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On a characteristic problem for a loaded hyperbolic equation

The paper studies a loaded hyperbolic equation with one-dimensional wave equation in its main part. In the loaded components there are two points, which are distributed respectively along a pair of intersecting characteristics at a constant speed. For such an equation we study the Cauchy problem with characteristics of the one dimensional wave equation belonging to any of the pair of intersecting straight lines. If to impose certain conditions at the boundary points on function presenting Cauchy data and the derivatives of the first, second and third orders we can prove the existence and uniqueness of the problem. The proof of the existence and uniqueness of the solution follows directly from the method of its production. We consider also issues concerning domains of dependence, influence, and definitions for Cauchy data that are specified on one of the characteristic curves. The above verifies once more the thesis of load influencing on posing of various initial - boundary value problem for partial differential equations.

Keywords: Cauchy problem, loaded equation, wave equation, characteristics, domain of influence, domain of dependence.

Introduction

A.M. Nakhushev first made the most general definition for a loaded equation in [1]. In [2] he had introduced concepts and a detailed classification for various loaded equations: loaded differential, loaded integral, loaded integrodifferential, loaded functional equations. Besides fundamental work on the study of loaded integral and loaded ordinary differential equations, we wish to acknowledge some of the proceedings [3–6]. Together with A.M. Nakhushev and his successors [7–16] a systematic researches and significant contribution to the boundary value problems for loaded differential equations had been made by the Kazakh mathematicians M.T. Dzhentaliev and M.I. Ramazanov, and their students [17, 18]. They investigated a wide class of homogeneous and inhomogeneous boundary value problems for essentially loaded parabolic and hyperbolic-elliptic equations, as well as the spectral issues on the corresponding homogeneous problems, when the load is specified with respect to the space variable and the load point moves at constant and variable speeds. Works [19–21] are devoted to uniqueness classes for solution of the Cauchy problem and non-trivial solutions of the homogeneous Cauchy problem for some classes of loaded differential equations and linear loaded systems of the first order.

The Cauchy problem and the Cauchy-Dirichlet problem for a spectrally loaded parabolic equation with a load at a fixed time variable are investigated in [22, 23].

The effect of the load on the convergence of spectral expansions for operators is considered in [24].

In [25–27] the Goursat problem for second-order strictly and weakly hyperbolic equations with two independent variables is investigated, where it is shown that the load allows eliminating the inequality between characteristics that are data in the Goursat problem.

This paper considers the following loaded equation

$$u_{xx} - u_{yy} = \lambda u(x + y, 0) + \mu u(x - y, 0), \quad (1)$$

where λ, μ are arbitrary real constants.

Using characteristic variables $\xi = x - y, \eta = x + y$ equation (1) has the form

$$v_{\xi\eta} = \frac{\lambda}{4} v(\eta, \eta) + \frac{\mu}{4} v(\xi, \xi), \quad (2)$$

where $v(\xi, \eta) = u\left(\frac{\xi+\eta}{2}, \frac{\eta-\xi}{2}\right)$.

By (2) it follows that

$$v(\xi, \eta) = f(\xi) + g(\eta) + \frac{\lambda}{4} \xi \int_0^\eta v(t, t) dt + \frac{\mu}{4} \eta \int_0^\xi v(t, t) dt, \quad (3)$$

where $f(\xi), g(\eta)$ are arbitrary smooth enough functions.

As $\eta = \xi$ by (3) obtain

$$\frac{d}{d\xi} \left[e^{-\frac{\lambda+\mu}{8}\xi^2} \int_0^\xi v(t, t) dt \right] = e^{-\frac{\lambda+\mu}{8}\xi^2} [f(\xi) + g(\xi)].$$

Hence

$$\int_0^\xi v(t, t) dt = \int_0^\xi e^{\frac{\lambda+\mu}{8}(\xi^2-t^2)} [f(t) + g(t)] dt. \quad (4)$$

Substituting into (3) instead of integrals values obtained using formula (4) and moving to x, y , coordinates we get

$$\begin{aligned} u(x, y) = f(x-y) + g(x+y) + \frac{\lambda}{4}(x-y) \int_0^{x+y} e^{\frac{\lambda+\mu}{8}[(x+y)^2-t^2]} [f(t) + g(t)] dt + \\ + \frac{\mu}{4}(x+y) \int_0^{x-y} e^{\frac{\lambda+\mu}{8}[(x-y)^2-t^2]} [f(t) + g(t)] dt. \end{aligned} \quad (5)$$

Formula (5) is an analogue of d'Alembert formula for equation (1) and obviously as $\lambda = \mu = 0$ coincides with the d'Alembert formula for equation (1) as $\lambda = \mu = 0$.

Assume $\Omega = \{(x, y) : 0 < x + y < 1, 0 < x - y < 1\}$ is a characteristic quadrilateral.

Cauchy problem. In the domain $\bar{\Omega}$ find a regular solution $u(x, y)$ of equation (1) continuous in $\bar{\Omega}$ and satisfying the conditions

$$u\left(\frac{x}{2}, \frac{x}{2}\right) = \varphi(x), \quad 0 \leq x \leq 1, \quad (6)$$

$$u_y\left(\frac{x}{2}, \frac{x}{2}\right) = \psi(x), \quad 0 \leq x \leq 1, \quad (7)$$

where $\varphi(x), \psi(x)$ – are specified functions.

It is well known that as $\lambda = \mu = 0$ this problem is not well posed. The necessary and sufficient condition for its solvability is

$$\varphi'(x) - \psi(x) = \varphi'(0) - \psi(0), \quad 0 \leq x \leq 1.$$

If the above condition is satisfied, the function $u(x, y)$ is a solution to problem (6), (7) for equation (1) and has the form

$$u(x, y) = f(x-y) - f(0) + \varphi(x+y),$$

where f is an arbitrary twice continuously differentiable function.

The following theorem is valid.

Theorem. Assume $\lambda \neq 0, \varphi \in C^3(\bar{J}) \cap C^4(J), \psi \in C^2(\bar{J}) \cap C^3(J)$ and the coordination conditions

$$\varphi(0) = \psi(0) = \varphi'(0) = 0, \quad (8)$$

$$\varphi''(0) - \psi'(0) = 0, \quad (9)$$

$$\varphi'''(0) - \psi''(0) = 0. \quad (10)$$

Then the solution for the Cauchy problem exists, is unique and representable in the form of

$$\begin{aligned} u(x, y) = \frac{4}{\lambda} [\varphi''(x-y) - \psi'(x-y)] - \left(x - \frac{\lambda+2\mu}{\lambda}y\right) [\varphi'(x-y) - \psi(x-y)] + \\ + (x-y) [\varphi'(x+y) - \psi(x+y)] + \varphi(x+y) - \varphi(x-y). \end{aligned} \quad (11)$$

Indeed the function $u(x, y)$ is a solution to the Cauchy problem if and only if it is representable as (5). Therefore, by substituting (5) into (6) and (7), we can get

$$f(0) + g(x) = \varphi(x), \quad (12)$$

$$-f'(0) + g'(x) - \frac{\lambda}{4} \int_0^x e^{\frac{\lambda+\mu}{8}(x^2-t^2)} [f(t) + g(t)] dt -$$

$$-\frac{\mu}{4}x[f(0) + g(0)] = \psi(x). \tag{13}$$

Taking into account (8) from (12) and (13) it is clear that

$$f(0) + g(0) = 0, \quad f'(0) = 0.$$

Hence, by (12) and (13) we have

$$g(x) = \varphi(x) - f(0), \tag{14}$$

$$\int_0^x e^{\frac{\lambda+\mu}{8}(x^2-t^2)} [f(t) + g(t)] dt = \frac{4}{\lambda} [\varphi'(x) - \psi(x)]. \tag{15}$$

By differentiation of (15) with respect to x and subtracting identity (15), previously multiplied by $\frac{\lambda+\mu}{4}x$ we can get

$$f(x) + g(x) = \frac{4}{\lambda} [\varphi''(x) - \psi'(x)] - \frac{\lambda + \mu}{\lambda} x [\varphi'(x) - \psi(x)]$$

or

$$f(x) = f(0) - \varphi(x) + \frac{4}{\lambda} [\varphi''(x) - \psi'(x)] - \frac{\lambda + \mu}{\lambda} x [\varphi'(x) - \psi(x)].$$

Substituting the obtained values into $f(x)$ and $g(x)$ in (5) and taking into account (15) when calculating the last two terms of formula (5), and after simple conversions we arrive at formula (11).

Substitution into equation (1) ensures us that the function $u(x, y)$ calculated by formula (11) is the solution to (1). It is easy to check that under conditions (8)–(10) the function $u(x, y)$ satisfies conditions (6), (7).

Note that the Cauchy problem as $\lambda = 0$ is not a well posed one. Indeed by (12), (13) the condition

$$\varphi'(x) - \psi(x) = \varphi'(0) - \psi(0) + \frac{\mu}{4}\varphi(0)$$

is necessary and sufficient for the solvability of the problem. In case this condition is satisfied the solution to the Cauchy problem is

$$u(x, y) = f(x - y) - f(0) + \varphi(x + y) + \frac{\mu}{4}(x + y) \int_0^{x-y} [f(t) - f(0) + \varphi(t)] e^{\frac{\mu}{4}[(x-y)^2-t^2]} dt,$$

where f — is an arbitrary twice continuously differentiable function.

If we make a replacement $u(x, y) = v(x, -y)$ in equation (1) then problem

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) = \varphi(x), \quad 0 \leq x \leq 1, \tag{16}$$

$$u_y\left(\frac{x}{2}, -\frac{x}{2}\right) = \psi(x) \quad 0 \leq x \leq 1, \tag{17}$$

becomes the problem

$$v\left(\frac{x}{2}, \frac{x}{2}\right) = \varphi(x), \quad v_y\left(\frac{x}{2}, \frac{x}{2}\right) = -\psi(x)$$

for the equation $v_{xx} - v_{yy} = \mu v(x + y, 0) + \lambda v(x - y, 0)$.

Therefore, assuming that $\mu \neq 0$ and

$$\varphi(0) = \psi(0) = \varphi'(0) = 0; \quad \varphi''(0) + \psi'(0) = 0; \quad \varphi'''(0) + \psi''(0) = 0,$$

the solution to the problem is representable as

$$\begin{aligned} v(x, y) = & \frac{\mu}{4} [\varphi''(x - y) + \psi'(x - y)] - \left(x - \frac{\mu + 2\lambda}{\mu}y\right) [\varphi'(x - y) + \psi(x - y)] + \\ & + (x - y) [\varphi'(x + y) + \psi(x + y)] + \varphi(x + y) - \varphi(x - y). \end{aligned}$$

This implies the solution to problem (16), (17) for equation (1) casts into the form

$$\begin{aligned} u(x, y) = & \frac{\mu}{4} [\varphi''(x + y) + \psi'(x + y)] - \left(x + \frac{\mu + 2\lambda}{\mu}y\right) [\varphi'(x + y) + \psi(x + y)] + \\ & + (x + y) [\varphi'(x - y) + \psi(x - y)] + \varphi(x - y) - \varphi(x + y). \end{aligned} \tag{18}$$

It is known [28] that in case with three spatial variables corresponding to the Cauchy problem a wave is completely determined by the Cauchy data on a sphere. This fact in the theory of sound is called Huygens principle. We also know that with two spatial variables in wave processes the Huygens principle does not hold since to determine the wave the Cauchy data must be specified not only on the circle but also at all points of the corresponding circle. In the case of one variable, to determine the value of the oscillation at the point (x, y) , one of the components of the Cauchy data must be set on the segment boundary $[x - y, x + y]$, and the second at all points of this segment.

The idea is that to determine $u(x, y)$ at the point (x, y) in formulas (11), (18) you need to know the Cauchy data only on the segment boundary $[x - y, x + y]$. That is, we can say that there is a one-dimensional version for the Huygens principle.

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А.Х. Аттаев

Жүктелген гиперболалық теңдеу үшін бір сипаттамалық есеп жайлы

Бұл жұмыста басты бөлігі шектің тербелісінің бірөлшемді теңдеуі болатын жүктелген гиперболалық теңдеу объект болып табылады. Жүктелген қосылғыштарда тұрақты жылдамдықпен қиылысатын сипаттаушы жұптарының бойына сәйкес таралатын жүктеменің екі нүктесі бар. Осындай теңдеу үшін бірөлшемді шектің тербелісінің теңдеуінің сипаттаушылары болып табылатын қиылысатын түзулердің кез келген жұбының деректерімен Коши есебін зерттеу жүргізілді. Коши деректерін беретін функцияға нүктелік сипаттағы белгілі шарттарда және олардың бірінші, екінші, үшінші туындылары қойылған есептің бар болуы және жалғыздығы дәлелденді, ал шешуі жайлы түсінік анық жазылды. Шешудің бар болуы және жалғыздығын дәлелдеу оны алу әдісінің өзінен шығады. Сонымен қоса бір сипаттаушыда берілетін, Коши деректерінің тәуелділік, әсер ету және анықталу облыстарымен байланысты сурақтар қарастырылды. Осылайша дербес туындылы жүктемені дифференциалдық теңдеулер үшін бастапқы-шектік есептердің қойылымына жүктеменің әсерінің тиімділігі жайлы тезис кезекті рет нақтыланды.

Кілт сөздер: Коши есебі, жүктемені теңдеу, шектің тербеліс теңдеуі, сипаттамалар, әсер ету облысы, тәуелсіздік облысы.

А.Х. Аттаев

Об одной характеристической задаче для нагруженного гиперболического уравнения

Объектом исследования статьи является нагруженное гиперболическое уравнение, главная часть которого представляет собой одномерное уравнение колебания струны. В нагруженных слагаемых присутствуют две точки нагрузки, которые распространяются соответственно вдоль пары пересекающихся характеристик с постоянной скоростью. Для такого уравнения проводится исследование задачи Коши с данными на любой из пары пересекающихся прямых, являющихся характеристиками уравнения колебания одномерной струны. При определенных условиях точечного характера на функции,

задающие данные Коши и производные первого, второго и третьего порядков от них, доказывается существование и единственность поставленной задачи, а представление самого решения выписывается в явном виде. Доказательство существования и единственности решения непосредственно следует из самого способа его получения. Также затрагиваются вопросы, связанные с областями зависимости, влияния и определения данных Коши, задаваемых на одной из характеристик. Этим самым в очередной раз подтверждается тезис об эффекте влияния нагрузки на постановку тех или иных начально-краевых задач для нагруженных дифференциальных уравнений с частными производными.

Ключевые слова: задача Коши, нагруженное уравнение, уравнение колебания струны, характеристики, область влияния, область зависимости.

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