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On eigenvalues of third order composite type equations with regular boundary value conditions

In the paper the question about distribution of eigenvalues of third-order composite type equations with regular, more precisely, with periodic boundary value conditions is studied. After, applying the Fourier method, the original problem splits into two problems on eigenvalues of third-order ordinary differential operators with periodic boundary value conditions in $L_2(0,1)$. Characteristic determinants are calculated and zeros of entire analytic functions are found, and their location on the complex plane is determined. Existence of an infinite number of eigenvalues of a third order composite type operator is proved. Distance between the neighboring eigenvalues of the third order composite type operator of each series, which lie on rays, perpendicular to sides of a conjugate indicator diagram, that is, a regular hexagon on the complex plane, is determined. Moreover, it is determined that zero is not an eigenvalue of a third order composite type operator, in other words, zero is a regular point of the operator that belongs to resolvent set of the original operator. Adjoint operator with periodic boundary value conditions is constructed.

Keywords: composite type equations, regular, periodic boundary value conditions, rectangular domain, Fourier method, characteristic determinant, entire analytic functions, eigenvalue, zeros of entire functions.

Introduction and Formulation of the problem

Series of spectral boundary value problems for composite type equations

$$\frac{\partial}{\partial x}\left(u_{xx}+u_{yy}\right)+\lambda u=0$$

has been investigated in [1, 2]. Solution of initial-boundary value problems for partial differential equations by the Fourier method is almost always reduced to the problem on determining eigenvalues and eigen-functions of some differential operators [3–6].

This paper is devoted to finding the eigenvalues of one boundary value problem for the equation

$$Lu\equiv u_{xxx}+u_{yyy}+\lambda u=0, \tag{1}$$

where λ is a spectral parameter and complex number, which is also composite type equation [1].

In a rectangular domain D we consider the problem on eigenvalues of the equation (1), satisfying the following boundary conditions:

$$u|_{\partial D}=0, \quad u_x(0,y)=u_x(1,y), \quad u_y(x,0)=u_y(x,1), \tag{2}$$

where $D=\{x,y:\ \ 0<x<1,\ \ 0<y<1\}$.

Solution of the problem

Looking for a solution of the problem (1), (2) by the Fourier method as follows:

$$u(x,y)=X(x)\cdot Y(y),$$

we come to the following spectral problems in the space $L_2(0,1)$ for ordinary differential operators

$$L_0X\equiv X'''+\mu X=0, \quad X(0)=X(1)=0, \quad X'(0)=X'(1), \tag{3}$$

$$L_1 Y \equiv Y''' + \nu Y = 0, \quad Y(0) = Y(1) = 0, \quad Y'(0) = Y'(1), \tag{4}$$

moreover, $\lambda = \mu + \nu$. Boundary value problems in (3),(4) are regular by G.D. Birkhoff [7]. In the monograph of M.A. Naimark [8; 67] a subclass of regular boundary conditions is distinguished, and it is noted that for an odd order of the equation, all strongly regular conditions are regular.

We solve the problem (3) (problem (4) is solved analogically). General solution of the equation (3) has the form

$$X(x) = C_1 e^{2ax} + \left(C_2 \cdot \cos\sqrt{3}ax + C_3 \cdot \sin\sqrt{3}ax\right) \cdot e^{-ax}, \tag{5}$$

where $C_1,\ C_2,\ C_3$ are arbitrary constants,

$$a = \frac{\sqrt[3]{-\mu}}{2} \neq 0. \tag{6}$$

Putting (5) into the boundary value condition (3), we will have the linear system concerning to the coefficients C_j:

$$\begin{cases} C_1 + C_2 = 0, \\ C_1 \cdot e^{2a} + C_2 \cdot e^{-a} \cdot \cos\sqrt{3}a + C_3 \cdot e^{-a} \cdot \sin\sqrt{3}a = 0, \\ C_1 \cdot \left(2a - 2a \cdot e^{2a}\right) + C_2 \cdot \left(-a + \sqrt{3}a \cdot e^{-a} \cdot \sin\sqrt{3}a + a \cdot e^{-a} \cdot \cos\sqrt{3}a\right) + \\ +C_3 \cdot \left(\sqrt{3}a - \sqrt{3}a \cdot e^{-a} \cdot \cos\sqrt{3}a + a \cdot e^{-a} \cdot \sin\sqrt{3}a\right) = 0. \end{cases}$$

Its determinant will be a characteristic determinant for the problem (3):

$$\Delta(a) = \begin{vmatrix} 1 & 1 & 0 \\ e^{2a} & e^{-a}\cos\sqrt{3}a & e^{-a}\sin\sqrt{3}a \\ 2a - 2ae^{2a} & ae^{-a}\left(\sqrt{3}\sin\sqrt{3}a + \cos\sqrt{3}a\right) - a & \sqrt{3}a - ae^{-a}\left(\sqrt{3}\cos\sqrt{3}a + \sin\sqrt{3}a\right) \end{vmatrix}. \tag{7}$$

From where by standard calculations and transformations the determinant (7) is reduced to the form:

$$\Delta(a) = \left(\sqrt{3} + 3i\right) e^{\left(1+i\sqrt{3}\right)a} + \left(\sqrt{3} - 3i\right) e^{\left(1-i\sqrt{3}\right)a} + \left(\sqrt{3} + 3i\right) e^{-\left(1+i\sqrt{3}\right)a} +$$

$$+ \left(\sqrt{3} - 3i\right) e^{-\left(1-i\sqrt{3}\right)a} - 2\sqrt{3}e^{2a} - 2\sqrt{3}e^{-2a}. \tag{8}$$

We formulate the obtained result as the following theorem.

Theorem 1. Characteristic determinant of the spectral problem (3) is represented as a form of quasi-polynomial (7) and is the entire analytic function of the variable a.

Connection of quasi-polynomials zeros with spectral problems is reflected in [9–13].

Sometimes entire analytical functions coincide with quasi-polynomials, zeros of which are investigated in [8, 14–18].

The papers [19, 20] are devoted to study of zeros of entire functions with an integral representation, related to spectral problems of a third-order differential operator with nonlocal boundary value conditions.

In [21, 22] the characteristic determinant of spectral problem for the Sturm-Liouville operator with perturbed regular boundary value conditions, which is an entire analytic function of the spectral parameter, is calculated. Also in this paper, they study stability problems of basis property of root functions systems of the original operator.

Zeros of the entire analytical function $\Delta(a)$ in (8) are eigenvalues of the operator L_0. Therefore, further we consider the question about distribution of eigenvalues of the entire function $\Delta(a)$ on the complex plane a.

Taking into account results of the monographs [9, 10, 16], the conjugate indicator diagram of the function $\Delta(a)$ will be a regular hexagon on the complex plane a. Sides of the hexagon consist of the following segments:

$$\left[\overline{1 - i\sqrt{3}};\ \overline{-1 - i\sqrt{3}}\right],\ \left[\overline{-1 + i\sqrt{3}};\ \overline{1 + i\sqrt{3}}\right],\ \left[\overline{-2};\ \overline{-1 - i\sqrt{3}}\right],$$

$$\left[\overline{-1 + i\sqrt{3}};\ \overline{-2}\right],\ \left[\overline{1 + i\sqrt{3}};\ \overline{2}\right],\ \left[\overline{2};\ \overline{1 - i\sqrt{3}}\right],$$

where lines mean complex conjugation, and they are commensurable numbers, that is, the length of each segment is equal to $d = 2$, and therefore they form the regular hexagon. From the origin we draw rays that are perpendicular to the sides of the regular hexagon.

Rays, which are perpendicular to the indicator diagram, are called critical. According to [10] the critical rays on the plane a are exactly six, that is

$$\arg \sqrt[3]{a} = \frac{\pi}{6} + \frac{\pi n}{3}, \quad n = 0,\ 1,\ 2,\ 3,\ 4,\ 5.$$

Along the ray, perpendicular to the segment passing through the points $\overline{2};\ \overline{1 - i\sqrt{3}}$, there are zeros of the quasi-polynomial $\left(\sqrt{3} + 3i\right) \cdot e^{\left(1+i\sqrt{3}\right)a} - 2\sqrt{3} \cdot e^{2a}$ from (8), which are majorizing exponents. Moreover, along this ray other exponents from (8) do not contribute.

We find zeros of the quasi-polynomial:

$$\left(\sqrt{3} + 3i\right) \cdot e^{\left(1+i\sqrt{3}\right)a} - 2\sqrt{3} \cdot e^{2a} = 0,$$

$$\left(\sqrt{3} + 3i\right) \cdot e^{\left(1+i\sqrt{3}\right)a} = 2\sqrt{3} \cdot e^{2ik\pi},$$

$$a_{k1} = \frac{2ik\pi}{-1 + i\sqrt{3}} + \frac{\ln\left|\frac{2\sqrt{3}}{\sqrt{3}+3i}\right| + iArg\left(\frac{2\sqrt{3}}{\sqrt{3}+3i}\right)}{-1 + i\sqrt{3}}, \quad k = 1,\ 2,\ 3,\ ...,$$

which are zeroes of the first series, where $\ln\left|\frac{2\sqrt{3}}{\sqrt{3}+3i}\right| + iArg\left(\frac{2\sqrt{3}}{\sqrt{3}+3i}\right) = const.$

Otherwise, from (7) if follows that eigenvalues of the first series of the operator L_0 will be

$$\mu_{k1} = -\left(\frac{4ik\pi}{-1 + i\sqrt{3}} + \frac{const}{-1 + i\sqrt{3}}\right)^3, \quad k = 1,\ 2,\ 3,\ ....$$

A similar procedure is carried out on the other sides of the hexagon, and along the other perpendicular rays we have the corresponding series of quasi-polynomials zeros from (8):

– segment $\left[\overline{-1 - i\sqrt{3}};\ \overline{1 - i\sqrt{3}}\right]$, 2-nd series of zeros

$$a_{k2} = \frac{ik\pi}{1 + i\sqrt{3}} + \frac{const}{2\left(1 + i\sqrt{3}\right)}, \quad k = 1,\ 2,\ 3,\ ...$$

$$\mu_{k2} = -\left(\frac{2ik\pi}{1 + i\sqrt{3}} + \frac{const}{1 + i\sqrt{3}}\right)^3, \quad k = 1,\ 2,\ 3,\ ...;$$

– segment $\left[\overline{-1 + i\sqrt{3}};\ \overline{1 + i\sqrt{3}}\right]$, 3-rd series of zeros

$$a_{k3} = ik\pi + const, \quad k = 1,\ 2,\ 3,\ ...,$$

$$\mu_{k3} = -(2ik\pi + const)^3, \quad k = 1,\ 2,\ 3,\ ....;$$

– segment $\left[\overline{-2};\ \overline{-1 - i\sqrt{3}}\right]$, 4-th series of zeros

$$a_{k4} = \frac{2ik\pi}{1 + i\sqrt{3}} + \frac{const}{1 + i\sqrt{3}}, \quad k = 1,\ 2,\ 3,\ ...,$$

$$\mu_{k4} = -\left(\frac{2ik\pi}{1 + i\sqrt{3}} + \frac{const}{1 + i\sqrt{3}}\right)^3, \quad k = 1,\ 2,\ 3,\ ....;$$

– segment $\left[\overline{2};\ \overline{1 + i\sqrt{3}}\right]$, 5-th series of zeros

$$a_{k5} = -\frac{2ik\pi}{1 + i\sqrt{3}} - \frac{const}{1 + i\sqrt{3}}, \quad k = 1,\ 2,\ 3,\ ...,$$

$$\mu_{k5} = \left(\frac{2ik\pi}{1 + i\sqrt{3}} + \frac{const}{1 + i\sqrt{3}}\right)^3, \quad k = 1,\ 2,\ 3,\ ....$$

– segment $\left[\overline{-1+i\sqrt{3}};\ \overline{-2}\right]$, 6-th series of zeros

$$a_{k6}=\frac{2ik\pi}{1-i\sqrt{3}}+\frac{const}{1-i\sqrt{3}},\quad k=1,\ 2,\ 3,\ ...,$$

$$\mu_{k6}=-\left(\frac{4ik\pi}{1-i\sqrt{3}}+\frac{const}{1-i\sqrt{3}}\right)^3,\quad k=1,\ 2,\ 3,\ ....$$

Thus,

Proposition 1.

1. There exists an infinite number of operator eigenvalue of the operator L_0.

2. Distance between neighboring eigenvalues of the operator L_0 of the each series is equal to $\frac{2\pi}{|d|}$.

3. Eigenvalues of each series of the operator L_0 lie on the rays, perpendicular to the segment containing the numbers

$$\left(\overline{1-i\sqrt{3}};\ \overline{-1-i\sqrt{3}}\right),\ \left(\overline{-1+i\sqrt{3}};\ \overline{1+i\sqrt{3}}\right),\ \left(\overline{-2};\ \overline{-1-i\sqrt{3}}\right),\ \left(\overline{-1+i\sqrt{3}};\ \overline{-2}\right),$$

$$\left(\overline{1+i\sqrt{3}};\ \overline{2}\right),\ \left(\overline{2};\ \overline{1-i\sqrt{3}}\right).$$

Similarly, by repeating the whole process of researching the problem (3), we solve the problem (4), and we obtain eigenvalues of the operator L_1:

$$\nu_{l1}=-\left(\frac{4il\pi}{-1+i\sqrt{3}}+\frac{const}{-1+i\sqrt{3}}\right)^3,\quad l=1,\ 2,\ 3,\ ....$$

$$\nu_{l2}=-\left(\frac{2il\pi}{1+i\sqrt{3}}+\frac{const}{1+i\sqrt{3}}\right)^3,\quad l=1,\ 2,\ 3,\ ....$$

$$\nu_{l3}=-(2il\pi+const)^3,\quad l=1,\ 2,\ 3,\ ....$$

$$\nu_{l4}=-\left(\frac{2il\pi}{1+i\sqrt{3}}+\frac{const}{1+i\sqrt{3}}\right)^3,\quad l=1,\ 2,\ 3,\ ....$$

$$\nu_{l5}=\left(\frac{2il\pi}{1+i\sqrt{3}}+\frac{const}{1+i\sqrt{3}}\right)^3,\quad l=1,\ 2,\ 3,\ ....$$

$$\nu_{l6}=-\left(\frac{4il\pi}{1-i\sqrt{3}}+\frac{const}{1-i\sqrt{3}}\right)^3,\quad l=1,\ 2,\ 3,\ ....$$

Analogically, all points of Proposition 1 are true for the operator L_1.

So, we have proved the following:

Theorem 2. Suppose that all conditions of Theorem 1 and Proposition 1 hold for the operators L_0 and L_1. Then eigenvalues of the operator L are $\lambda_{klj}=\pm\left(\mu_{kj}+\nu_{lj}\right)$, where $k=1,2,3,...,l=1,2,3,...,j=\overline{(1-6)}$ mean the each series.

Remark. In the case $a=\frac{\sqrt[3]{-\mu}}{2}=0$, representing general solution (3) as $X\left(x\right)=ax^2+bx+c$ and satisfying the boundary value conditions in (3), we have $X\left(x\right)=0$, that is, $\mu_0=0$ is not eigenvalue of the operator L_0. Similarly, $\nu_0=0$ is a regular point of the operator L_1. Thus, $\lambda_0=0$ is not eigenvalue of the operator L.

Conjugate problems

$L_0X\equiv l_0\left(X\right)=X'''\left(x\right)$. Applying the method of integration by parts, we get the Lagrange formula:

$$\int_0^1 l_0\left(X\right)\overline{\upsilon\left(x\right)}dx+\int_0^1 X\left(x\right)\overline{l_0^*\left(\upsilon\right)}dx=X''\left(1\right)\overline{\upsilon\left(1\right)}-X''\left(0\right)\overline{\upsilon\left(0\right)}-$$
$$-\left[\overline{\upsilon'\left(0\right)}-\overline{\upsilon'\left(1\right)}\right]\cdot X'\left(0\right)+X\left(1\right)\cdot\overline{\upsilon''\left(1\right)}-X\left(0\right)\cdot\overline{\upsilon''\left(0\right)}.$$

Here $l_0^*\left(\upsilon\right)$ is the conjugate differential expression:

$$l_0^*\left(\upsilon\right)=-\upsilon'''\left(x\right),\quad 0<x<1. \tag{9}$$

Consequently, the operator оператор L_0^*, conjugate to the operator L_0, is given by the differential expression (9) and the boundary value conditions

$$\upsilon(1) = \upsilon(0) = 0, \quad \upsilon'(0) - \upsilon'(1) = 0. \tag{10}$$

Analogically, for the operator L_1 conjugate operator is

$$L_1 Y \equiv l_1(Y) = Y'''(y), \quad L_1^* : l_1^*(\upsilon) = -\upsilon'''(y), \quad 0 < y < 1$$

with the boundary value conditions (10). From this, it follows that in the domain D conjugate problem to the problem (1), (2) will be

$$L^* V = V_{xxx} + V_{yyy} - \lambda V = 0,$$

satisfying the boundary value conditions

$$V|_{\partial D} = 0, \quad V_x(1, y) = V_x(0, y), \quad V_y(x, 0) = V_y(x, 1).$$

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Н.С. Иманбаев, М.Н. Оспанов

Регулярлық шеттік шарттармен берілген үшінші ретті құрамдас типті теңдеудің меншікті мәндері жайлы

Мақалада регулярлық, дәлірек айтқанда, периодтық шеттік шарттармен берілген үшінші ретті құрамдас типті теңдеудің меншікті мәндерінің орналасуы туралы мәселе зерттелді. Бастапқы есепті Фурье әдісімен шешуді қолданғаннан кейін, $L_2(0,1)$ кеңістігінде периодтық шарттармен берілген үшінші ретті жай дифференциалдық теңдеудің меншікті мәндерін зерттеуге арналған екі есепке тармақталған. Осы есептердің сипаттамалық анықтауыштарының бүтін аналитикалық функциялар болатындығы дәлелденіп, олардың нөлдері табылып, кешенді жазықтықтағы орындары анықталған. Бастапқы оператордың меншікті мәндерінің саналымды, шексіз екендігі көрсетілген. Түйіндес индикаторлық диаграммасы құрылып, әр сериядағы меншікті мәндердің перпендикуляр сәулелердің бойында арақашықтықтары анықталған. Спектралдық параметрдің нөлдік мәні оператордың меншікті мәні болмайтындығы көрсетілген. Түйіндес операторы құрылған.

Кілт сөздер: құрамдас типті теңдеу, регулярлық периодтық шеттік шарттар, төртбұрыш аймақ, Фурье әдісі, характеристикалық анықтауыш, бүтін аналитикалық функция, меншікті мәндер, бүтін функцияның нөлдері.

Н.С. Иманбаев, М.Н. Оспанов

О собственных значениях уравнений третьего порядка составного типа с регулярными краевыми условиями

В статье исследован вопрос распределения собственных значений уравнений третьего порядка составного типа с регулярными, точнее, с периодическими краевыми условиями. После применения метода Фурье исходная задача распадается на две задачи, т.е. на собственные значения обыкновенных дифференциальных операторов третьего порядка с периодическими краевыми условиями в $L_2(0,1)$. Вычислены характеристические опеределители и найдены нули целых аналитических функций и определено их расположение на комплексной плоскости. Доказано существование бесконечного числа собственных значений оператора третьего порядка составного типа. Определено расстояние между соседними собственными значениями оператора третьего порядка составного типа каждой серии, которое лежит на лучах перпендикулярно сторонам сопряженной индикаторной диаграммы, то есть правильного шестиугольника на комплексной плоскости. Доказано, что нуль не является собственным значением оператора третьего порядка составного типа, иначе говоря, нуль является регулярной точкой оператора, которая принадлежит резольвентному множеству исходного оператора. Построен сопряженный оператор с периодическими краевыми условиями.

Ключевые слова: уравнения составного типа, регулярные, периодические краевые условия, прямоугольная область, метод Фурье, характеристический определитель, целые аналитические функции, собственные значения, нули целых функций.

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