UDC 510.67

O.I. Ulbrikht

Ye.A. Buketov Karaganda State University, Kazakhstan
(E-mail: ulbrikht@mail.ru)

Forking and independence for fragments of Jonsson sets

The concept of independence plays a very important role in Model Theory for classification of a fixed complete theory. In this paper, we study the Jonsson theories, which, generally speaking, are not complete. For such theories, the concept of forking is introduced axiomatically in the framework of the study of the Jonsson subsets of the semantic model of this theory. Equivalence of forking by Shelah, by Laskar-Poizat and an axiomatically given forking for existential types over subsets of the semantic model of the Jonsson theory is given. Further, as and for complete theories, independence is defined through the notion of non-forking.

Keywords: Jonsson theory, semantic model, existential type, Jonsson set, a fragment of the Jonsson set, forking, independence.

One of the most important concepts of modern Model Theory is the concept of forking. With the help of this concept, we can evaluate the dependence of the properties of an element on each other in a first-order language. It should be noted that this concept was introduced by S. Shelah [1] to solve a very important problem of the spectrum of an arbitrary complete theory. Over time, experts in the theory of models, evaluating the depth and significance of the concept of forking, began to seek new approaches for its simpler explanation. One of the well-known sources in this direction is the well-known work of French mathematicians D.Laskar and B.Poizat [2], in which the concept of forking was redefined in the framework of a certain order. Later, other mathematicians observed that it is possible to consider the abstract properties of the independence of the model elements from each other and to associate this with the properties of the first order of the types of these elements for the subject of non-forking. In particular, as an example, we can cite the following monograph by D. Baldwin [3], where he considered a system of axioms that defines an abstract property of independence.

The study of Jonsson theories is inherently back to the tasks of the so-called «eastern» Model Theory, founded by Abraham Robinson, who lived on the eastern coast of the United States, unlike Alfred Tarski, who lived on the west coast of the United States. And accordingly, the tasks that were determined at the time by A.Tarsky's theoretical-model problems became the basis for the so-called «Western» Model Theory. All the main differences between these two trunk directions of Model Theory of that time can be found in the well-known book by J. Barwise [4].

The Jonsson theories, generally speaking, are not complete and the morphisms that serve them, as a rule, are isomorphic embeddings and homomorphisms. At the same time, the semantic aspect of these theories, in view of certain theoretical-model circumstances, reflects the class of existential-closed models of the Jonsson theory under consideration. In [5], homomorphisms in positive Model Theory were defined. In [6], a variant of the study of the Jonsson theories was proposed in the framework of the positive Model Theory. In an earlier work, A.R. Yeshkeyev [7] considered positive analogs of Jonsson theories and their particular cases - the Robinson theories.

Let's give the basic definitions necessary to understand the content of this article.

Definition 1 [4]. The theory T is called Jonsson if:

1) T has an infinite model;

2) T is inductive, i.e. T is equivalent to the set $\forall\exists$-propositions;

3) T has the joint embedding property (JEP), that is, any two models $\mathfrak{A} \models T$ and $\mathfrak{B} \models T$ are isomorphically embedded in a certain model $\mathfrak{C} \models T$;

4) T has the property of amalgamation (AP), that is, if for any $\mathfrak{A}, \mathfrak{B}, \mathfrak{C} \models T$ such that $f_1 : \mathfrak{A} \to \mathfrak{B}$, $f_2 : \mathfrak{A} \to \mathfrak{C}$ are isomorphic embeddings, exist $\mathfrak{D} \models T$ and isomorphic embeddings $g_1 : \mathfrak{B} \to \mathfrak{D}$, $g_2 : \mathfrak{C} \to \mathfrak{D}$ such that $g_1 f_1 = g_2 f_2$.

Definition 2 [8]. Let $\kappa \geqslant \omega$. The model $\mathfrak{M}$ of theory T is said to be κ-universal for T if every model T of cardinality is strictly less than κ is isomorphically embedded in $\mathfrak{M}$.

Definition 3 [8]. Let $\kappa \geqslant \omega$. The model $\mathfrak{M}$ of theory T is said to be κ-homogeneous for T if for any two models $\mathfrak{A}$ and $\mathfrak{A}_1$ of T, which are submodels of $\mathfrak{M}$, the cardinality is strictly less than κ, and the isomorphism

$f : \mathfrak{A} \to \mathfrak{A}_1$, for each extension $\mathfrak{B}$ of the model $\mathfrak{A}$, which is a submodel of $\mathfrak{M}$ and a model T of cardinality strictly less than κ, there exists an extension $\mathfrak{B}_1$ of the model $\mathfrak{A}_1$, which is a submodel of $\mathfrak{M}$, and an isomorphism $g : \mathfrak{B} \to \mathfrak{B}_1$ that extends f.

A homogeneous-universal model for T is a κ-homogeneous-universal model for T of cardinality κ, where $\kappa \geqslant \omega$.

Definition 4 [8]. The semantic model C of Jonsson theory T is the ω^+-homogeneous-universal model of theory T.

Definition 5 [8]. The Jonsson theory T is said to be perfect if its semantic model C is saturated.

The central concept of this paper is the notion of a fragment of the Jonsson set that was defined in [9] and some of its model-theoretic properties were considered in [10–12]. In this paper we carry over the main results from [13, 14], and, as can be seen from the following definition, the concept of the Jonsson set is very well coordinated with the concept of a basis of a linear space. We note that linear spaces are a particular case of modules, and the theory of modules is a Jonsson theory.

In definition 6 we changed the point a), in contrast to the definition of the Jonsson set in [15]. In the original definition there was a requirement of the existential definability of this set, now we require simply definability.

Definition 6 [8]. The set X is called the Jonsson set in the theory T if it satisfies the following properties:

a) X is a definable subset of C, where C is the semantic model of the theory T;

b) $dcl(X)$ is the carrier of some existentially closed submodel of C, where $dcl(X)$ is the definable closure of the set X.

Definition 7 [5]. We say that all $\forall\exists$-consequences of an arbitrary theory create a Jonsson fragment of this theory if the deductive closure of these $\forall\exists$-consequences is a Jonsson theory.

Consider the countable language L, complete for existential sentences the perfect Jonsson theory T in the language L and its semantic model C. Let X be the Jonsson set in T and M an existentially closed submodel of the semantic model C, where $dcl(X) = M$. Then let $Th_{\forall\exists}(M) = Fr(X)$, where $Fr(X)$ is the Jonsson fragment of the Jonsson set X.

Since the concept of forking is central to stability theory, it is natural to want to study it from different points of view. For this purpose, we first describe forking axiomatically. We recall the definition of forking.

Definition 8. a) It is said that formula $\varphi(\bar{x}, \bar{b})$ divide over A, if there exists a sequence $\langle \bar{b}_n : n < \omega \rangle$ and a number $k < \omega$, satisfying the following conditions: 1) $\bar{b}_n \equiv_A \bar{b}$, $n < \omega$; 2) $\{\varphi(\bar{x}, \bar{b}_n) : n < \omega\}$ k-inconsistent.

b) It is said that the type p (not necessarily complete) forks over A, if there exists a finite set Σ of formulas that are divisible over A such that $p \vdash \vee\{\varphi : \varphi \in \Sigma\}$;

Let T be Jonsson theory, $S^J(X)$ be the set of all existential complete n-types over X, that are compatible with T, for each finite n.

Definition 9. We say that the Jonsson theory T is J-λ-stable if for any T-existentially closed model A, for any subset X of the set A, $|X| \leqslant \lambda \Rightarrow |S^J(X)| \leqslant \lambda$. We will call the Jonsson theory J-stable if it is J-λ-stable for some λ.

Let $\mathcal{A}$ be the class of all subsets of the semantic model $\mathfrak{M}$, $\mathcal{P}$ be the class of all existential complete types, $JNF \subseteq \mathcal{P} \times \mathcal{A}$ is some binary relation. We write in the form of axioms some conditions imposed on JNF.

Axiom 1. If $(p, A) \in JNF$, $f : A \to B$ are isomorphic embeddings, then $(f(p), f(A)) \in JNF$.

Axiom 2. If $(p, A) \in JNF$, $q \subseteq p$, then $(q, A) \in JNF$.

Axiom 3. If $A \subseteq B \subseteq C$, $p \in S^J(C)$, then $(p, A) \in JNF \Leftrightarrow (p, B) \in JNF \ \& \ (p \restriction B, A) \in JNF$.

Axiom 4. If $A \subseteq B$, $dom(p) \subseteq B$, $(p, A) \in JNF$, then $\exists q \in S^J(B)$ $(p \subseteq q \ \& \ (q, A) \in JNF)$.

Axiom 5. There exists a cardinal μ such that if $A \subseteq B \subseteq C$, $p \in S^J(B)$, $(p, A) \in JNF$, then $\left|\{q \in S^J(C) : p \subseteq q \ \& \ (q, A) \in JNF\}\right| < \mu$.

Axiom 6. There exists a cardinal $\varkappa$ such that $\forall p \in \mathcal{P}$, $A \in \mathcal{A}$ if $(p, A) \in JNF$, then $\exists A_1 \subseteq A$, $(|A_1| < \varkappa \ \& \ (p, A_1) \in JNF)$.

Axiom 7. If $p \in S^J(A)$, then $(p, A) \in JNF$.

Let F be the fragment of some Jonsson set D, where D is a subset of the semantic model $\mathfrak{M}$ of some Jonsson theory T, i.e. $F = Th_{\forall\exists}(dcl(D))$, $dcl(D) = M' \in E_T$.

Theorem 1. The following conditions are equivalent:

1. In the theory F, the relation JNF satisfies axioms 1–7.

2. T^* is stable for any $p \in \mathcal{P}$, $A \in \mathcal{A}$ $((p, A) \in JNF \Leftrightarrow p$ does not fork over $A)$, where $T^* = Th(M')$.

Proof. It follows from Theorem 10 [14].

Consider the strengthening of Lemma 19.7 from [13]. For this we give the following known definitions.

Definition 10. If $A \subseteq M \cap N$, $p \in S(M)$, $q \in S(N)$, then $p \geqslant_A q$ means that $\forall \varphi(\bar{x}, \bar{y}) \in L(A)$ $(\exists \bar{m} \in M\ p \ni \varphi(\bar{x}, \bar{m}) \Rightarrow \exists \bar{n} \in N\ q \ni \varphi(\bar{x}, \bar{n}))$; $p \sim_A q$ means that $p \geqslant_A q$ & $q \geqslant_A p$.

$$[p]^A \rightleftharpoons \{q : \exists N \models T,\ q \in S(N),\ p \sim_A q\}.$$

It is easy to understand that the relation $\geqslant_A$ induces an analogous relation between classes, which is a partial order relation. If $A = \varnothing$, then the index A for $\geqslant_A$ and $\sim_A$ will be omitted. Each equivalence class in $\sim_A$ is uniquely determined by the set of formulas from $L(A)$, representable in each type of this class.

Definition 11. The formula $\varphi(\bar{x}, \bar{y}) \in L(A)$ is said to be representable in $p \in S(M)$, $A \subseteq M$ если $\exists \bar{m} \in M$ $(p \vdash \varphi(\bar{x}, \bar{m}))$. Obviously, in this way, the number of equivalence classes in $\sim_A$ is at most $2^{|L(A)|} = 2^{|L| \cdot |A|}$.

The equivalence classes with respect to $\sim_A$ will be denoted by ξ^A. If $p \in S(A)$, then Ω_p denotes the partially ordered set $\langle \{\xi^A : \exists M \supseteq A,\ p_1 \in S(M),\ p \subseteq p_1,\ p_1 \in \xi^A\}; \geqslant \rangle$.

The following results are known, their proof can be extracted from [13].

Lemma 1. In Ω_p there is a maximal element.

Lemma 2. If T is stable, $p \in S(M)$, $M \subseteq B$, $p' \in S(B)$ is the successor of p, then $p \sim_M p'$.

Lemma 3. If T is stable, $p \in S(M)$, $M \subseteq B$, $q \in S(B)$, $p \subseteq q$ and $p \sim_M q$, then q is the successor of p.

Lemma 4. If $A \subseteq M \cap N$, $p \in S(M)$, $q \in S(N)$, $p \upharpoonright A = q \upharpoonright A$ and p, q do not fork over A, then $p \sim_A q$.

Lemma 5. If T is stable, $p \in S(A)$, then Ω_p has a unique maximal (i.e., greatest) element.

The following definition belongs to A.E.Yeshkeyev.

If T is a J-stable, existentially complete Jonsson theory, $p \in S^J(A)$, then $\beta^J(p)$ is the largest element of Ω_p.

Now we can introduce the following relation *JNFLP* (Jonsson non-forking by Lascar-Poizat) on $\mathcal{P} \times \mathcal{A}$.

Definition 12. Let T be a J-stable, existentially complete Jonsson theory.

1. If $p \in S^J(B)$, $A \subseteq B$, then $(p, A) \in JNFLP \Leftrightarrow \beta^J(p) = \beta^J(p \upharpoonright A)$.

2. If p is an arbitrary existential type, then $(p, A) \in JNFLP \Leftrightarrow$ there exists a $p' \in S^J(A \cup dom(p))$ such that $p \subseteq p'$ and $(p', A) \in JNFLP$.

Theorem 2. In the J-stable existentially complete Johnson theory, the relation *JNFLP* satisfies axioms 1–7.

The axioms 1, 2, 3, 4, 7 are trivially verified. Axiom 6 is satisfied for $\varkappa = |L|^+$. Suppose the contrary. Let $p \in S^J(A)$ and $\forall A_1 \subseteq A$, if $|A_1| < \varkappa$, then $(p, A_1) \notin JNFLP$. Obviously, $|A| \geqslant \varkappa = |L|^+$. There exists a sequence $\langle A_\alpha : \alpha < |L|^+ \rangle$ such that $|A_\alpha| \leqslant |L|$, $A_\alpha \subseteq A_\beta$ for $\alpha < \beta < |L|^+$ and $(p \upharpoonright A_{\alpha+1}, A_\alpha) \notin JNFLP$. Let $M \supseteq \bigcup\limits_\alpha A_\alpha$ be an arbitrary existentially closed submodel of the semantic model of the theory T of cardinality $|T|$, $p_\alpha \supseteq p \upharpoonright A_\alpha$ such that $p_\alpha \in S^J(M)$ and $[p_\alpha]^{A_\alpha}$ is the largest element in $\Omega_{(p \upharpoonright A_\alpha)}$. Then $\langle \{p_\alpha : \alpha < |L|^+\}; \geqslant \rangle$ is strictly decreasing sequence. Hence, there exist the formulas $\varphi_\alpha(\bar{x}, \bar{y}_\alpha) \in L$, $\alpha < |L|^+$ such that $\varphi_\alpha(\bar{x}, \bar{y}_\alpha)$ is representable in p_α, but is not representable in $p_{\alpha+1}$. It is clear that for $\alpha \neq \gamma$ $\varphi_\alpha(\bar{x}, \bar{y}_\alpha) \neq \varphi_\gamma(\bar{x}, \bar{y}_\gamma)$ since there is no power set $> |L|$ of formulas of the language L. Contradiction.

Axiom 5 is satisfied for $\mu = \left(2^{|T|}\right)^+$. In fact, let $p \in S^J(B)$, $(p, A) \in JNFLP$, $A \subseteq B \subseteq C$. By axiom 6, there exists is $A_0 \subseteq A$ such that $|A_0| \leqslant |L|$, $(p, A_0) \in JNFLP$.

1 case: Let C be an existentially closed submodel of the semantic model $\mathfrak{M}$ of the theory T. $C \models T$. Let $A_0 \subseteq M_0 \preccurlyeq_{E_1} C$. If $p' \in S^J(C)$, $p \subseteq p'$, $(p', B) \in JNFLP$, then $(p', A_0) \in JNFLP$. Therefore, $(p', M_0) \in JNFLP$. Hence p' is the successor of $p' \upharpoonright M_0$. There are no more such types than $|S^J(M_0)| \leqslant 2^{|T|}$.

2 case: $C \not\models T$. Then we take $N \in E_T$ such that $N \supseteq C$. $|\{q \in S^J(C) : p \subseteq q$ & $(q, A) \in JNFLP\}| \leqslant$ $\leqslant |\{q \in S^J(N) : p \subseteq q$ & $(q, A) \in JNFLP\}| \leqslant 2^{|T|}$.

The following theorem is an extension of Theorem 19.8 of [13] and is the main result of this paper.

Theorem 3. If F is J-stable, then the concepts of *JNF* and *JNFLP* are the same.

The proof follows from Theorem 1 and Theorem 2.

Next, we define the concept of independence. Non-forking extensions will in some sense be «free», i.e. independent. So in what follows we will talk about the concept of forking when we are dealing with some type in the Jonsson theory, which satisfies the relation *JNF*. We will follow the following definition.

Definition 13. We say that $\bar{a}$ does not depend on B over A if $tp(\bar{a}/A)$ does not fork over $A \bigcup B$. We denote this fact by $\bar{a} \bot_A B$.

In particular, one can note that the concept of independence for Jonsson sets has many good properties: monotonicity, transitivity, finite basis, symmetry, etc., similarly to complete theories.

Forking, as in Theorem 1, can be used to give the notion of independence in J-ω-stable theories [8].

Summarizing, we note that in [14] was obtained a result, where for the Jonsson theories the binary relation *JNF* was determined and it was proved that the notion *JNF* in the class of J-stable theories coincides with the concept of non-forking in stable theories in the sense of S. Shelah. In this paper we obtain the following result: for a fixed fragment of a certain Jonsson subset of the semantic model of some fixed J-stable existentially complete Jonsson theory, we prove both equivalences of the binary relations *JNF* and *JNFLP*. Moreover, for *JNF* in this class of theories, we have obtained a more detailed version of Theorem 10 [14]. Namely, we get the assertion that the binary relation *JNF* is also equivalent to the condition obtained in [13] with respect to some definable closure of the Jonsson subset of the semantic model of the Jonsson theory under consideration. The results obtained with these binary relations provide an additional opportunity to characterize the behavior of existential types in the framework of the study of the examined fragment of the Jonsson subset of the semantic model of this Jonsson theory.

All concepts that are not defined here can be extracted from [8].

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О.И. Ульбрихт

Йонсондық жиындардың фрагменттері үшін форкинг пен тәуелсіздік

Тәуелсіздік ұғымы бекітілген толық теорияның модельдерді классификациялау теориясында өте маңызды рөлін атқарады. Мақалада йонсондық теориялар қарастырылды, олар, жалпы айтқанда, толық емес болып табылады. Мұндай теориялар үшін форкинг ұғымы берілген теорияның семантикалық моделінің йонсондық ішкі жиындарының зерттеу аясында аксиоматикалық түрде енгізіледі. Шелах, Ласкар-Пуаза форкингі және йонсондық теорияның семантикалық моделінің йонсондық ішкі жиындарының экзистенциалды түрлері үшін аксиоматикалық түрде берілген форкингінің эквиваленттілігі келтірілді. Әрі қарай, толық теориялардағыдай, тәуелсіздік форкинг ете алмайтындылылық ұғымы арқылы анықталады.

Кілт сөздер: йонсондық теория, семантикалық модель, экзистенциалды түр, йонсондық жиын, йонсондық жиынның фрагменті, форкинг, тәуелсіздік.

О.И. Ульбрихт

Форкинг и независимость для фрагментов йонсоновских множеств

Понятие независимости играет очень важную роль в теории классификации моделей фиксированной полной теории. В статье изучены йонсоновские теории, которые, вообще говоря, не полны. Для таких теорий аксиоматически вводится понятие форкинга в рамках изучения йонсоновских подмножеств семантической модели данной теории. Приведена эквивалентность форкинга по Шелаху, Ласкара-Пуаза и аксиоматически заданного форкинга для экзистенциальных типов над подмножествами семантической модели йонсоновской теории. Далее, как и для полных теорий, определяется независимость через понятие нефоркуемости.

Ключевые слова: йонсоновская теория, семантическая модель, экзистенциальный тип, йонсоновское множество, фрагмент йонсоновского множества, форкинг, независимость.

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