 and 2.7 from [12]) in the framework of study of (n_1, n_2)-positive Jonsson theories.

Theorem 3. Let T be perfect (n_1, n_2)-positive Jonsson theory, where $n_1 = n_2 + 1; n_1, n_2 > 0$. Then the following conditions are equivalent:

1) T is n_2-E.Q.;

2) any n_2-chain of models which belong to $\Sigma_{n_2}(T)$ is Jonsson eventually elementary;

3) T is n_1-model complete;

4) any n_2-chain of models, which belong to $\Sigma_{n_2}(T)$, whose union is also a model of $\Sigma_{n_2}(T)$, is Jonsson eventually elementary.

It is easy to see that since all homomorphisms are immersions, we can use all the necessary results from work [11] without loss of generality for positive (n_1, n_2)-Jonsson theory.

Essential point in the proof of following result is a perfectness of Jonsson theory in the sense of work [11].

Proof. A equivalence of the aforecited statements follows from following three equivalences:

a) (2) equivalent to (4) follows from Proposition 5 and Lemma 4;

b) from (1) in (3) proof is obvious;

c) from (3) in (1) follows from Theorem 2;

d) (1) equivalent to (2) follows from Theorem 2.6. [12].

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Позитивті (n_1, n_2)-йонсондық теориялардың экзистенциалды-тұйық модельдерінің тізбелері

Мақалада (n_1, n_2)-йонсондық теориялар тізбелерінің модельді-теоретикалық қасиеттері қарастырылған. Сонымен қатар қарастырылып отырған теориялар модельді компаньоннің мағынасына сәйкес келеді. Негізгі алынған нәтижелер мынадай: жаңа ұғымдар енгізілді; n_2 позитивті теориялар үшін n_2-кванторлар элиминациясы, (n_1, n_2)-йонсондық теориясы; n_1-йонсондық тізбе; (n_1, n_2)-позитивті йонсондық теориясының ерекшелігі көрсетілген.

Кілт сөздер: (n_1, n_2)-йонсондық теория, позитивті (n_1, n_2)-йонсондық теория, n_2-кванторлар элиминациясы, n_1-йонсондық тізбе.

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Цепи экзистенциально-замкнутых моделей позитивных (n_1, n_2)-йонсоновских теорий

В статье рассмотрены теоретико-модельные свойства цепей позитивных (n_1, n_2)-йонсоновских теорий. При этом рассматриваемые теории являются совершенными в смысле существования соответственного модельного компаньона. Основные полученные результаты следующие: введены новые понятия; n_2-элиминация кванторов для позитивной теории, (n_1, n_2)-йонсоновская теория, n_1-йонсоновская цепь; указана особенность совершенной (n_1, n_2)-позитивной йонсоновской теории.

Ключевые слова: (n_1, n_2)-йонсоновская теория, позитивная (n_1, n_2)-йонсоновская теория, n_2-элиминация кванторов, n_1-йонсоновская цепь.

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