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∇-cl-atomic and prime sets

In this article the model-theoretic properties of special formula subsets of the semantic model of some fixed Jonsson theory are considered. The main purpose of this paper is the study of concepts of models' primeness and atomness in the study of inductive theories which admit the property of joint embedding and amalgama property. For this purpose is determined special sets, each element of which realise some type which is which is the main type in the sense of an existential formulas. Definable closures of such sets form an existential closed model. The main result obtained in this paper describes the properties of atomic and prime sets regarding strongly convex Jonsson theories.

Keywords: strongly convex theory, center of Jonsson theory, semantic model, atomic set, algebraically prime set, core set.

In the well-known paper [1], R. Vaught have proved the fundamental theorem-criterion on the behavior of countable prime and atomic models for complete theories in countable language. The essence of this criterion is that in a complete theory any countable-prime model is at the same time an atomic model of this theory. After some time A. Robinson in [2] have defined the concept of an algebraically prime model. This concept is a generalization of the concept of a prime model. Further, in the well-known work [3], D. Baldwin and D. Kueker considered the concept of new types of atomicity of a countable model. Naturally, appear the question about an analogue of Vaught's theorem for an algebraically prime model. We denote this problem by AAP (atomicity & algebraically primeness).

After some time, A. Robinson in [2] defined the concept of an algebraically prime model, and this concept is a generalization of the concept of a prime model. Further, in the well-known paper [3], J.T. Baldwin and D.W. Kueker considered the concept of new types of atomic of a countable model. Obviously, the question arose about the analogue of the theorem of Vaught for an algebraically prime model. We denote this problem symbolically by AAP (atomicness & algebraically primeness). Unfortunately, in [3], the authors were unable to obtain a criterion for an algebraically prime model in the language of new types of atomicity; moreover, a sufficient number of examples given in this paper suggests that this issue is unlikely to be resolved positively, i.e. a criterion or some conditions connecting the concepts of algebraic simplicity and the corresponding form of atomicity from [3] are obtained.

In this paper, we transfer the main ideas from [3] to countable models of some fixed Jonsson theory. Interest in the study of Jonsson theories is due to the following factors. Firstly, the class of Jonsson theories contains a sufficient number of well-known classical examples of algebras that are widely used in various sections of mathematics. For example, to Jonsson theories we can relate the theory of groups, Abelian groups, a large number of different types of rings, in particular, fields of fixed characteristic, also linear orders and Boolean algebras and such universal object as polygons over a monoid or S-actions, where S is a monoid. Secondly, arbitrary Jonsson theory is, generally speaking, not complete, and since the technical apparatus of the modern Model theory is adapted for the study of complete theories, the conditions that determine the jonssonness, naturally, distinguish among all, generally speaking, incomplete theories, which more or less adapted to the model-theoretic study of the class of theories. Nevertheless, some completeness of the considered Jonsson theory is necessary and, as a rule, it does not exceed $\forall$, $\exists$ or $\forall\exists$ completeness. Thirdly, when studying Jonsson theories, an important role is played the types of morphisms, with the help of which the classes of models of these theories are studied. If in the case of a complete theory, we are dealing with elementary monomorphisms (embeddings or extensions), then in the case of a Jonsson theory we will deal with an isomorphic and homomorphic morphisms (embeddings or extensions). When studying the Jonsson theories, we distinguish some special subclasses in which the behavior of countable models is more predictable with respect to the AAP problem. These are the following classes of theory: the class of convex theories defined by A. Robinson [2] and the class of existentially prime theories [4].

Studing the latest results of the modern model theory, it became clear, that a model-theoretic approach to the study of formula-definable subsets of some considered model is great importance. For complete theories, this model is associated with the monster model; in the Jonsson's case, analog of theory is the semantic model of considered theory. In this article, we will consider special formula subsets that, on the one hand, define atomicity in the sense of [3], but, on the other hand, firstly, give some geometrical interpretation in the sense of pregeometry given on the Boolean of semantic model, secondly, gives a new tool for study of the corresponding type of atomicity. So in this paper, we continue to investigate the AAP problem within the above paper and restrictions. We give the necessary definitions and related ones for further paper in this article.

We give the definitions [5] and related results necessary for further work in this article. Recall that

Definition 1. A theory T is Jonsson if:

1) theory T has infinite models;

2) theory T is inductive;

3) theory T has the joint embedding property (JEP);

4) theory T has the property of amalgam (AP).

Examples of Jonsson theories are:

1) group Theory;

2) theory of Abelian groups;

3) theory of fields of fixed characteristics;

4) theory of Boolean algebras;

5) theory of polygons over a fixed monoid;

6) theory of modules over a fixed ring;

7) theory of linear order.

When studying the model-theoretic properties of Jonsson theory, the semantic method plays an important role. It consists in the following: the elementary properties of the center of Jonsson theory are in a certain sense associated with the corresponding first-order properties of Jonsson theory itself. The center of Jonsson theory is a syntactic invariant and its properties are well defined in the case when Jonsson theory is perfect. The following concepts define the essence of the semantic model and the center of Jonsson theory [6].

Definition 2. Let $\kappa \geq \omega$. Model M of theory T is called κ-universal for T, if each model T with the power strictly less κ isomorphically imbedded in M; κ- homogeneous for T, if for any two models A and A_1 of theory T, which are submodels of M with the power strictly less then κ and for isomorphism $f : A \rightarrow A_1$ for each extension B of model A, wich is a submodel of M and is model of T with the power strictly less then κ there is exist the extension B_1 of model A_1, which is a submodel of M and an isomorphism $g : B \rightarrow B_1$ which extends f.

Definition 3. Model C of Jonsson theory T is called semantic model, if it is ω^+-homogeneous-universal.

Definition 4. The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^*, i.e. $T^* = Th(C)$.

The following two facts speak about the «good» exclusivity of the semantic model.

Fact 1 [6; 160]. Each Jonsson theory T has k^+-homogeneous-universal model of power 2^k. Conversely, if a theory T is inductive and has infinite model and ω^+-homogeneous-universal model then the theory T is a Jonsson theory.

Fact 2 [6; 160]. Let T is a Jonsson theory. Two k-homogeneous-universal models M and M_1 of T are elementary equivalent.

Definition 5. Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^*.

The following theorem is a criterion of perfectness of Jonsson theory.

Theorem 1 [6; 158]. Let T is a Jonsson theory. Then the following conditions are equivalent:

1) Theory T is perfect;

2) Theory T^* is a model companion of theory T.

Theorem 2 [6; 162]. If T is a perfect Jonsson theory then $E_T = ModT^*$.

We will select some special subsets of the semantic model.

Definition 6. Let $X \subseteq C$. We will say that a set X is $\nabla - cl$-Jonsson subset of C, if X satisfies the following conditions:

1) X is ∇-definable set (this means that there is a formula from ∇, the solution of which in the C is the set X, where $\nabla \subseteq L$, that is ∇ is a view of formula, for example $\exists, \forall, \forall\exists$ and so on.);

2) $cl(X) = M$, $M \in E_T$, where cl is some closure operator defining a pregeometry over C (for example $cl = acl$ or $cl = dcl$).

When studying the model-theoretic properties of an inductive theory, so called existentially closed models play an important role. Recall their definitions.

Definition 7. Model A of a theory T is called existentially closed if for any model B and any existential formula $\varphi(\overline{x})$ with constants of A we have $A \models \exists\overline{x}\varphi(\overline{x})$ provided that A is a submodel of B and $B \models \exists\overline{x}\varphi(\overline{x})$.

Through E_T we denote the class of all existentially closed models of the theory T.

In connection with this definition in the frame of the study of inductive theories, the following two remarks are true:

Remark 1: For any inductive theory E_T is not empty.

Remark 2: Any countable model of the inductive theory is isomorphically embedded in some countable existentially closed model of this theory.

An analogue of a prime model (in the sense of a complete theory) for an inductive model, generally speaking, incomplete theory, is the concept of an algebraically prime model, which introduced A. Robinson [2].

Definition 8. A is an algebraically prime model of theory T, if A is a model of T and A may be isomorphically embedded in each model of the theory T.

Note that since the class of Jonsson theories of a fixed signature is a subclass of inductive theories of this signature, then the above remarks 1,2 are true for Jonssons theories and, by criterion of Jonsson theory's perfectness, class of existentially closed models of considered Jonsson theory coincides with the class of center's model of this theory.

In connection with the interest to the AAP problem in the frame of the study of Jonsson theory in [7] a new class of theories was defined, in which there is an algebraically prime model which is existentially closed.

Recall the definition of this class.

Definition 9. The inductive theory T is called the existentially prime if: 1) it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T; 2) class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq 0$.

The following definition of a theory's convexity belongs to A. Robinson [2].

Definition 10. The theory is called convex if for any its model A and for any family $\{B_i \mid i \in I\}$ of substructures of A, which are models of the theory T, the intersection $\bigcap_{i\in I} B_i$ is a model of T, provided it is non-empty. If in addition such an itnersection is never empty, then T is called strongly convex.

The concept of a core model which introduced by A. Robinson is also an example of a particular case of an algebraically prime model.

Definition 11. A signature model of a given theory (hereinafter structure) is called core if it is isomorphic to the unique substructure of each model of the given theory. The core structure that is the model of the theory of a given signature will be called the core model of the theory.

The following result from Kueker's paper [8] gives a criterion of the existence of a core structure.

Theorem 3. For any T the following conditions are equivalent:

(1) C is a core structure for T;

(2) C is a model of every universal sentence consistent with T, and there are existential formulas $\varphi_i(x)$ and $k_i \in \omega$, for $i \in I$, such that

$$C, T \models \exists^{=k_i} x\varphi_i \quad \text{for all} \quad i \in I,$$

and

$$C \models \forall x \bigvee_{i\in I} \varphi_i.$$

The following definitions are taken from J. Baldwin and D. Kueker's work [3]. These definitions distinguish a whole class of new types of atomic models, and this new type of atomic models differs significantly from the concept of the atomic model from [1].

Definition 12. A formula $\varphi(\overline{x})$ is a Δ-formula, if exist existential formulas (from Σ) $\psi_1(\overline{x})$ and $\psi_2(\overline{x})$ such that

$$T \models (\varphi \leftrightarrow \psi_1) \quad \text{и} \quad T \models (\neg\varphi \leftrightarrow \psi_2).$$

Definition 13.

(i) $(A, a_0, ..., a_{n-1}) \Rightarrow_\Gamma (B, b_0, ..., b_{n-1})$ means that for every formula $\varphi(x_1, ..., x_{n-1})$ of Γ, if $A \models \varphi(\overline{a})$, then $B \models \varphi(\overline{b})$.

(ii) $(A, \overline{a}) \equiv_\Gamma (B, \overline{b})$ means that $(A, \overline{a}) \Rightarrow_\Gamma (B, \overline{b})$ and $(B, \overline{b}) \Rightarrow_\Gamma (A, \overline{a})$.

As classes Γ we consider $\triangle$ or Σ.

The following definition of an atomic model refers to [1].

Consider a complete theory T in L. A formula $\varphi(x_1...x_n)$ is said to be complete (in T) iff for every formula $\psi(x_1...x_n)$ exactly one of

$$T \models \varphi \to \psi, \quad T \models \varphi \to \neg\psi$$

holds. A formula $\theta(x_1...x_n)$ is said to be completable (in T) iff there is a complete formula $\varphi(x_1...x_n)$ with $T \models \varphi \to \theta$. If $\theta(x_1...x_n)$ is not completable it is said to be incompleatable.

A theory T is said to be atomic iff every formula of L which is consistent with T is completable in T. A model A is said ti be an atomic model iff every n- tuple $a_1...a_n \in A$ satisfies a complete formula in $Th(A)$

Definition 14. A model is called atomic if every tuple of its elements satisfies some complete formula. In connection with the new conceptof atomicity from [3], the following concept will be analogous to the definition of a complete formula

Definition 15. A formula $\varphi(x_1, ..., x_n)$ is complete for Γ-formulas(w.r.t T) if φ is consistent with $T$and for every formula $\psi(x_1, ..., x_n)$ in Γ, having no more free variables than φ, either

$$T \models \forall \bar{x} (\varphi \to \psi) \quad \text{or} \quad T \models \forall \bar{x} (\varphi \to \neg\psi).$$

Equivalently, a consistent $\varphi(\overline{x})$ is complete for Γ — formulas provided whenever as $\psi(\overline{x})$ is a Γ — formula and $(\varphi \wedge \psi)$ is consistent with T, then $T \models (\varphi \to \psi)$.

And the concept of the atomic model from [1] is transformed into the following concept from [3].

Definition 16. B is a $(\Gamma_1, \Gamma_2,)$ — atomic model of T, if B is a model of T and for every n every n-tuple of elements of A satisfies some formula from B in Γ_1, which is complete for Γ_2-formulas.

The following notion of a weakly atomic model from [3] is a generalization of above definition.

Definition 17. B is a weak (Γ_1, Γ_2) — atomic model of T, if B is a model of T and for every n every n-tuple $\overline{a}$a of elements of A satisfies in B some formula $\varphi(\overline{x})$ of Γ_1 such that $T \models (\varphi \to \psi)$ as soon as $\psi(\overline{x})$ of Γ_2 and $B \models \psi(\overline{a})$.

In this paper we will not give examples of the (Γ_1, Γ_2) — atomic model and the weak (Γ_1, Γ_2) atomic model, leaving the reader to do this on their own, referring to a sufficient the number of examples of these concepts given in [3].

Before discussing the results obtained, concerning to $\nabla - cl$ atomic models, we note that we fix some Jonsson theory T and its semantic model C in the countable language L and $\nabla \subseteq L$:∇ is consistent with T, that is, any finite subset of formulas from ∇ is consistent with T. Let $A \subseteq C$.

Let cl be, as in Definition 6, and it is true that $cl = acl$ and at the same time $cl = dcl$. It is clear that such the operator is a special case of the closure operator and its example is the a closure operator defined on any linear space as a linear shell.

We also assume that the pregeometry given by the cl operator is modular [9].

Definition 18. The set A will be called $(\nabla_1, \nabla_2) - cl$ atomic in the theory T, if

1) $\forall a \in A, \exists \varphi \in \nabla_1$ such that for any formula $\psi \in \nabla_2$ follows that φ is complete formula for ψ and $C \models \varphi(a)$;

2) $cl(A) = M, M \in E_T$.

Definition 19. A set A will be called weakly $(\nabla_1, \nabla_2) - cl$ is atomic in T, if

1) $\forall a \in A, \exists \varphi \in \nabla_1$ such that in $C \models \varphi(a)$ for any formula $\psi \in \nabla_2$ follow that $T \models (\varphi \to \psi)$ whenever $\psi(x)$ of ∇_2 and $C \models \psi(a)$;

2) $cl(A) = M, M \in E_T$.

It is easy to understand that definitions 18 and 19 are naturally generalized the notion of atomicity and weak atomicity to be ∇_1 atomic and weak ∇_1 atomic for any tuple of finite length from set A.

Thus, we have generalized the concepts (Γ_1, Γ_2) of the atomic model and weakly (Γ_1, Γ_2) of the atomic model dividing in to $(\nabla_1, \nabla_2) - cl$ atomic and weakly $(\nabla_1, \nabla_2) - cl$ atomic set. Also note that the concept $(\nabla_1, \nabla_2) - cl$ atomic and weakly $(\nabla_1, \nabla_2) - cl$-atomic sets are some special modifications of definition 6.

Let $i \in \{1, 2\}$, $M_i = cl(A_i)$, where $A_i = (\nabla_1, \nabla_2)$ is a $cl-$ atomic set . $a_0, ..., a_{n-1} \in A_1$, $b_0, ..., b_{n-1} \in A_2$.

Definition 20.

(i) $(M_1, a_0, ..., a_{n-1}) \Rightarrow_\nabla (M_2, b_0, ..., b_{n-1})$ means that for every formula $\varphi(x_1, ..., x_{n-1})$ of ∇, if $M_1 \models \varphi(\overline{a})$, then $M_2 \models \varphi(\overline{b})$.

(ii) $(M_1, \overline{a}) \equiv_\nabla (M_2, \overline{b})$ means that $(M_1, \overline{a}) \Rightarrow_\nabla (M_2, \overline{b})$ and $(M_1, \overline{b}) \Rightarrow_\nabla (M_1, \overline{a})$.

Definition 21. A set A is said to be $(\nabla_1, \nabla_2) - cl$-algebraically prime in the theory T, if

1) If A is $(\nabla_1, \nabla_2) - cl$-atomic set in T;

2) $cl(A) = M, M \in AP_T$.

From the definition of an algebraically prime set in the theory T follows that the Jonsson theory T which has an algebraically prime set is automatically existentially prime. It is easy to understand that an example of such a theory is the theory of linear spaces.

Definition 22. The set A is said to be $(\nabla_1, \nabla_2) - cl$-core in the theory T, if

1) If A is (∇_1, ∇_2) a cl - atomic set in the theory T;

2) $cl(A) = M$, M is the core model of the T theory.

We formulate some obtained results regarding these new concepts.

Lemma 1. Let T be complete for existential sentences perfect Jonsson theory. 1) If A is weakly $(\nabla, \Delta) - cl$-atomic set in the theory T, then A is $(\nabla, \Delta) - cl$-atomic set, 2) If A is weak $(\nabla, \Delta) - cl$-atomic set in the theory T, then A is $(\nabla, \Delta) - cl$-atomic set.

Proof. Note, that due to the perfectness of the theory T we use theorems 1,2 and definition 19. Since $dcl(A) = M \in E_T$, then $M \in ModT^*$, where T^* is a center of T. Since the theory T is perfect, then T^* is model companion of T, and accordingly is a model complete theory. So any formula of T^* is equivalent to some Σ-formula.

It follows that any $(\nabla_1, \nabla_2) - cl$ set A is $(\Delta, \Delta) - cl$ set A. It follows that both points of Lemma 1 are satisfied.

Let $i \in \{1, 2\}$, $M_i = cl(A_i)$, where $A_i = (\Sigma, \Sigma) - cl$-is a atomic set. $a_0, ..., a_{n-1} \in A_1$, $b_0, ..., b_{n-1} \in A_2$.

Theorem 4. Let T - be complete for $\exists$-sentences a strongly convex Jonsson perfect theory and let A is $(\nabla_1, \nabla_2) - cl$-atomic set in T.

Then $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv) \wedge (vi), (i) \Rightarrow (i)^* \Rightarrow (v) \Rightarrow (vi), (ii) \Rightarrow (ii)^* \Rightarrow (vi)$, $(i)^* \Rightarrow (ii)^*$ and $(iv)^* \Rightarrow (iv)$, where:

(i) A is $(\Delta, \Sigma) - cl$-atomic set in theory T,

$(i)^*$ A is weakly $(\Delta, \Pi) - cl$-atomic set in theory T,

(ii) A is $(\Sigma, \Sigma) - cl$-atomic set in theory T,

$(ii)^*$ A is weakly $(\Sigma, \Pi) - cl$-atomic set in theory T,

(iii) A is weakly $(\Sigma, \Sigma) - cl$-atomic set in theory T,

(iv) $cl(A) \in AP_T$,

$(iv)^*$ A is core in theory T,

(v) A is weakly $(\Delta, \Delta) - cl$-atomic set in theory T,

(vi) A is weakly $(\Sigma, \Delta) - cl$-atomic set in theory T,

Lemma 2. Let A_1 will be weak $(\Sigma, \Sigma) - cl$-atomic set of T. Assume that

$$(M_1, a_0, ..., a_{n-1}) \Rightarrow_\exists (M_2, b_0, ..., b_{n-1}).$$

Then for any $a_n \in M_1$ there is some $b_n \in M_2$ such that

$$(M_1, a_0, ..., a_n) \Rightarrow_\exists (M_2, b_0, ..., b_n).$$

Proof. Let $\varphi(x_0, ..., x_{n-1})$ be existential, satisfied by $a_0, ..., a_{n-1}$ in M_1, and which imply every existential formula satisfied by M_1 $a_0, ..., a_{n-1}$. It follows from the definition 19. Let $\psi(x_0, ..., x_n)$ be satisfy for the some $a_0, ..., a_n$. Then $T \models (\varphi \rightarrow \exists x_n \psi)$ and $M_2 \models \varphi(b_0, ..., b_{n-1})$, is follows, that exists some b_n, such that $M_2 \models \psi(b_0, ..., b_n)$, and this b_n, will be what we need.

We can show, that (iii)⇒(iv). Let M_1 be countable and weak (Σ, Σ)-atomic, and let M_2 be any model of T. Then $M_1 \Rightarrow_\exists M_2$ since T is a complete theory for existential sentences, and Lemma 2 can be applied repeatedly where $A = \{a_i : i \in \omega\}$ to build step by step an embedding of A_1 into M_2.

Remark 3: By the perfectness of T, we can apply Lemma 1 and then, by Lemma 1, we can replace ∇_i on Δ, where $i \in \{1, 2\}$. Due to the strongly convexity of the theory, the theory T has a unique core model. This follows from the fact that if the theory satisfies the property of joint embedding and is additionally strongly convex, then its core model in the theory T is unique up to isomorphism [8]. Based on this fact, we can conclude that under the conditions of this theorem we have a unique core model, since its existence follows from strongly convexity, and its uniqueness follows from the combination with Jonssonness.

Proof. The only implication that is not follows directly from the definitions is $(iii) \Rightarrow (iv)$, which is a consequence of the previous Lemma 2, and $(iv) \Rightarrow (iv)^*$ follows from the remark 3.

All concepts that are not defined here can be extracted from [6].

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∇-cl-атомдық және жай жиындар

Мақалада белгілі бекітілген йонсондық теориясының семантикалық моделінің арнайы формулалар ішкі жиындарының модельді-теориялық қасиеттері қарастырылған. Бұл жұмыстың негізгі мақсаты жай және атомдық модельдердің индуктивті теориялар аясында үйлесімді енгізу және амальгама қасиеттерінің түсініктері болып табылады. Бұл үшін арнайы жиындар анықталды, олардың әр элементі экзистенционалдық формулалар аясында кейбір басты типті жүзеге асырады. Осындай жиындарының анықталған тұйықталуы экзистенционалды тұйық модельді қалыптастырады. Осы мақалада алынған негізгі нәтиже салыстырмалы түрде дөңес йонсондық теориясының атомдық және жай жиынтығының қасиеттерін сипаттайды.

Кілт сөздер: қатты дөнес теория, йонсон теориясының орталығы, семантикалық модель, атомдық жиын, алгебралық жай жиын, ядролық жиын.

А.Р. Ешкеев, А.К. Исаева

∇-*cl*-атомные и простые множества

В работе рассмотрены теоретико-модельные свойства специальных формульных подмножеств семантической модели некоторой фиксированной йонсоновской теории. Основной целью данной работы является изучение понятий простоты и атомности моделей в раках изучения индуктивных теорий, допускающих свойства совместного вложения и свойства амальгамы. Для этой цели определяются специальные множества, каждый элемент которых реализует некоторый тип, являющийся главным в смысле экзистенциальных формул. Определимые замыкания таких множеств образуют экзистенциальную замкнутую модель. Основной результат, полученный в этой работе, описывает свойства атомных и простых множеств относительно сильно выпуклых йонсоновских теорий.

Ключевые слова: сильно выпуклая теория, центр йонсоновской теории, семантическая модель, атомное множество, алгебраически простое множество, ядерное множество.

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