G. Shaikhova[1], G. Shaikhova[2]

[1]*Karaganda State Technical University, Kazakhstan;*
[2]*L.N. Gumilyov Eurasian National University, Astana, Kazakhstan*
(E-mail: g.shaikhova@gmail.com)

Traveling wave solutions for the two-dimensional Zakharov-Kuznetsov-Burgers equation

In this paper, the two-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation is investigated. The basic set of fluid equations is reduced to ZKB equation. This equation is a two-dimensional analog of the well-known Korteweg-de Vries-Burgers equation, and also is typical example of so-called dispersive equations which attract the considerable attention of both pure and applied mathematicians in the past decades. We obtain traveling wave solutions for two-dimensional Zakharov-Kuznetsov-Burgers equation by modified Kudryashov method which is a powerful method for obtaining exact solutions of integrable and non-integrable nonlinear evolution equations. Graphical representation of obtained solutions is demonstrated.

Keywords: modified Kudryashov method, Zakharov-Kuznetsov-Burgers equation, kink, nonlinear equation, traveling wave.

Introduction

The two-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation [1] in given by:

$$u_t + u_{xxx} + u_{xyy} + uu_x + \delta(u_{xx} + u_{yy}) = 0, \tag{1}$$

where $\delta = const > 0$. The equation (1) is referred as Zakharov-Kuznetsov-Burgers equation because in case of $\delta = 0$ it will be Zakharov-Kuznetsov equation. And also this equation is a two-dimensional analog of the well-known Korteweg-de Vries-Burgers (KdV) equation which includes dissipation and dispersion and has been studied by various researchers due to its applications in mechanics and physics [2]. The two-dimensional ZKB equation on a strip was studied by [3]. The author had proved the existence and uniqueness results for regular and weak solutions. In work [4] Lie symmetry analysis, nonlinear self-adjointness and conservation laws to the extended two-dimensional ZKB equation were studied. Application of the ZKB equation in dusty plasma was studied in [5].

The aim of the paper is to obtain new traveling wave solutions of the two-dimensional ZKB equation by using the modified Kudryashov method. This method is a powerful method for obtaining exact solutions of nonlinear evolution equations [6–8]. The modified Kudryashov method was applied to the generalized Kuramoto-Sivashinsky equation [9], the Kudryashov-Sinelshchikov equation [10], the generalized Fisher equation [11].

The paper is organized as follows. In Section 2, the key idea of the method is described. In Section 3, the proposed method is applied to the two-dimensional ZKB equation. We conclude this paper in Section 4.

The modified Kudryashov method

Let us present the algorithm of the modified Kudryashov method for finding exact solutions of nonliner partial differential equation (NPDE). We consider the NPDE in the following form:

$$E_1(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, ...) = 0, \tag{2}$$

where E_1 is a polynomial of u(x, y, t) and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Using the traveling transformation

$$u(x, y, t) = u(\xi), \quad \xi = kx + ry + \omega t, \tag{3}$$

where k, r, ω are constants, the NPDE (2) is reduced to nonlinear ordinary differential equation (ODE)

$$E_2(u, \omega u', ku', ru', \omega^2 u'', k^2 u'', r^2 u'', ...) = 0, \tag{4}$$

where prime denotes the derivation with respect to ξ. We look for exact solutions of (4) in the following form:

$$u(\xi) = \sum_{n=0}^{N} a_n Q(\xi)^n, \tag{5}$$

where $a_0, a_1, a_2, a_3, ..., a_N$ are unknown constants and $Q(\xi)$ is the following function:

$$Q(\xi) = \frac{1}{1 + e^\xi}. \tag{6}$$

The function satisfies the first-order ordinary differential equation

$$Q_\xi = Q^2 - Q. \tag{7}$$

Equation (7) is necessary to calculate the derivatives of function $u(\xi)$. We can calculate the necessary number of derivatives of function u. For instance, we consider the general case when N is arbitrary. Differentiating (5) with respect to ξ and taking into account (7) we have

$$u' = \sum_{n=0}^{N} a_n n Q^n (Q - 1); \tag{8a}$$

$$u'' = \sum_{n=0}^{N} a_n n Q^n (Q - 1)[(n + 1)Q - n]; \tag{8b}$$

$$u''' = \sum_{n=0}^{N} a_n n Q^n (Q - 1)[(n^2 + 3n + 2)Q^2 - (2n^2 + 3n + 1)Q + n^2]. \tag{8c}$$

Next, substitute equations (8) in (4). Then we collect all terms with the same powers of function $Q(\xi)$ and equate the resulting expressions to zero. Finally, we obtain algebraic system of equations. Solving this system, we get values for the unknown parameters $a_0, a_1, a_2, a_3, ..., a_N$.

Traveling wave solutions for the Zakharov-Kuznetsov-Burgers equation

In this section, we will find the traveling wave solutions for the two-dimensional ZKB equation through the modified Kudryashov method. Let us consider two-dimensional ZKB equation (1). Using the traveling wave transformation of the form (3), equation (1) is converted into the following ordinary differential eqution

$$\omega u' + (k^3 + kr^2)u''' + kuu' + \delta(k^2 + r^2)u'' = 0. \tag{9}$$

Equation (9) is integrable, therefore, integrating once equation (9) with respect to ξ, we obtain

$$\omega u + (k^3 + kr^2)u'' + \frac{k}{2}u^2 + \delta(k^2 + r^2)u' + C_1 = 0, \tag{10}$$

where C_1 is constant integration. Considering the homogeneous balance between the highest order derivatives u'' and the nonlinear terms u^2 in the equation (10), we can get $N = 2$. Then the equation (5) reduces to

$$u = a_0 + a_1 Q + a_2 Q^2, \tag{11}$$

where a_0, a_1, a_2 are constants to be determined later. Now substituting (11) into (10), and setting coefficients of the same power of Q^n equal to zero, we obtain these algebraic equations:

$$Q^4 : 6k(r^2 + k^2)a_2 + \frac{1}{2}ka_2^2 = 0; \tag{12a}$$

$$Q^3 : (2\delta - 10k)(k^2 + r^2)a_2 + 2k(k^2 + r^2)a_1 + ka_1 a_2 = 0; \tag{12b}$$

$$Q^2 : (k^2 - r^2)(\delta - 3k)a_1 + ((k^2 + r^2)(4k - 2\delta) + \omega)a_2 + \frac{1}{2}ka_1^2 + ka_0 a_2 = 0; \tag{12c}$$

$$Q^1 : ((k^2 + r^2)(k - \delta) + \omega)a_1 + ka_0 a_1 = 0; \tag{12d}$$

$$Q^0 : C_1 + \omega a_0 + \frac{1}{2}ka_0^2 = 0. \tag{12e}$$

Solving system of equations (12) with the aid of Maple, we obtain the following results:

Case A:

$$a_0 = \frac{1}{25\delta}(6\delta^3 + 150\delta r^2 - 125\omega), \quad a_1 = 0, a_2 = -\frac{12}{25}\delta^2 - 12r^2; \tag{13}$$

$$k = \frac{\delta}{5}, C_1 = -\frac{1}{6250}[(6\delta^3 + 150\delta r^2)^2 - (125\omega)^2]. \tag{14}$$

Substituting (13)–(14) into (11) with (6), respectively, we obtain new traveling wave solution for ZKB equation (1) as follows:

$$u_1(x,y,t) = \frac{1}{25\delta}(6\delta^3 + 150\delta r^2 - 125\omega) - \left(\frac{12}{25}\delta^2 + 12r^2\right)\left(\frac{1}{1+e^\xi}\right)^2, \tag{15}$$

where $\xi = kx + ry + \omega t$. The graphical representation of obtained solution (15) is depicted on Figure 1.
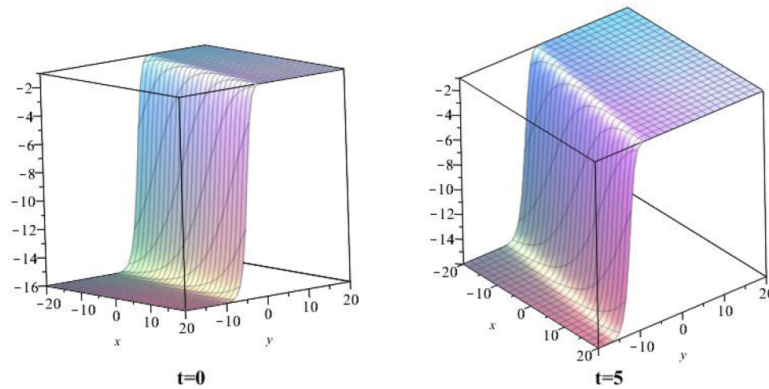

Figure 1. Dynamics the solution of $u_1(x,y,t)$ for ZKB equation when $\delta = 1, r = 1.1, \omega = 1.7$

Case B:

$$a_0 = -\frac{1}{25\delta}(6\delta^3 + 150\delta r^2 - 125\omega), \quad a_1 = \frac{24}{25}\delta^2 + 24r^2, a_2 = -\frac{12}{25}\delta^2 - 12r^2; \tag{16}$$

$$k = -\frac{\delta}{5}, C_1 = \frac{1}{6250}[(6\delta^2 + 150\delta r^2)^2 - (125\omega)^2]. \tag{17}$$

Substituting (16)–(17) into (11) with (6), respectively, we obtain new traveling wave solution for ZKB equation (1) as follows:

$$u_2(x,y,t) = -\frac{1}{25\delta}(6\delta^3 + 150\delta r^2 - 125\omega) + \left(\frac{24}{25}\delta^2 + 24r^2\right)\left(\frac{1}{1+e^\xi}\right) - \left(\frac{12}{25}\delta^2 + 12r^2\right)\left(\frac{1}{1+e^\xi}\right)^2, \tag{18}$$

where $\xi = kx + ry + \omega t$. We demonstrate solution (18) on Figure 2.
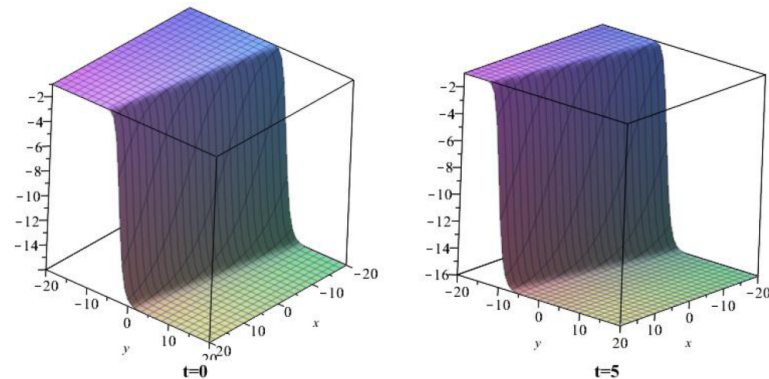

Figure 2. Dynamics the solution of $u_2(x,y,t)$ for ZKB equation when $\delta = 1, r = 1.1, \omega = 1.7$

Conclusion

In this work, we have demonstrated the efficiency of the modified Kudryashov method for finding exact solutions of the two-dimensional Zakharov-Kuznetsov-Burgers equation. We have obtained new traveling wave solution. Graphical representation of these exact solutions is presented. This method can be more successfully applied to study nonlinear evolution equations, which frequently arise in nonlinear sciences.

References

1 Faminskii, A.V. (2014). An initial-boundary value problem in a strip for two-dimensional Zakharov-Kuznetsov-Burgers equation. *Retrieved from arXiv:1408.5847v1 [math.AP]*.

2 Amick, C.I., Bona, J.L., & Schonbek, M.F. (1989). Decay of solutions of some nonlinear wave equations. *Journal of Differential Equations*, 81, 1–49.

3 Larkin, N.A. (2016). The 2D Zakharov-Kuznetsov-Burgers equation on a strip. *Boletim da Sociedade Paranaense de Matematica*, 34(1), 151–172.

4 Wang, G., & Fakhar, K. (2015). Lie symmetry analysis, nonlinear self-adjointness and conservation laws to an extended (2+1)-dimensional Zakharov–Kuznetsov–Burgers equation. *Computers and Fluids*, 119, 143–148.

5 Seadawy, A.R. (2015). Nonlinear wave solutions of the three-dimensional Zakharov–Kuznetsov–Burgers equation in dusty plasma. *Physica A*. 439, 124–131.

6 Kudryashov, N.A. (2004). *Analytical theory of nonlinear differential equations.* Moscow; Izhevsk: Institute of Computer Investigations.

7 Ryabov, P.N. Sinelshchikov, D.I., & Kochanov, M.B. (2011). Application of the Kudryashov method for finding exact solutions of the high order nonlinear evolution equations. *Applied Mathematics and Computation, 218,* 3965–3972.

8 Kabir, M., Khajeh, A., Abdi Aghdam, E., & A. Yousefi Koma (2011). Modified Kudryashov method for finding exact solitary wave solutions of higher order nonlinear equations. *Mathematical methods in the Applied Sciences, 34,* 213–219.

9 Kudryashov, N.A. (1990). Exact solutions of the generalized Kuramoto-Sivashinsky equation. *Physics Letters A, 147,* 287–291.

10 Ryabov, P.N. (2010). Exact solutions of the Kudryashov-Sinelshchikov equation. *Applied Mathematics and Computation, 217,* 3585-3590.

11 Kudryashov, N.A., & Zakharchenko, A.S. (2014). A note on solutions of the generalized Fisher equation. *Applied Mathematics Letters, 32,* 53–56.

Г. Шаихова, Г. Шаихова

Екіөлшемді Захаров-Кузнецов-Бюргерс теңдеуінің қозғалатын толқындық шешімдері

Мақалада екіөлшемді Захаров-Кузнецов-Бюргерстің (ЗКБ) теңдеуі зерттелді. Сұйықтық теңдеулерінің базалық жинағы осыған келеді. ЗКБ теңдеуі белгілі екіөлшемді Кортевег-де Фриз-Бюргерс теңдеуіне ұқсас болып табылады, сонымен қатар соңғы жылдықта іргелі және қолданбалы математиктердің елеулі назарын аударып отырған дисперсионды теңдеулердің әдеттегі мысалы. Кудряшовтың модификациялық әдісі арқылы екіөлшемді Захаров-Кузнецов-Бюргерстің теңдеуі үшін қозғалатын толқындық шешімдер алынды. Бұл әдіс интегралданатын және интегралдалданбайтын сызықты емес эволюциялық теңдеулердің нақты шешімдерін алу үшін үздік әдіс болып табылады. Алынған шешімдердің суреттері ұсынылған.

Кілт сөздер: модификацияланған Кудряшов әдісі, Захаров-Кузнецов-Бюргерс теңдеуі, кинк, сызықты емес теңдеу, қозғалатын толқын.

Г. Шаихова, Г. Шаихова

Перемещающиеся волновые решения двумерного уравнения Захарова-Кузнецова-Бюргерса

В статье исследовано двумерное уравнение Захарова-Кузнецова-Бюргерса (ЗКБ). Базовый набор уравнений жидкости сводится к уравнению ЗКБ. Это уравнение является двумерным аналогом известного уравнения Кортевега-де Фриза-Бюргерса, а также типичным примером так называемых дисперсионных уравнений, которые привлекают значительное внимание как фундаментальных, так и прикладных математиков в последние десятилетия. Получены перемещающиеся волновые решения для двумерного уравнения Захарова-Кузнецова-Бюргерса с помощью модифицированного метода Кудряшова, который является мощным методом для получения точных решений интегрируемых и неинтегрируемых нелинейных эволюционных уравнений. Показано графическое представление полученных решений.

Ключевые слова: модифицированный метод Кудряшова, уравнения Захарова-Кузнецова-Бюргерса, кинк, нелинейное уравнение, перемещающаяся волна.