МАТЕМАТИКА
MATHEMATICS

DOI 10.31489/2019M4/8-14

UDC 517.968

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On strongly loaded heat equations

The article is devoted to the research of boundary value problems for the spectrum - loaded operator of heat conduction with the moving point of loading to the temporary axle in zero or on infinity. For strongly loaded parabolic 2k-order equations the adjoint boundary value problems, when order of loaded term is greater then one of differential part of equation, is studied. In this article we continue a investigation of the boundary value problems for spectrally loaded parabolic equations in unbounded domains.The boundary value problem for the spectral-loaded equation of thermal conductivity, which on the one hand is quite close to the problems with the load containing the second derivative of the spatial variable, and is of independent interest on the other hand in this work, is considered.

Keywords: loaded heat equation, class of essentially bounded functions, inverse Laplace transformation, residue.

1 Statement of the problem

We consider the first boundary value problem of heat conduction in the degenerating domain $Q = \{x \in (0,\infty),\ t \in (0,\infty)\}$ the cogeralized boundary value problems for a heavily loaded heat equation (which generally is called a heat equation order $2k$) in the domain :

$$L_\lambda u = f \Leftrightarrow \begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} + \lambda \dfrac{\partial^{2k} u}{\partial x^{2k}} \mid_{x=a} = f, \\ u(x,0) = 0, u(0,t) = 0; \end{cases} \tag{1}$$

$$L^*_\lambda \upsilon = g \Leftrightarrow \begin{cases} -\dfrac{\partial \upsilon}{\partial t} - \frac{\partial^2 \upsilon}{\partial x^2} + \overline{\lambda}\delta^{(2k)}(x-a) \bigotimes \int_0^\infty \upsilon(\xi,t)d\xi = g, \\ u(x,\infty) = 0, \upsilon(0,t) = \upsilon(\infty,t) = 0, \end{cases} \tag{2}$$

where $a = const, a > 0, k \geq 2, \lambda = \lambda_1 + \iota\lambda_2 \in \mathcal{C}$ is the parameter

$$f, u \in L_1(Q), \frac{\partial^{2k} u}{\partial x^{2k}} \mid_{x=a} \in L_1(0,\infty); \qquad g, \upsilon, \int_0^\infty \upsilon(\xi,t)d\xi \in L_\infty(Q). \tag{3}$$

2 Reducing the problem to an integral equation

By inverting the differential part in the boundary value problem we obtain the following (1), we will have:

$$u(x,t) = -\lambda \int_0^t erf(\frac{x}{2\sqrt{t-\tau}}) * \frac{\partial^{2k}\upsilon(\eta,\tau)}{\partial\eta^{2k}} \mid_{\eta=a} d\tau + \\ + \int_0^t \int_0^\infty G(x,\xi,t-\tau) f(\xi,\tau) d\xi d\tau, \tag{4}$$

where

$$G(x,\xi,t) = \frac{1}{2\sqrt{\Pi t}} exp(-\frac{(x-\xi)^2}{4t}) - exp(-\frac{(x+\xi)^2}{4t}) \tag{5}$$

Then differentiating (4) x by $2k$ times and assuming $x = a$, we contain the integral equation Volterra of the second kind

$$K_{\lambda\mu} \equiv \mu(t) - \lambda \int_0^t K_{2k}(t-\tau)\mu(\tau)d\tau = f_1(t) \tag{6}$$

where the following notation is used:

$$\mu(t) = \frac{\partial^{2k}u}{\partial x^{2k}} \mid_{x=a}, f_1(t) = \left(\frac{\partial^{2k}}{\partial x^{2k}} \int_0^t \int_0^\infty G(x,\xi,t-\tau) f(\xi,\tau) d\xi d\tau \right)_{\mid x=a}$$

$$K_{2k}(t-\tau) = \frac{d^{2k}}{dx^{2k}} \left\{ erf\left(\frac{x}{2\sqrt{t-\tau}} \right) \right\}_{\mid x=a}$$

or for the kernel $K_{2k}(\theta)$, you can use the ratio

$$K_{2k(\theta)} = \frac{1}{\sqrt{\pi\theta}} \cdot \frac{d^{2k-1}}{dx\sqrt{2k-1}} \left\{ exp\left(-\frac{x^2}{4\theta} \right) \right\} \mid_{x=a}$$

For example, we write the explicit form of kernels for $k = 2, 3, 4$:

$$K_{4(\theta)} = \frac{1}{4\sqrt{\pi}} \left[-\frac{a\sqrt{3}}{2\theta^{7/2}} + \frac{3a}{\theta^{5/2}} \right] exp\left(-\frac{a^2}{4\theta} \right),$$

$$K_{6(\theta)} = \frac{1}{8\sqrt{\pi}} \left[-\frac{a\sqrt{5}}{4\theta^{11/2}} + \frac{5a^3}{\theta^{9/2}} - \frac{15a}{\theta^{7/2}} \right] exp\left(-\frac{a^2}{4\theta} \right),$$

$$K_{8(\theta)} = \frac{1}{16\sqrt{\pi}} \left[-\frac{a\sqrt{7}}{8\theta^{15/2}} + \frac{21a^5}{4\theta^{13/2}} - \frac{105a^3}{2\theta^{11/2}} + \frac{105a}{\theta\theta^{9/2}} \right] exp\left(-\frac{a^2}{4\theta} \right),$$

Inverting the differential part to problem (2) in the same way as in problem (1), we will have:

$$\upsilon(x,t) = -\overline{\lambda} \int_t^\infty \int_0^\infty G(x,\xi,\tau-t)\delta^{(2k)}(\xi-\tau) \bigotimes \int_0^\infty \upsilon(\eta,\tau) d\eta d\xi d\tau + \\ + \int_t^\infty \int_0^\infty G(x,\xi,\tau-t) g(\xi,\tau) d\xi d\tau. \tag{7}$$

Integrating the relation (7) over the variable x from 0 to ∞ and denoting

$$\upsilon(t) = \int_0^\infty \upsilon(\eta,t) d\eta, \tag{8}$$

we will obtain the integral equation

$$K_\lambda^* \upsilon \equiv \upsilon(t) - \overline{\lambda} \int_t^\infty K_{2k}(\tau-t)\upsilon(\tau)d\tau = g_1(t). \tag{9}$$

where

$$g_1(t) = \int_t^\infty \int_0^\infty erf\left(\frac{\xi}{2\sqrt{\tau-t}} \right) g(\xi,\tau) d\xi d\tau.$$

3 Laplace transformation. The partition plane of the spectral parameter

The equation (9) is an equation with a difference kernel, so you can use a transformation of Laplace to hear. In this case we use the following formulas [1–3]:

$$L\left\{\int_0^t K(t-\tau)\varphi(\tau)d\tau\right\} = \widehat{K}(p)\cdot\widehat{\varphi}L(p),$$

$$L\left\{\int_t^\infty K(t-\tau)\varphi(\tau)d\tau\right\} = \widehat{K}(-p)\cdot\widehat{\varphi}L(p),$$

where

$$\widehat{K}(-p) = \int_0^\infty K(t)e^{pt}dt,$$

we also use an easily checkable equality.

$$K_{2k}(t-\tau) = \frac{d^{2k}}{dx^{2k}}\left\{erf\left(dfracx2\sqrt{t-\hat{\tau}}\right)\right\}|_{x=a} = \frac{d^k}{dt^k}\left\{erf\left(\frac{x}{2\sqrt{t-\tau}}\right)\right\}|_{x=a}$$

Then if we apply the Laplace transform to the homogeneous equation (9), we obtain the following transcendental equation

$$\widehat{\varphi}L(p)\cdot[1-\overline{\lambda}\cdot\widehat{K}(-p)] = 0. \tag{10}$$

If we assume $\widehat{\varphi}L(p)\neq 0$ then the following equality must hold.

$$1-\overline{\lambda}\cdot\widehat{K}(-p) = 0. \tag{11}$$

Let equation (11) have one simple root -p_0 i.e.

$$1-\overline{\lambda}\cdot\widehat{K}(-p)] = (p-p_0)\cdot\psi(p),$$

where $\widehat{\psi}L(p)\neq 0$. Then equation (10) takes the form $\widehat{\varphi}L(p)\cdot(p-p_0) = 0$, therefore $\widehat{\varphi}L(p) = \delta(p-p_0)$, a, so $\varphi(t) = e^{p_0 t}$, $R_{ep_0} < 0$. From [2; 390, theorem 146] it follows that functions of this kind are the only solutions of the homogeneous equation (9) [4–6].
In our case

$$\widehat{K}(-p) = (-p)^{k-1}e^{-a\sqrt{-p}}.$$

Therefore, we need to find the roots of the transcendental equation

$$1-\overline{\lambda}\cdot(-p)^{k-1}e^{-a\sqrt{-p}} = 0, Re(-p) > 0. \tag{12}$$

In contrast to the previously considered case (spectral-loaded, k1), the roots of equations (12) can be found only approximately (for each numerically given X), in roots cannot be found clearly. To clarify the existence and number of roots of equation (12) for concretes – of the values of parameter A we rewrite it as follows:

$$\lambda = \frac{e^{a\sqrt{-p}}}{(-p)^{k-1}}. \tag{13}$$

Considering this equation as a function $\lambda = \lambda(p)$ whose domain is $Re(-p) > 0$, that, is as a conformal map, we find in what is displayed (on the complex plane λ) the domain of the variable p . By requirement $Re(-p) > 0$, $-\pi/2 < arg(-p) < \pi/2$,means $-\pi/4 < arg\sqrt{(-p)} < \pi/4$, if $z = \sqrt{(-p)} = x+\iota y$, means the boundary of the domain of definition the variable $-p$ is the lines $y = \pm x$. According to the law of correspondence of boundaries it is enough to find the images this line [7–9].

We have

$$|\lambda| = \frac{|e^{az}|}{|z|^{2(k-1)}} = \frac{e^{ax}}{(x^2+y^2)^{k-1}}, \tag{14}$$

$$\arg\lambda = \arg e^{ax+i(ay+2\pi n)} - 2(k-1)\arg z = ay+2\pi n-2(k-1)\operatorname{arctg}\frac{y}{x}.$$

Considering $y = \pm x$, we have

$$\arg\lambda = ax + 2\pi n - 2(k-1)\frac{\pi}{4} = ax + (2n - \frac{k-1}{2})\pi, n \in Z,$$

$$ax = \arg\lambda - (2n - \frac{k-1}{2})\pi, n \in Z. \quad (15)$$

Thus, from equation (14) (talking into account (15) and the fact that $y = \pm x$) we obtain that the lines defined by the equation

$$|\lambda| = \frac{(a/\sqrt{2})^{2k-2}}{|\arg\lambda + \left(2n + \frac{k-1}{2}\right)\pi|^{2k-2}} \cdot exp|\arg\lambda + \left(2n + \frac{k-1}{2}\right)\pi|,$$

where $n = 0, 1, 2, ...$ divide the complex λ– plane into disjoint domains $D_m, m = 0, 1, 2, ...$

Comment.

Note that in addition to the areal domain D_0, which has only the outer boundary of $\Gamma_0 = \partial D_0$ each of the domains D_m has a boundary∂D_m consisting of Γ_m an external Γ_{m-1} part:

$$\partial D_m = \Gamma_{m-1} \bigcup \Gamma_m, where, \Gamma_{m-1} \bigcap \Gamma_m = (-1)^m exp\{m\pi\},$$

those the external Γ_m and internal Γ_{m-1} parts of the boundary ∂D_m of the domain D_m have one common point, lying on the real axis of the complex plane of parameter λ. D_0 is the area into which that part of the plane of the complex variable p I displayed, for which $-\pi/4 < argp < 7\pi/4$,, those are the exterior of the angle lying between the lines $y = -x$ and $y = x$ This just means that if $\lambda \in D_0$, then equation (12) does not have the roots we need, i.e. such for which $Re(-p) > 0$.

Obviously, for $k = 1$ we get our previously established picture of the partition of the complex plane λ [10].

Difference is that for each domain D_m, that is, when $\lambda \in D_m$ equation (12) will have exactly $2m$ roots, (this is easy to trace, for example, for real values λ, p) [11–13].

4 Solution of integral equations

With $\forall\lambda \in D_m,\ m = 1, 2, ...$, homogeneous equation (9) has a General solution of the species.

$$\upsilon(t) = \Sigma_{k=1}^{2m} c_k \cdot e^{p_k t}, \quad (16)$$

where c_k - is an arbitrary constants, p_k- is the corresponding roots of equation (12).

We find a proprietary solution of the in homogeneous equation (9). Applying Laplace's transformation to it we get

$$\widehat{\upsilon}(p) \cdot \left[1 - \overline{\lambda}(-p)^{k-1} e^{-a\sqrt{-p}}\right] = \widehat{g_1}(p), \text{ with } Rep \leq 0, \quad (17)$$

where $\widehat{\upsilon}(p), \widehat{g_1}(p)$ - is the Laplace transformation, corresponding to the $\upsilon(t)$ and $g_1(t)$ functions; Since the function

$$\widehat{A}(p, \overline{\lambda}) \equiv 1 - \overline{\lambda}(-p)^{k-1} e^{-a\sqrt{-p}}.$$

It is determined only at $Rep \leq 0$, then we will continue it analytically on the whole complex plane with a cut along the positive real axis. Suppose that the Laplace transform of functions $g(t)$ is analytic in the band $-\varepsilon < Rep < \varepsilon$. Then from equality (17) at $\forall\lambda\Gamma_m$,
$(m = 0, 1, 2, ...,)$ we get

$$\widehat{\upsilon}(p) = \widehat{g_1}(p) + \overline{\lambda}\frac{(-p)^{k-1} exp(-a\sqrt{-p})}{1 - \overline{\lambda}(-p)^{k-1} exp(-a\sqrt{-p})} \cdot \widehat{g_1}(p).$$

Passing in this balance to the originals we get

$$\upsilon(t) = g_1(t) + \overline{\lambda}\int_t^{\infty} r_\lambda - (t-\tau)g_1(\tau)d\tau, \quad (18)$$

where

$$r_\lambda - (y) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \frac{(-p)^{k-1} exp(-a\sqrt{-p})}{1 - \overline{\lambda}(-p)^{k-1} exp(-a\sqrt{-p})} exp(yp)dp. \quad (19)$$

If the roots of the equation

$$1 - \overline{\lambda}(-p)^{k-1} exp(-a\sqrt{-p}) = 0.$$

lie on the imaginary axis integration will perform along the contour, bypassing these points on the left. At the sometime integral should be understood in the sense of main value for Cauchy. Thus, the General solution of equation (9) at $\lambda \in D_m$ has the form

$$\upsilon(t) = g_1(t) + \overline{\lambda} \int_t^{\infty} r_\lambda - (t-\tau) g_1(\tau) d\tau + \Sigma_{k=1}^{2m} c_k \cdot e^{p_k t}. \tag{20}$$

We formulate the results in the form of the following lemmas.

Lemma 1. The $\lambda \in D_0$ values are regular numbers of the $\mathbf{K}_{\lambda^*}$ (9) operator.

Lemma 2. The set $C \setminus D_0$ consists of the characteristic numbers of the $\mathbf{K}_{\lambda^*}$ operator (9). And if $\lambda \in D_m \cup \Gamma_{m-1} \setminus \{(-1)^m e^{m\pi}\}$, $m = 1, 2, ...$, then $Ker(\mathbf{K}_{\lambda^*}) = 2m$ and the corresponding own functions have the form:

$$\upsilon_{\lambda k}(t) = exp(p_k t), \quad k = 1, ..., 2m.$$

where p_k- is the corresponding roots of equation (12).

Now consider the integral equation (6), which is usually called the *recovery equation* [14, 15]. *This name is explained by the fact that such equations arise in the theory of restoration - a section of probability theory, which describes a wide range of phenomena related to with the Failure and recovery of elements of the any system. The recovery equation is also of great importance in the study of problems of both applied and theoretical nature in reliability, queueing theory, in the theory of stocks, in the theory of branching processes and so on.* [16–18].

Applying the Laplace transform to equation (6)and using the convolution theorem we obtain

$$\widehat{\mu}(p) = \widehat{f}(p) + \frac{\lambda}{p}^{k-1} e^{-a\sqrt{p}} 1 - \lambda p^{k-1} e^{-a\sqrt{p}} \cdot \widehat{f}_1(p), \quad p = s + i\sigma, \quad Rep = s > 0,$$

Using the inverse Laplace transform we have

$$\mu(p) = f_1(t) + \lambda \int_0^t r_\lambda + (t-\tau) f_1(\tau) dt, \tag{21}$$

here the resolvent $r_{\lambda+}(\theta)$ is defined by the formula

$$r_\lambda + (\theta) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\lambda p^{k-1} e^{-a\sqrt{p}}}{1 - \lambda p^{k-1} e^{-a\sqrt{p}}} \cdot e^{p\theta} dp, \quad p = s + i\sigma. \tag{22}$$

Where the integration path is parallel to the imaginary axis of the complex plane, to the Right of all singular points of the integrand, that is, to the right of all zeros of the function

$$\widehat{A}(p, \lambda) = 1 - \lambda \cdot p^{k-1} e^{-a\sqrt{p}}.$$

In order for the $\mu(t)$function defined by equality (21) to be substantially limited, it is necessary and sufficient that the conditions are satisfied

$$\int_0^{\infty} f_1(t) \cdot exp(-p_k t) dt = 0, \quad 1 \le k \le 2m, \tag{23}$$

where p_k the roots of the function $\widehat{A}(p, \lambda)$ for which $Rep_k > 0$ and they coincide with the roots of equation (12) with the opposite sign. Thus, the right side of equation (6) must be orthogonal to the eigenfunctions (16) of the conjugate integral equation (9). Thus, the fair

Lemma 3. If $\lambda \in D_0$, then the inhomogeneous equation (6) is definitely torn; if $\lambda \in C \setminus D_0, \lambda \in D_m$, then for the unambiguous solvability of equation (6), it is necessary and sufficient fulfillment of $m-$ the conditions of solvability (23). The conditions (23) mean that the free member of the integral equation (6) must be orthogonal to the solutions of the homogeneous conjugate integral (9).

The validity of these statements, as well as conditions (23), can be shown as follows.

The image of the solution of the integral equation (6) is determined by the equality

$$\widehat{\mu}(p) = \frac{\widehat{f}_1(p)}{1 - \lambda p^{k-1} e^{-a\sqrt{p}}}. \tag{24}$$

The following options are possible.

1. The $\widehat{A}(p,\lambda) = 1 - \lambda \cdot p^{k-1} e^{-a\sqrt{p}}$. function doesn't have zeros in the right half-plane (this means that the $\lambda \in D_0$). In this case, the equation for any right part of $f(t)$ has the only solution, which is expressed through the $r_{\lambda+}(\theta)$ resolvent defined by the formula (22)

$$\mu(t) = f(t) + \lambda \int_0^t r_\lambda + (t-\tau) f_1(\tau) d\tau, \quad t \in \Re_+. \tag{25}$$

2. Function $\widehat{A}(p,\lambda) = 1 - \lambda \cdot p^{k-1} e^{-a\sqrt{p}}$. has a $p_k, (k = 1, 2, ...2m)$ zeroes in the right half plane (it means $\lambda \in D_m,\ m = 1, 2, ...,$). The $\widehat{f}_1(t)$ function must then be zero at these points p_k. In this Case, the function (24) again will not have pluses in the $Re\ p > 0$ area, so the equation (6) also has the only solution of the species (25). Condition $\widehat{f}_1(t) = 0$, on the conversion of the $\widehat{f}_1(t)$ function to zero at points $p = p_k$ just is exactly the some as the condition:

$$\int_0^\infty f(t) \cdot e^{-p_k t} dt = 0, \quad k = 1, 2, ..., 2m. \tag{26}$$

So we proved the following statement.

Lemma 4. On the complex plane C there are no characteristic numbers of the operator $\mathbf{K}_{\lambda^*}$ (6).

5 Main result

Directly from lemmas and integral representations (4)–(7) follows

Theorem. Boundary value problems (1)–(2) are Noetherian if $\lambda \in \{C \setminus D_0\}$ in Addition, if $\lambda \in D_m$, then $dimKer(L_{\lambda^*} = 2mm)$ and $dimKer(L_\lambda) = -2m$.

References

1 Krasnov, M.L. (1975). *Integral Equations.* Moscow: Nauka.

2 Titchmarsh, E. (1986). *Introduction to the theory of Fourier integrals.* (3d ed.) Chelsea Pub Co.

3 Koks, D.V., & Smith, E. (1967). *Recovery theory.* Moscow.

4 Akhmanova, D.M., Dzhenaliev, M.T., & Ramazanov, M.I. (2011). On a particular second kind Volterra integral equation with a spectral parameter. *Siberian Mathematical Journal, 52, 1,* 1–10. DOI: 10.1134/S0037446606010010

5 Amangaliyeva, M.M., Akhmanova, D.M., Dzhenaliev, M.T., & Ramazanov, M.I. (2011). Boundary value problems for a spectrally loaded heat operator with load line approaching the time axis at zero or infinity. *Differential Equations, 47, 2,* 231–243. DOI: 10.1134/S0012266111020091

6 Kosmakova, M.T. (2016). On an integral equation of the Dirichlet problem for the heat equation in the degenerating domain. *Bulletin of the Karaganda University. Mathematic sriies, 1 (81),* 62–67.

7 Amangaliyeva, M.M., Dzhenaliev, M.T., Kosmakova, M.T., & Ramazanov, M.I. (2015). On one homogeneous problem for the heat equation in an infinite angular domain. *Siberian Mathematical Journal, Vol. 56, No. 6,* 982–995. DOI: 10.1134/S0037446615060038

8 Kosmakova, M.T., Akhmanova, D.M., Iskakov, S.A., Tuleutaeva, Zh.M., & Kasymova, L.Zh. (2019). Solving one pseudo-Volterra integral equation. Bulletin of the Karaganda University. Mathematics seriies, 1 (93), 72–77. DOI 10.31489/2019M1/72-77.

9 Dzhenaliev, M.T., & Ramazanov, M.I. (2006). On the boundary value problem for the spectrally loaded heat conduction operator. *Siberian Mathematical Journal, 47, 3,* 433–451. DOI: 10.1007/s11202-006-0056-z

10 Dzhenaliev, M.T., & Ramazanov, M.I. (2010). *Loaded equations as perturbations of differential equations.* Almaty: Nauka.

11 Dzhenaliev, M.T., & Ramazanov, M.I. (2007). On a boundary value problem for a spectrally loaded heat operator: I. *Differential Equations, 43, 4,* 513–524. DOI: 10.1134/S0012266107040106

12 Dzhenaliev, M.T., & Ramazanov, M.I. (2007). On a boundary value problem for a spectrally loaded heat operator: II. *Differential Equations, 43, 6,* 806–812. DOI: 10.1134/S0012266107060079

13 Ramazanov, M.I., Kosmakova, M.T., Romanovsky, V.G., Zhanbusinova, B.H., & Tuleutaeva, Z.M. (2018). Boundary value problems for essentially-loaded parabolic equation. *Bulletin of the Karaganda University. Mathematics seriies, 4 (92),* 79–86. DOI: 10.31489/2018M4/79-86

14 Kosmakova, M.T., Orumbayeva, N,T., Medeubaev, N.K., & Tuleutaeva, Zh.M. (2018). Problems of Heat Conduction with Different Boundary Conditions in Noncylindrical Domains. AIP Conference Proceedings. 1997. UNSP 020071-1. DOI: 10.1063/1.5049065

15 Akhmanova D.M., Ramazanov M.I., & Yergaliyev M.G. (2018). On an integral equation of the problem of heat conduction with domain boundary moving by law of t = x(2). *Bulletin of the Karaganda University-Mathematics, 1 (89),* 15–19. DOI: 10.31489/2018M1/15-19

16 Kosmakova, M.T., Ramazanov, M.I., Tokesheva, A.S., & Khairkulova, A.A. (2016). On the non-uniqueness of solution to the homogeneous boundary value problem for the heat conduction equation in an angular domain. *Bulletin of the Karaganda University-Mathematics, 4 (84),* 80–87. DOI: 10.31489/2016M4/80-87

17 Jenaliyev, M.T., Iskakov, S.A., & Ramazanov, M.I. (2017). On a parabolic problem in an infinite corner domain. *Bulletin of the Karaganda University-Mathematics, 1 (85),* 28–35. DOI: 10.31489/2017M1/28-35

18 Jenaliyev, M.T., Iskakov, S.A., Ramazanov, M.I., & Tuleutaeva, Z.M. (2018). On the solvability of the first boundary value problem for the loaded equation of heat conduction. *Bulletin of the Karaganda University. Mathematics series, 1 (89),* 33-41.

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Жылуөткізгіштіктің әлді жүктелген теңдеуі туралы

Нөлге немесе шексіздікте жүктеме сызығының уақыт осіне жуықталумен берілген спектралды — жүктелген параболалық операторлар үшін шекаралық есептерді зерттеуге арналған. $2k$ ретті қатты жүктелген параболалық теңдеу үшін жүктелген қосындылардың тәртібі теңдеудің дифференциалды бөлігінің тәртібінен асып кеткен жағдайда ұштасқан шекаралық есептер зерттелді. Мақалада шексіз аймақтардағы спектрлік жүктелген параболалық теңдеулер үшін шеттік есептерді зерттеу жалғастырылған. Жылуөткізгіштіктің спектралды-жүктелген теңдеуі үшін шеттік есеп қарастырылды, ол бірінші жағынан екінші ретті туындыны қамтитын жүктемелерге жеткілікті жақын кеңістіктік айнымалы бойынша анықталса, екінші жағынан өз бетінше қызығушылықты білдірді.

Кілт сөздер: жүктелген теңдеу, маңызды шектелген функциялар классы, Лапластың кері түрлендіруі, шегерім.

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О сильно-нагруженных уравнениях теплопроводности

Даны постановки граничных задач для спектрально-нагруженных параболических уравнений в четверти плоскости, когда порядок производной в нагруженном слагаемом равен порядку дифференциальной части уравнения с движущейся пространственной точкой нагрузки по степенному закону. Для сильно нагруженного параболического уравнения порядка 2k исследованы сопряженные граничные задачи в случае, когда порядок нагруженного слагаемого превышает порядок дифференциальной части уравнения. В статье продолжено исследование краевых задач для спектрально-нагруженных параболических уравнений в неограниченных областях. Рассмотрена краевая задача для спектрально-нагруженного уравнения теплопроводности, которая, с одной стороны, достаточно близка к задачам с нагрузкой, содержащей вторую производную по пространственной переменной, и представляющая самостоятельный интерес — с другой.

Ключевые слова: нагруженное уравнение, класс существенно ограниченных функций, обратное преобразование Лапласа, вычет.