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A nonlocal problem for loaded partial differential equations of fourth order

A nonlocal problem for the fourth order system of loaded partial differential equations is considered. The questions of a existence unique solution of the considered problem and ways of its construction are investigated. The nonlocal problem for the loaded partial differential equation of fourth order is reduced to a nonlocal problem for a system of loaded hyperbolic equations of second order with integral conditions by introducing new functions. As a result of solving nonlocal problem with integral conditions is applied a method of introduction functional parameters. The algorithms of finding the approximate solution to the nonlocal problem with integral conditions for the system of loaded hyperbolic equations are proposed and their convergence is proved. The conditions of the unique solvability of the nonlocal problem for the loaded hyperbolic equations are obtained in the terms of initial data. The results also formulated relative to the original problem.

Keywords: nonlocal problem, loaded partial differential equations of fourth order, integral condition, system of loaded hyperbolic equations, algorithm, unique solvability.

Introduction

Many problems of dynamics and kinetics of gas sorption, processes of drying by air stream, movement of adsorbed mixtures and others lead to the study of nonlocal problems for the systems of hyperbolic equations with loading [1–10] and also for nonlocal problems with integral conditions for equations of hyperbolic type [11–16]. In order to solve these problems, the theoretical methods of ordinary differential equations, loaded differential equations, numerical-analytical method are applied, and new approaches and methods are developed as well. Conditions for solvability are received and ways for finding the approximate solutions are offered. Mathematical modeling of various processes in physics, chemistry, biology, technology, ecology, economics and others are leaded to nonlocal problems for the higher order loaded differential equations with variable coefficients and parameters. Despite the presence of numerous works, general statements of nonlocal problems for the higher order loaded partial differential equations remain poorly studied up to now. Therefore, the problems of solvability of nonlocal problems for the fourth order partial differential equations with and without loading remain important for applications [17–21].

The Goal of this paper is to study boundary value problems with data on the characteristics for the fourth order system of hyperbolic equations with loading and to establish coefficient criteria for unique solubility and to construct algorithms for finding their approximate solutions. Therefore, in the present paper we study of a questions the existence and uniqueness of classical solutions to nonlocal problem for the fourth order system of loaded partial differential equations and the methods of finding its approximate solutions. For these purposes, we are applied method of introduction a new functions [22, 23] for solve of this problem.

We consider on the domain $\Omega = [0,T] \times [0,\omega]$ a nonlocal problem for the fourth order system of the loaded partial differential equations with two independent variables

$$\frac{\partial^4 u}{\partial x^3 \partial t} = \sum_{i=1}^{3} \Big\{ A_i(t,x) \frac{\partial^{4-i} u}{\partial x^{4-i}} + B_i(t,x) \frac{\partial^{4-i} u}{\partial x^{3-i} \partial t} \Big\} + C(t,x)u +$$

$$+ \sum_{i=1}^{3} \sum_{k=1}^{m} \Big\{ K_{i,k}(t,x) \frac{\partial^{4-i} u(t,x)}{\partial x^{4-i}} + L_{i,k}(t,x) \frac{\partial^{4-i} u(t,x)}{\partial x^{3-i} \partial t} \Big\}\Big|_{t=t_k} + \sum_{k=1}^{m} M_{i,k}(t,x) u(t_k,x) + f(t,x), \qquad (1)$$

$$P(x) \frac{\partial^3 u(0,x)}{\partial x^3} + S(x) \frac{\partial^3 u(T,x)}{\partial x^3} = \varphi(x), \qquad x \in [0,\omega], \qquad (2)$$

$$u(t,0) = \psi_0(t), \qquad \frac{\partial u(t,x)}{\partial x}\Big|_{x=0} = \psi_1(t), \qquad \frac{\partial^2 u(t,x)}{\partial x^2}\Big|_{x=0} = \psi_2(t), \qquad t \in [0,T]. \qquad (3)$$

Here $u(t,x) = col(u_1(t,x), u_2(t,x), ..., u_n(t,x))$ is unknown function; the $n \times n$ matrices $A_i(t,x)$, $B_i(t,x)$, $C(t,x)$, $K_{i,k}(t,x)$, $L_{i,k}(t,x)$, $M_{i,k}(t,x)$, $i = \overline{1,3}$, $k = \overline{1,m}$, and n vector function $f(t,x)$ are continuous on Ω; $0 \le t_1 < t_2 < ... < t_m \le T$; the $n \times n$ matrices $P(x)$, $S(x)$, and n vector function $\varphi(x)$ are continuous on $[0,\omega]$; the n vector-functions $\psi_0(t)$, $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$.

Let $C(\Omega, R^n)$ be the space of continuous vector functions $u : \Omega \to R^n$ on Ω with norm
$||u||_0 = \max_{(t,x)\in\Omega} ||u(t,x)||$.

A function $u(t,x) \in C(\Omega, R^n)$, having partial derivatives $\dfrac{\partial^{i+j} u(t,x)}{\partial x^i \partial t^j} \in C(\Omega, R^n)$, $i = \overline{1,3}$, $j = 0,1$, is called a *classical solution* to problem (1)–(3) if it satisfies to system of loaded equations (1) for all $(t,x) \in \Omega$ and meets the conditions (2) and (3).

We will investigate the existence of a unique solution to the nonlocal problem for the fourth order loaded partial differential equation (1)–(3). We use method of introduction a new functions for solve of the problem (1)–(3) and construct of its approximate solutions. The nonlocal problem for the fourth order system of loaded partial differential equations is reduced to a nonlocal problem for a system of loaded hyperbolic equations of second order with integral conditions by introducing new functions. An algorithms of finding the approximate solution to the equivalent nonlocal problem with integral conditions are constructed and their convergence is proved. The conditions of the unique solvability of the nonlocal problem for the system of loaded hyperbolic equations with integral conditions are established in the terms of initial data. The results also formulated relative to the original of the nonlocal problem for the fourth order system of loaded partial differential equations.

1 Scheme of the method

Introduce a new unknown functions $\quad w(t,x) = \dfrac{\partial^2 u(t,x)}{\partial x^2}, \quad v(t,x) = \dfrac{\partial u(t,x)}{\partial x}.$

Taking into account of first and second conditions in (3), we have

$$v(t,x) = \psi_1(t) + \int_0^x w(t,\xi)d\xi, \qquad u(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x \int_0^\xi w(t,\xi_1)d\xi_1 d\xi.$$

Then problem (1)–(3) is reduced to a following problem

$$\frac{\partial^2 w}{\partial x \partial t} = A_1(t,x) \frac{\partial w}{\partial x} + B_1(t,x) \frac{\partial w}{\partial t} + A_2(t,x) w +$$

$$+ \sum_{k=1}^{m} \Big\{ K_{1,k}(t,x) \frac{\partial w(t_k,x)}{\partial x} + L_{1,k}(t,x) \frac{\partial w(t,x)}{\partial t}\Big|_{t=t_k} + K_{2,k}(t,x) w(t_k,x) \Big\} + f(t,x) + g(t,x,v,u), \qquad (4)$$

$$P(x) \frac{\partial w(0,x)}{\partial x} + S(x) \frac{\partial w(T,x)}{\partial x} = \varphi(x), \qquad (5)$$

$$w(t,0) = \psi_2(t), \qquad t \in [0,T], \tag{6}$$

$$v(t,x) = \psi_1(t) + \int_0^x w(t,\xi)d\xi, \qquad u(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x \int_0^\xi w(t,\xi_1)d\xi_1 d\xi, \tag{7}$$

where

$$g(t,x,v,u) = A_3(t,x)v(t,x) + B_2(t,x)\frac{\partial v}{\partial t} + B_3(t,x)\frac{\partial u}{\partial t} + C(t,x)u +$$

$$+\sum_{k=1}^{m}\Big\{K_{3,k}(t,x)v(t_k,x) + L_{2,k}(t,x)\frac{\partial v(t,x)}{\partial t}\Big|_{t=t_k} + L_{3,k}(t,x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=t_k} + M_{i,k}(t,x)u(t_k,x)\Big\}.$$

From (7) it follows

$$\frac{\partial v(t,x)}{\partial t} = \dot{\psi}_1(t) + \int_0^x \frac{\partial w(t,\xi)}{\partial t}d\xi, \qquad \frac{\partial u(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t)x + \int_0^x \int_0^\xi \frac{\partial w(t,\xi_1)}{\partial t}d\xi_1 d\xi. \tag{8}$$

A triple functions $(w(t,x), v(t,x), u(t,x))$, where $w(t,x) \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial w(t,x)}{\partial x} \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial w(t,x)}{\partial t} \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial^2 w(t,x)}{\partial x \partial t} \in C(\Omega, \mathbb{R}^n)$, and $v(t,x) \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial v(t,x)}{\partial t} \in C(\Omega, \mathbb{R}^n)$, $u(t,x) \in C(\Omega, \mathbb{R}^n)$, $\frac{\partial u(t,x)}{\partial t} \in C(\Omega, \mathbb{R}^n)$, is called a solution to problem (4)–(7), if it satisfies the system of loaded hyperbolic equations second order (4) for all $(t,x) \in \Omega$, the boundary conditions (5), (6), and integral relations (7).

The problem (4)–(6) at fixed $v(t,x)$, $u(t,x)$, is a nonlocal problem for system of loaded hyperbolic equations of second order with respect to $w(t,x)$ on Ω. The integral relations (7) allow us to determine the unknown functions $v(t,x)$ and $u(t,x)$.

From (8) we define the partial derivatives $\frac{\partial v(t,x)}{\partial t}$ and $\frac{\partial u(t,x)}{\partial t}$ for all $(t,x) \in \Omega$.

The problem (4)–(6) can be interpreted:

• as a nonlocal problem for the system of loaded hyperbolic equations of second order with distributed parameters $v(t,x)$ and $u(t,x)$;

• as an inverse problem for the system of loaded hyperbolic equations of second order, where the unknown functions $v(t,x)$, $u(t,x)$ determine from integral relations (7);

• as a control problem for the system of loaded hyperbolic equations of second order, where the control functions $v(t,x)$, $u(t,x)$ satisfy integral constrains (7).

Since the function $w(t,x)$ and the functions $v(t,x)$, $u(t,x)$ are unknown together to find a solution to problem (4)–(7) we use an iterative method.

2 Algorithm for finding of solution to problem (4)–(7)

A triple functions $(w^*(t,x), v^*(t,x), u^*(t,x))$ we determine as a limit of sequences of triple functions $(w^{(p)}(t,x)$, $v^{(p)}(t,x)$, $u^{(p)}(t,x))$ and $p = 0,1,2,...$, by the following algorithm:

Step - 0. 1) Let $v(t,x) = \psi_1(t)$, $u(t,x) = \psi_0(t) + \psi_1(t)x$, $\frac{\partial v(t,x)}{\partial t} = \dot{\psi}_1(t)$, $\frac{\partial u(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t)x$ in right-hand side of system (4). Then from nonlocal problem for the system of loaded hyperbolic equations (4)–(6) we find $w^{(0)}(t,x)$ for all $(t,x) \in \Omega$. Also we find its partial derivatives $\frac{\partial w^{(0)}(t,x)}{\partial x}$, $\frac{\partial w^{(0)}(t,x)}{\partial t}$ and $\frac{\partial^2 w^{(0)}(t,x)}{\partial x \partial t}$ for all $(t,x) \in \Omega$;

2) From integral relations (7) we determine $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$:

$$v^{(0)}(t,x) = \psi_1(t) + \int_0^x w^{(0)}(t,\xi)d\xi, \quad u^{(0)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x \int_0^\xi w^{(0)}(t,\xi_1)d\xi_1 d\xi, \qquad (t,x) \in \Omega.$$

Then from (8) we find $\frac{\partial v^{(0)}(t,x)}{\partial t}$ and $\frac{\partial u^{(0)}(t,x)}{\partial t}$:

$$\frac{\partial v^{(0)}(t,x)}{\partial t} = \dot{\psi}_1(t) + \int_0^x \frac{\partial w^{(0)}(t,\xi)}{\partial t}d\xi, \quad \frac{\partial u^{(0)}(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t) + \int_0^x \int_0^\xi \frac{\partial w^{(0)}(t,\xi_1)}{\partial t}d\xi_1 d\xi,$$

And so on.

Step - p. 1) Suppose that $v(t,x) = v^{(p-1)}(t,x)$, $u(t,x) = u^{(p-1)}(t,x)$, $\dfrac{\partial v(t,x)}{\partial t} = \dfrac{\partial v^{(p-1)}(t,x)}{\partial t}$ and $\dfrac{\partial u(t,x)}{\partial t} = = \dfrac{\partial u^{(p-1)}(t,x)}{\partial t}$ in right-hand side of system (4). Then from nonlocal problem for the system of hyperbolic equations (4)–(6) we find $w^{(p)}(t,x)$ for all $(t,x) \in \Omega$. Also we find its partial derivatives $\dfrac{\partial w^{(p)}(t,x)}{\partial x}$, $\dfrac{\partial w^{(p)}(t,x)}{\partial t}$ and $\dfrac{\partial^2 w^{(p)}(t,x)}{\partial x \partial t}$ for all $(t,x) \in \Omega$.

2) From integral relations (7) we determine $v^{(p)}(t,x)$ and $u^{(p)}(t,x)$:

$$v^{(p)}(t,x) = \psi_1(t) + \int_0^x w^{(p)}(t,\xi)d\xi, \quad u^{(p)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x \int_0^\xi w^{(p)}(t,\xi_1)d\xi_1 d\xi, \qquad (t,x) \in \Omega.$$

Then from (8) we find $\dfrac{\partial v^{(p)}(t,x)}{\partial t}$ and $\dfrac{\partial u^{(p)}(t,x)}{\partial t}$:

$$\frac{\partial v^{(p)}(t,x)}{\partial t} = \dot{\psi}_1(t) + \int_0^x \frac{\partial w^{(p)}(t,\xi)}{\partial t} d\xi, \quad \frac{\partial u^{(p)}(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t) + \int_0^x \int_0^\xi \frac{\partial w^{(p)}(t,\xi_1)}{\partial t} d\xi_1 d\xi,$$

$p = 1, 2, \ldots.$

3 Nonlocal problem for system of loaded hyperbolic equations

We also consider an auxiliary nonlocal problem for system of loaded hyperbolic equations second order

$$\frac{\partial^2 w}{\partial x \partial t} = A_1(t,x)\frac{\partial w}{\partial x} + B_1(t,x)\frac{\partial w}{\partial t} + A_2(t,x)w +$$

$$+ \sum_{k=1}^m \Big\{ K_{1,k}(t,x)\frac{\partial w(t_k,x)}{\partial x} + L_{1,k}(t,x)\frac{\partial w(t,x)}{\partial t}\Big|_{t=t_k} + K_{2,k}(t,x)w(t_k,x)\Big\} + F(t,x), \tag{9}$$

$$P(x)\frac{\partial w(0,x)}{\partial x} + S(x)\frac{\partial w(T,x)}{\partial x} = \varphi(x), \qquad x \in [0,\omega], \tag{10}$$

$$w(t,0) = \psi_2(t), \qquad t \in [0,T]. \tag{11}$$

Here the functions $F(t,x) \in C(\Omega, \mathbb{R}^n)$.

Let $t_0 = 0$, $t_{m+1} = T$.

By lines of loading $t = t_k$, $k = \overline{1,m}$, we divide of domain $\Omega = \bigcup_{r=1}^{m+1} \Omega_r$, where $\Omega_r = [t_{r-1}, t_r] \times [0,\omega]$, $r = \overline{1,m+1}$. By $w_r(t,x)$ denote the restriction of function $w(t,x)$ to the subdomain Ω_r such that $w_r : \Omega_r \to R^n$ and $w_r(t,x) = w(t,x)$ for all $(t,x) \in \Omega_r$ and $r = \overline{1,m+1}$.

Further, by $\lambda_r(x)$ denote the value of $w_r(t,x)$ under $t = t_{r-1}$, $r = \overline{1,m+1}$. We replace $w_r(t,x)$ by $\widetilde{w}_r(t,x) + \lambda_r(x)$ in each domain Ω_r, $r = \overline{1,m+1}$. This implies $\widetilde{w}_r(t_{r-1},x) = 0$, and $\dfrac{\partial \widetilde{w}_r(t_{r-1},x)}{\partial x} = 0$, for all $x \in [0,\omega]$ and $r = \overline{1,m+1}$.

Then the problem (9)–(11) is equivalent to the problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial^2 \widetilde{w}_r}{\partial x \partial t} = A_1(t,x)\frac{\partial \widetilde{w}_r}{\partial x} + A_1(t,x)\dot{\lambda}_r(x) + B_1(t,x)\frac{\partial \widetilde{w}_r}{\partial t} + A_2(t,x)\widetilde{w}_r + A_2(t,x)\lambda_r(x) +$$

$$+ \sum_{k=1}^m \Big\{ K_{1,k}(t,x)\dot{\lambda}_{k+1}(x) + K_{2,k}(t,x)\lambda_{k+1}(x)\Big\} + \sum_{k=1}^m L_{1,k}(t,x)\frac{\partial \widetilde{w}_{k+1}(t,x)}{\partial t}\Big|_{t=t_k} + F(t,x), \tag{12}$$

$$\widetilde{w}_r(t_{r-1},x) = 0, \qquad x \in [0,\omega], \qquad r = \overline{1,m+1}, \tag{13}$$

$$\widetilde{w}_r(t,0) = \psi_2(t) - \psi_2(t_{r-1}), \qquad t \in [t_{r-1}, t_r], \qquad r = \overline{1,m+1}, \tag{14}$$

$$P(x)\dot{\lambda}_1(x) + S(x)\dot{\lambda}_{m+1}(x) + S(x)\frac{\partial \widetilde{w}_{m+1}(t_{m+1},x)}{\partial x} = \varphi(x), \qquad x \in [0,\omega], \tag{15}$$

$$\frac{\partial \widetilde{w}_s(t_s,x)}{\partial x} + \dot{\lambda}_s(x) = \dot{\lambda}_{s+1}(x), \qquad x \in [0,\omega], \qquad s = \overline{1,m}. \tag{16}$$

Here relations (16) are conditions of continuity at interior lines $t = t_s$, $s = \overline{1,m}$ of desired function $w(t,x)$.

The problems (9)–(11) and (12)–(16) are equivalent in the following sense. If the function $w(t,x)$ is a classical solution to (9)–(11), then system of pairs $(\lambda_r(x), \widetilde{w}_r(t,x))$, where $\lambda_r(x) = w(t_{r-1},x)$ and $\widetilde{w}_r(t,x) = w(t,x) - w(t_{r-1},x)$, and $(t,x) \in \Omega_r$, and $r = \overline{1,m+1}$ is a solution to problem (12)–(16). Conversely, if the system of pairs $(\lambda_r^*(x), \widetilde{w}_r^*(t,x))$, $(t,x) \in \Omega_r$, and $r = \overline{1,m+1}$, is a solution to (12)–(16), then the function $w^*(t,x)$ defined by the equalities

$$w^*(t,x) = \lambda_r^*(x) + \widetilde{w}_r^*(t,x) \quad \text{for all} \quad (t,x) \in \Omega_r, \quad \text{and} \quad r = \overline{1,m+1},$$

is a classical solution to problem (9)–(11).

From compatibility condition at $(0,0)$ we obtain:

$$\lambda_r(0) = \psi_2(t_{r-1}), \qquad r = \overline{1,m+1}. \tag{17}$$

At fixed λ_r problem (12)–(14) is Goursat problem for system of loaded hyperbolic equations of second order on Ω_r with respect to $\widetilde{w}_r(t,x)$, $r = \overline{1,m+1}$.

Let $\widetilde{V}_r(t,x) = \frac{\partial \widetilde{w}_r(t,x)}{\partial x}$, $\widetilde{W}_r(t,x) = \frac{\partial \widetilde{w}_r(t,x)}{\partial t}$.

Goursat problem (12)–(14) is equivalent to the system of three integral equations on Ω_r at fixed $\lambda_r(x)$

$$\widetilde{V}_r(t,x) = \int_{t_{r-1}}^{t} \Big\{ A_1(\tau,x)\widetilde{V}_r(\tau,x) + B_1(\tau,x)\widetilde{W}_r(\tau,x) + A_2(\tau,x)\widetilde{w}_r(\tau,x) + \sum_{k=1}^{m} L_{1,k}(\tau,x)\widetilde{W}_{k+1}(t_k,x) +$$

$$+ F(\tau,x) + A_1(\tau,x)\dot{\lambda}_r(x) + A_2(\tau,x)\lambda_r(x) + \sum_{k=1}^{m} \Big\{ K_{1,k}(\tau,x)\dot{\lambda}_{k+1}(x) + K_{2,k}(\tau,x)\lambda_{k+1}(x) \Big\} \Big\} d\tau, \tag{18}$$

$$\widetilde{W}_r(t,x) = \dot{\psi}_2(t) + \int_0^x \Big\{ A_1(t,\xi)\widetilde{V}_r(t,\xi) + B_1(t,\xi)\widetilde{W}_r(t,\xi) + A_2(t,\xi)\widetilde{w}_r(t,\xi) + \sum_{k=1}^{m} L_{1,k}(t,\xi)\widetilde{W}_{k+1}(t_k,\xi) +$$

$$+ F(t,\xi) + A_1(t,\xi)\dot{\lambda}_r(\xi) + A_2(t,\xi)\lambda_r(\xi) + \sum_{k=1}^{m} \Big\{ K_{1,k}(t,\xi)\dot{\lambda}_{k+1}(\xi) + K_{2,k}(t,\xi)\lambda_{k+1}(\xi) \Big\} \Big\} d\xi, \tag{19}$$

$$\widetilde{w}_r(t,x) = \psi_2(t) - \psi_2(t_{r-1}) + \int_{t_{r-1}}^{t} \widetilde{W}_r(\tau,x) d\tau. \tag{20}$$

Substituting $\widetilde{V}_r(\tau,x) = \frac{\partial \widetilde{w}_r(\tau,x)}{\partial x}$ in the right-hand side (18) and repeating the process ν times, and $\nu \in \mathbb{N}$, we obtain

$$\widetilde{V}_r(t,x) = D_{\nu,r}(t,x)\dot{\lambda}_r(x) + \sum_{k=1}^{m} \widetilde{D}_{\nu,r,k}(t,x)\dot{\lambda}_{k+1}(x) + E_{\nu,r}(t,x)\lambda_r(x) + \sum_{k=1}^{m} \widetilde{E}_{\nu,r,k}(t,x)\lambda_{k+1}(x) +$$

$$+ G_{\nu,r}(t,x,\widetilde{V}_r) + H_{\nu,r}(t,x,\widetilde{W}_r,\widetilde{w}_r) + F_{\nu,r}(t,x), \tag{21}$$

where

$$D_{\nu,r}(t,x) = \int_{t_{r-1}}^{t} A_1(\tau,x)d\tau + \int_{t_{r-1}}^{t} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} A_1(\tau_2,x)d\tau_2 d\tau_1 + ... +$$

$$+ \int_{t_{r-1}}^{t} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} A_1(\tau_2,x) ... \int_{t_{r-1}}^{\tau_{\nu-1}} A_1(\tau_\nu,x)d\tau_\nu d\tau_{\nu-1} ... d\tau_2 d\tau_1,$$

$$\widetilde{D}_{\nu,r,k}(t,x) = \int_{t_{r-1}}^{t} K_{1,k}(\tau,x)d\tau + \int_{t_{r-1}}^{t} A_1(\tau,x)\int_{t_{r-1}}^{\tau} K_{1,k}(\tau_1,x)d\tau_1 d\tau +$$

$$+ \int_{t_{r-1}}^{t} A_1(\tau,x)\int_{t_{r-1}}^{\tau} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} K_{1,k}(\tau_2,x)d\tau_2 d\tau_1 d\tau + ... +$$

$$+\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-1}} A_1(\tau_\nu,x)\int_{t_{r-1}}^{\tau_\nu} K_{1,k}(\tau_{\nu+1},x)d\tau_{\nu+1}d\tau_\nu...d\tau_1,$$

$$E_{\nu,r}(t,x)=\int_{t_{r-1}}^{t} A_2(\tau,x)d\tau+\int_{t_{r-1}}^{\tau} A_1(\tau,x)\int_{t_{r-1}}^{\tau_1} A_2(\tau,x)d\tau_1 d\tau+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau,x)\int_{t_{r-1}}^{\tau} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} A_2(\tau_2,x)d\tau_2 d\tau_1 d\tau+...+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-1}} A_1(\tau_\nu,x)\int_{t_{r-1}}^{\tau_\nu} A_2(\tau_{\nu+1},x)d\tau_{\nu+1}d\tau_\nu...d\tau_1,$$

$$\widetilde{E}_{\nu,r,k}(t,x)=\int_{t_{r-1}}^{t} K_{2,k}(\tau,x)d\tau+\int_{t_{r-1}}^{t} A_1(\tau,x)\int_{t_{r-1}}^{\tau} K_{2,k}(\tau_1,x)d\tau_1 d\tau+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau,x)\int_{t_{r-1}}^{\tau} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} K_{2,k}(\tau_2,x)d\tau_2 d\tau_1 d\tau+...+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-1}} A_1(\tau_\nu,x)\int_{t_{r-1}}^{\tau_\nu} K_{2,k}(\tau_{\nu+1},x)d\tau_{\nu+1}d\tau_\nu...d\tau_1,$$

$$G_{\nu,r}(t,x,\widetilde{V}_r)=\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-1}} A_1(\tau_\nu,x)\widetilde{V}_r(\tau_\nu,x)d\tau_\nu...d\tau_2 d\tau_1,$$

$$H_{\nu,r}(t,x,\widetilde{W}_r,\widetilde{w}_r)=\int_{t_{r-1}}^{t}\Big[B_1(\tau,x)\widetilde{W}_r(\tau,x)+A_2(\tau,x)\widetilde{w}_r(\tau,x)+\sum_{k=1}^{m} L_{1,k}(\tau,x)\widetilde{W}_{k+1}(t_k,x)\Big]d\tau+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1}\Big[B_1(\tau_2,x)\widetilde{W}_r(\tau_2,x)+A_2(\tau_2,x)\widetilde{w}_r(\tau_2,x)+\sum_{k=1}^{m} L_{1,k}(\tau_2,x)\widetilde{W}_{k+1}(t_k,x)\Big]d\tau_2 d\tau_1+$$

$$+...+\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-2}} A_1(\tau_{\nu-1},x)\int_{t_{r-1}}^{\tau_{\nu-1}}\Big[B_1(\tau_\nu,x)\widetilde{W}_r(\tau_\nu,x)+A_2(\tau_\nu,x)\widetilde{w}_r(\tau_\nu,x)+$$

$$+\sum_{k=1}^{m} L_{1,k}(\tau_\nu,x)\widetilde{W}_{k+1}(t_k,x)\Big]d\tau_\nu d\tau_{\nu-1}...d\tau_1,$$

$$F_{\nu,r}(t,x)=\int_{t_{r-1}}^{t} F(\tau,x)d\tau+\int_{t_{r-1}}^{t} A_1(\tau_1,x)\int_{t_{r-1}}^{\tau_1} F(\tau_2,x)d\tau_2 d\tau_1+...+$$

$$+\int_{t_{r-1}}^{t} A_1(\tau_1,x)...\int_{t_{r-1}}^{\tau_{\nu-2}} A_1(\tau_{\nu-1},x)\int_{t_{r-1}}^{\tau_{\nu-1}} F(\tau_\nu,x)d\tau_\nu d\tau_{\nu-1}...d\tau_1,$$

$(t,x)\in\Omega_r,\quad r=\overline{1,m+1},\quad \nu\in\mathbb{N},\quad k=\overline{1,m}.$

From (21) we find $\widetilde{V}_r(t_r,x)=\frac{\partial\widetilde{w}_r(t_r,x)}{\partial x}$ for all $x\in[0,\omega]$, and $r=\overline{1,m+1}$. Then, substituting their into (15) and (16), and multiplying both sides (15) by $h_m=t_{m+1}-t_m$, we obtain the system of differential equations with respect to functions $\lambda_r(x)$, and $r=\overline{1,m+1}$:

$$h_m P(x)\dot{\lambda}_1(x)+h_m S(x)\sum_{k=1}^{m}\widetilde{D}_{\nu,m+1,k}(t,x)\dot{\lambda}_{k+1}(x)+$$

$$+h_m S(x)[I+D_{\nu,m+1}(t_{m+1},x)]\dot{\lambda}_{m+1}(x)=$$

$$=-h_m S(x)\Big[E_{\nu,m+1}(t_{m+1},x)\lambda_{m+1}(x)+\sum_{k=1}^{m}\widetilde{E}_{\nu,m+1,k}(t_{m+1},x)\lambda_{k+1}(x)\Big]-$$

$$-h_m S(x)G_{\nu,m+1}(t_{m+1},x,\widetilde{V}_{m+1})-h_m S(x)H_{\nu,m+1}(t_{m+1},x,\widetilde{W}_{m+1},\widetilde{w}_{m+1})-$$

$$-h_m S(x)F_{\nu,m+1}(t_{m+1},x)+h_m\varphi(x),\qquad x\in[0,\omega], \tag{22}$$

$$[I+D_{\nu,s}(t_s,x)]\dot{\lambda}_s(x)+\sum_{k=1}^{m}\widetilde{D}_{\nu,s,k}(t_s,x)\dot{\lambda}_{k+1}(x)-\dot{\lambda}_{s+1}(x)=-E_{\nu,s}(t_s,x)\lambda_s(x)-\sum_{k=1}^{m}\widetilde{E}_{\nu,s,k}(t_s,x)\lambda_{k+1}(x)-$$

$$-G_{\nu,s}(t_s,x,\widetilde{V}_s)-H_{\nu,s}(t_s,x,\widetilde{W}_s,\widetilde{w}_s)-F_{\nu,s}(t_s,x), \qquad s=\overline{1,m}, \qquad x\in[0,\omega]. \tag{23}$$

We denote by $Q_\nu(x)$ and $E_\nu(x)$ the $n(m+1)\times n(m+1)$ matrices composed of the coefficients $\dot{\lambda}_r(x)$ and $\lambda_r(x)$ in (22), (23), respectively, $r=\overline{1,m+1}$.

So, we can rewrite the equations (12) and (13) in the compact form

$$Q_\nu(x)\dot{\lambda}(x)=-E_\nu(x)\lambda(x)-F_\nu(x)-G_\nu(x,\widetilde{V})-H_\nu(x,\widetilde{W},\widetilde{w}), \tag{24}$$

where $F_\nu(x)=\big(h_m S(x)F_{\nu,m+1}(t_{m+1},x)-h_m\varphi(x),F_{\nu,1}(t_1,x),...,F_{\nu,m}(t_m,x)\big)'$,

$G_\nu(x,\widetilde{V})=\big(h_m S(x)\Big[G_{\nu,m+1}(t_{m+1},x,\widetilde{V}_{m+1}),G_{\nu,1}(t_1,x,\widetilde{V}_1),...,G_{\nu,m}(t_m,x,\widetilde{V}_m)\big)'$,

$H_\nu(x,\widetilde{W},\widetilde{w})=\big(h_m S(x)H_{\nu,m+1}(t_{m+1},x,\widetilde{W}_{m+1},\widetilde{w}_{m+1}),H_{\nu,1}(t_1,x,\widetilde{W}_1,\widetilde{w}_1),...,H_{\nu,m}(t_m,x,\widetilde{W}_m,\widetilde{w}_m)\big)'$.

System (24) with conditions (17) given us Cauchy problem for ordinary differential equations with respect to $\lambda_r(x)$, $r=\overline{1,m+1}$.

If we know $\widetilde{w}_r(t,x)$ and its partial derivatives $\widetilde{V}_r(t,x)$, $\widetilde{W}_r(t,x)$, then from Cauchy problem (24), (17) we find $\dot{\lambda}_r(x)$ and $\lambda_r(x)$ for all $x\in[0,\omega]$, where $r=\overline{1,m+1}$. Conversely, if we know $\lambda_r(x)$ and its derivative $\dot{\lambda}_r(x)$, then from Goursat problem (12)–(14) we can find $\widetilde{w}_r(t,x)$ and its partial derivatives $\widetilde{V}_r(t,x)$, $\widetilde{W}_r(t,x)$ for all $(t,x)\in\Omega_r$, $r=\overline{1,m+1}$. For solve Goursat problem (12)–(14) we use equivalent system of three integral equations (18)–(20).

Since the $\widetilde{w}_r(t,x)$ and $\lambda_r(x)$ are unknown to find a solution to problem (12)–(16) we use the iterative method: 1) At fixed $\widetilde{w}_r(t,x)$ from the Cauchy problem (24), (17) we find the introducing parameters $\lambda_r(x)$ and their derivative $\dot{\lambda}_r(x)$ for all $x\in[0,\omega]$, $r=\overline{1,m+1}$; 2) At fixed $\lambda_r(x)$ from the Goursat problem (12)–(14) we find the unknown functions $\widetilde{w}_r(t,x)$ and their partial derivatives $\widetilde{V}_r(t,x)$, $\widetilde{W}_r(t,x)$ for all $(t,x)\in\Omega_r$, $r=\overline{1,m+1}$.

Let $h=\max\limits_{i=\overline{1,m+1}}(t_i-t_{i-1})$, $\alpha(x)=\max\limits_{t\in[0,T]}||A_1(t,x)||$, $\beta_k(x)=\max\limits_{t\in[0,T]}||K_{1,k}(t,x)||$, $k=\overline{1,m}$.

The following assertion given us a sufficient conditions of unique solvability to problem (12)–(16) and a convergence this iterative process.

Theorem 1. Let for some ν, $\nu\in\mathbb{N}$ the $(n(m+1)\times n(m+1))$ matrix $Q_\nu(x)$ is invertible for all $x\in[0,\omega]$ and the following conditions are valid:

1) $||[Q_\nu(x)]^{-1}||\le\gamma_\nu(x)$, where $\gamma_\nu(x)$ is positive and continuous on $[0,\omega]$ function;

2) $q_\nu(x)=\gamma_\nu(x)\cdot\left\{e^{\alpha(x)h}-\sum\limits_{j=0}^{\nu}\frac{[\alpha(x)h]^j}{j!}+\Big[e^{\alpha(x)h}-\sum\limits_{j=0}^{\nu-1}\frac{[\alpha(x)h]^j}{j!}\Big]h\sum\limits_{k=1}^{m}\beta_k(x)\right\}\le\chi<1,$

where χ - const.

Then problem with parameters (12)–(16) has unique solution.

Theorem 2. Let for some ν, $\nu\in\mathbb{N}$ the $(n(m+1)\times n(m+1))$ matrix $Q_\nu(x)$ is invertible for all $x\in[0,\omega]$ and conditions 1)-2) of Theorem 1 are fulfilled.

Then nonlocal problem for system of loaded hyperbolic equations of the second order (9)–(11) has unique classical solution.

The proofs of Theorem 1 and 2 are similar of proof Theorem 1 in [22].

Therefore, for problem (4)–(7) we have the following statement.

Theorem 3. Let

i) the $n\times n$ matrices $A_i(t,x)$, $B_i(t,x)$, $K_{i,k}(t,x)$, $L_{i,k}(t,x)$, $M_{i,k}(t,x)$, $i=\overline{1,3}$, $k=\overline{1,m}$, $C(t,x)$, and n vector function $f(t,x)$ are continuous on Ω;

ii) the $n\times n$ matrices $P(x)$, $S(x)$, and n vector function $\varphi(x)$ are continuous on $[0,\omega]$;

iii) the n vector-functions $\psi_0(t)$, $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$;

iv) the nonlocal problem for system of loaded hyperbolic equations of the second order (9)–(11) is uniquely solvable for any $F(t,x)\in C(\Omega,\mathbb{R}^n)$, $\varphi(x)\in C([0,\omega],\mathbb{R}^n)$ and $\psi_2(t)\in C^1([0,T],\mathbb{R}^n)$.

Then problem with integral conditions (4)–(7) has a unique solution.

This Theorem is proved on the basis of the above algorithm and is similar of proof Theorem 2 [23].

From equivalence of problem (1)–(3) and (4)–(7) it follows

Theorem 4. Let

1) the conditions i)–iii) of Theorem 3 are fulfilled;

2) for some ν, $\nu\in\mathbb{N}$ the $(n(m+1)\times n(m+1))$ matrix $Q_\nu(x)$ is invertible for all $x\in[0,\omega]$ and $||[Q_\nu(x)]^{-1}||\le\gamma_\nu(x)$, where $\gamma_\nu(x)$ is positive and continuous on $[0,\omega]$ function;

3) the following inequality holds:

$$q_\nu(x) = \gamma_\nu(x) \cdot \left\{ e^{\alpha(x)h} - \sum_{j=0}^{\nu} \frac{[\alpha(x)h]^j}{j!} + \Big[e^{\alpha(x)h} - \sum_{j=0}^{\nu-1} \frac{[\alpha(x)h]^j}{j!} \Big] h \sum_{k=1}^{m} \beta_k(x) \right\} \le \chi < 1,$$

where χ *- const.*

Then problem (1)–(3) has a unique classical solution.

So, the nonlocal problem for system of loaded partial differential equations of the fourth order (1)–(3) is reduced to an equivalent nonlocal problem with integral conditions for system of loaded hyperbolic equations of the second order. For solve of the nonlocal problem with integral conditions for system of loaded hyperbolic equations of the second order results of articles [22–23] are used. Algorithms of finding solutions to the nonlocal problem with integral conditions for system of loaded hyperbolic equations of the second order are constructed and their convergence is proved. The conditions of the unique solvability to the nonlocal problem for system of loaded partial differential equations of the fourth order are established.

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А.Т. Асанова, А.Е. Иманчиев, Ж.М. Кадирбаева

Төртінші ретті дербес туындылы жүктелген дифференциалдық теңдеу үшін бейлокал есеп

Төртінші ретті дербес туындылы жүктелген дифференциалдық теңдеулер жүйесі үшін бейлокал есеп қарастырылған. Қарастырылып отырған есептің жалғыз шешімінің бар болуы мәселелері мен оны табу жолдары зерттелді. Жаңа функциялар енгізу әдісі арқылы төртінші ретті дербес туындылы жүктелген дифференциалдық теңдеулер жүйесі үшін бейлокал есеп екінші ретті жүктелген гиперболалық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есепке келтіріледі. Нәтижесінде алынған интегралдық шарттары бар бейлокал есепті шешу үшін функционалдық параметрлер енгізу әдісі қолданылды. Жүктелген гиперболалық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есептің жуық шешімдерін табу алгоритмдері ұсынылған және оның жинақтылығы дәлелденген. Жүктелген гиперболалық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есептің бірмәнді шешілімділігінің шарттары бастапқы берілімдер терминінде алынған. Нәтижелер сәйкесінше бастапқы төртінші ретті дербес туындылы жүктелген дифференциалдық теңдеулер жүйесі үшін бейлокал есепке қатысты тұжырымдалған.

Кілт сөздер: бейлокал есеп, төртінші ретті дербес туындылы жүктелген дифференциалдық теңдеулер, интегралдық шарт, жүктелген гиперболалық теңдеулер жүйесі, алгоритм, бірмәнді шешілімділік.

А.Т. Асанова, А.Е. Иманчиев, Ж.М. Кадирбаева

Нелокальная задача для нагруженного дифференциального уравнения в частных производных четвертого порядка

Рассмотрена нелокальная задача для системы нагруженных дифференциальных уравнений в частных производных четвертого порядка. Исследованы вопросы существования единственного решения рассматриваемой задачи и способы его построения. Методом введения новых функций нелокальная задача для системы нагруженных дифференциальных уравнений в частных производных четвертого порядка сведена к нелокальной задаче с интегральными условиями для системы нагруженных гиперболических уравнений второго порядка. Для решения полученной нелокальной задачи с интегральными условиями применен метод введения функциональных параметров. Предложены алгоритмы нахождения приближенного решения нелокальной задачи с интегральными условиями для системы нагруженных гиперболических уравнений второго порядка и доказана их сходимость. Получены условия однозначной разрешимости нелокальной задачи с интегральными условиями для системы нагруженных гиперболических уравнений в терминах исходных данных. Результаты сформулированы относительно исходной нелокальной задачи для системы нагруженных дифференциальных уравнений в частных производных четвертого порядка.

Ключевые слова: нелокальная задача, нагруженные дифференциальные уравнения в частных производных четвертого порядка, интегральное условие, система нагруженных гиперболических уравнений, алгоритм, однозначная разрешимость.

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