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Companions of the fragments in the Jonsson enrichment

In this article we consider the properties of central types for the existentially prime strongly convex Jonsson theories in some extension. This class of theories is a subclass of a broad class of Jonsson theories. In particular, the Jonsson theories include the class of all fields of a fixed characteristic. In the given work, problems related to the classical problems of the general Model Theory concerning the following topics were considered. First of all, we note the values of enrichment. Using the one-place predicate, the Jonsson subset is singled out and the concepts of P -stability and various kinds of similarities are considered for the Jonsson completion. The following results were obtained:Coincidence of P - stability for a prototype of the central type and its center. Equivalence of syntactic similarity of companions of fragments of Jonsson enrichment and syntactic similarity of their centers.The above notion of stability has an applied value for studying the properties of the central types in this enrichment. In the second place, it is necessary to note the significance of the concept of the central type in this enrichment.The very idea of a central type presupposes an additional description of the properties of incomplete Jonsson theories by means of central completion. The Jonsson subsets of the semantic model of the existentially prime convex Jonsson theory have good theoretic-model properties. This concerns the Morley rank and it is preserved in the syntactic and semantic similarity of the above theories.

Keywords: Jonsson theory, Jonsson set, fragment of Jonsson sets, Existentially Prime Strongy Convex Jonsson theories.

One of the classic problem science is the study of the problems of classification of objects for some the general featured. In the math role performing such objects play sets with determined on them relationships. With using mathematical logic, these objects have been associated with some sets formula language calculus of predicate. This relationship between the syntax and semantics of the fixing language itself is the essence of model theory. Therefore, it is clear that finding syntax and semantics similarity signs can be useful in classification of the object model theory. Our research related to the concepts of convexity of the theory in the class of existential simple Jonsson theories. The main results obtained for the central types of fragments Jonsson subsets of semantic models of some fixed Jonsson theory. Next, we enrich the signature of this Jonsson set in a single predicate.We give the necessary definitions associated with new subclasses Jonsson theories and enriched signatures.

Let L be a countable first-order language.

Definition 1. The inductive theory T called existential-prime, if:

1. It has a simple algebraic model and the class of all algebraically simple models it is denoted by AP.

2. The class (E_T) of model theory T has nonempty intersection with an AP class, ie, $T_{AP} \cap E_T \neq \emptyset$.

Definition 2. The theory T is called convex if for any model $\mathfrak{A}$ and any family $\{\mathfrak{B}_i | i \in I\}$ of its substructures, which are models of the theory T, the intersection $\bigcap_{i\in I} \mathfrak{B}_i$ is a model theory T. It is assumed that this intersection is not empty. If this intersection is never empty, then the theory is called the strongly convex.

We give the necessary definitions related to Jonsson theories and enriched signatures.

Definition 3. We say that a set $X - \Sigma$-definable, if it is definable some existential formula.

Definition 4. The set X is said Jonsson in theory T if it satisfies the following properties:

1) X is a Σ-definable subset of C;

2) $dsl(X)$ is the carrier some existentially closed submodel C.

For more information on Jonsson sets can obtain in the works [1–3].

Let T is an arbitrary Jonsson theory in the language of the first order signature σ. Let C is a semantic model of theory T. Let $A \subseteq C$ is a Jonsson set of theory T. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$.

Let $T_A^C = T \cup Th_{\forall\exists}(C, a)_{a\in A} \cup \{P(c_a) | a \in A\} \cup \{P(c)\} \cup \{''P \subseteq''\}$ where $\{''P \subseteq''\}$ is an infinite set of sentences expressing the fact that the interpretation of symbol P is existentially closed submodel in the language of the signatures $\sigma_\Gamma(A)$ and this model is a definable closure of the set A. It is understood that the consideration the set of sentences is Jonsson theory and this theory generally is not complete.

Let T^* is the center of the Jonsson theory T_A^C and $T^* = Th(C')$ where C' is a semantic model of the theory T_A^C. By restriction theory T_A^C to signatures $\sigma_\Gamma(A)\backslash\{c\}$ the theory T_A^C becomes a complete type. This type we call a central type of the theory T relatively the Jonsson set A and denoted by P_A^C.

We say that all $\forall\exists$ - corollary of the arbitrary theory form a Jonsson fragment of this theory, if the deductive closure of these $\forall\exists$ - corollary is Jonsson Theories. Obtained in this case Jonsson theories will be called Jonsson fragment (further fragment). Accordingly, it is determined by the fragment of Jonsson set. In both cases, we can carry out research Jonsson fragments on the connection with an initial theory that the new formulation of the problem research is Jonsson's theory.

Let X Jonsson set in the theory T and M is existentially closed submodel semantic model C, considered Jonsson theory T where $dcl(X) = M$. Then let $Th_{\forall\exists} = Fr(X)$, $Fr(X)$ is Jonsson fragment of Johnson sets X.

On similarity in Jonsson theoties

T.G. Mustafin in his work [4] define a precise notion of syntactic [4; Def. 1] and semantic similarity [4; Def. 4] complete theory That in the language this determination and the respective regulations of concepts (for example, shell of theory [4; Def. 12], semantic property (theory, model, element) [4; Def. 8]), he proved that for an arbitrary complete there is syntax similary for it some theory of polygons [4, Th. 4, Th. 5]. In the class Jonsson theory this approach to classification the respective regulations of the objects correctly but requires certain changes in the definition relevant similarity theory. This is connected, firstly, so that, in generally speaking, Jonsson Theory is not complete, and, secondly, that in the class of models Jonsson theory is uniform and universal models, generally speaking not saturated. This paragraph is connected with differences concepts similarity between Jonsson theories. Through generalizations some definitions in the work [4] and the technique of work with Jonsson theories received, that in the class of ideal $\exists$-complete Jonsson theory the concepts entered similarities Jonsson theories match with relevant completes in total theory of the meaning.

To give the following definition.

Let T is complete theory, then $F(T) = \bigcup_{n<\omega} F_n(T)$, where $F_n(T)$ is Boolean algebra of formula with n free variables.

Definition 5. [4; Def. 1]. Let T_1 and T_2 are complete theory.

We will say, that T_1 and T_2 are syntax similarity, if there is bijection $f : F(T_1) \to F(T_2)$ such that

1) restriction f to $F_n(T_1)$ is isomorphism Boolean algebra F_nT_1 and F_nT_2, $n < \omega$;

2) $f(\exists v_{n+1}\varphi) = \exists\varphi_{n+1} f(\varphi), \varphi \in F_{n+1}(T)$, $n < \omega$;

3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 6 [4; Def. 2].

1) Pure triple is called $\langle A, \Gamma, M\rangle$ where A is nonempty, Γ is group permutations A and M is family subset A, such that $M \in \mathbb{M} \Rightarrow g(M) \in \mathbb{M}$ for every $g \in \Gamma$;

2) If $\langle A_1, \Gamma_1, M_1\rangle$ and $\langle A_2, \Gamma_2, M_2\rangle$ are pure triple and $\psi : A_1 \to A_2$ is bijection, then ψ is isomorphism, if:

(i) $\Gamma_2 = \{\psi g \psi^{-1} : g \in \Gamma_1\}$;

(ii) $M_2 = \{\psi(E) : E \in \mathbb{M}_1\}$.

Definition 7 [4; Def. 3]. Pure triple $\langle |C|, G, \mathbb{N}\rangle$ is called semantic triple of complete theory T , where $|C|$ is carrier monster-model C of theory T, $G = Aut(C)$, N is class all subset $|C|$, every of which carrier corresponding elementary of submodel C.

Definition 8 [4; Def. 4]. The complete theory T_1 and T_2 are called semantic similarity, if their semantic triple are isomorphic between itself.

The following definitions will be generalizations previous definitions.

Let T is an arbitrary Jonsson theory, then $E(T) = \bigcup_{n<\omega} E_n(T)$ where $E_n(T)$ is lattice $\exists$–formula with n free variables, T^* is center of Jonsson theory T, i.e. $T^* = Th(C)$, where C is semantic model of Jonsson theory T in [5].

Definition 9. Let T_1 and T_2 are Jonsson theory.

We will say, that T_1 and $T_2 - J$ is syntactically similar, if there is bijection $f : E(T_1) \to E(T_2)$ such that:

1) restriction f to $E_n(T_1)$ is isomorphism lattice $E_n(T_1)$ and $E_n(T_2)$, $n < \omega$;

2) $f(\exists v_{n+1}\varphi) = \exists\varphi_{n+1} f(\varphi)$, $\varphi \in E_{n+1}(T)$, $n < \omega$;

3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 10. Pure triple $\langle C, AutC, SubC\rangle$ is called J is semantical triple, where C is semantical model T, $AutC$ is group of automorphism C, $SubC$ is class all subset of carrier C, which there are carrier relevant submodel C.

Definition 11. Two Jonsson theories T_1 and T_2 are called J is semantical similar, if their J is semantical triple similar how pure triple.

It is clear that the definition 11 is a generalization [4; Def. 1], and definition 11 is a generalization of [4; Def. 4] in the following sense:

a) in the definition 9 for each $n < \omega$ instead of Boolean algebra $F_n(T)$ considered lattice $\exists$-formula of $E_n(T)$;

b) in the definition of 10 instead of the monster model complete theory T, considered semantic model of Jonsson theory T and as N of the definition [4; Def. 3] considered $SubC_i$ is the class of all subsets of the carrier C_i, which are carriers of the relevant submodels $SubC_i$, which the satisfies M of [4; Def. 2].

B due to the new definition of the semantic model of [1], we introduce the following definition.

Definition 12. Jonsson theory T is called perfect if each semantic model T is the saturated model of T^*.

The main results of of the work is the following result is associated with the above definitions.

Let A_1 and A_2 are Jonsson subset of the semantic model the some of $EPPCJ$ – theory. Where $Fr(A_1)$ and $Fr(A_2)$ are Jonsson sets of fragments A_1 and A_2. Then let $T_1 = Fr(A_1)$, $T_2 = Fr(A_2)$. Respectively $T^C_{A_1}$ and $T^C_{A_2}$ are the enrichment of Jonsson sets A_1 and A_2 the corresponding fragments T_1 and T_2.

We have the following results.

Theorem. Let T_1 and T_2 are $\exists$ – complete perfect Jonsson theory. Then following conditions are equivalent:

1) T_1 and T_2 are J – syntactically similar;

2) T_1^* and T_2^* are syntactically similar, T_1^* and T_2^* respectively centers enrichment of fragments consideration sets A_1 and A_2.

Proof. For the proof should be necessary in the following two facts.

Fact 1. For any Jonsson theory T the following conditions are equivalent:

1) T is perfect;

2) T^* is model complete.

Proof follows from the fact that perfect Jonsson theory T the equivalent, that T^* is a model companion of theory T [6].

Fact 2. For any complete for $\exists$ – sentences Jonsson theory T the following conditions are equivalent:

1) T^* is model complete;

2) for each $n < \omega$, $E_n(T)$ is Boolean algebra, where $E_n(T)$ is a lattice $\exists$ – formula with n free variables.

Proof. 1)$\Rightarrow$2) Let T^* is the model complete $\Rightarrow E_n(T^*)$ is Boolean algebra, because T^* is complete theory (the elementary theory of the semantic model), but $E_n(T) \subseteq E_n(T^*)$, because $T \subseteq T^*$.

We have 2 cases:

1) T is complete, then $T = T^* \Rightarrow T$ is model complete, $\Rightarrow E_n(T)$ is Boolean algebra;

2) If $T \subset T^* \Leftrightarrow T^* = Th(C)$, where C is semantic model of T, then $\forall \varphi \in T \Rightarrow \varphi \in T^*$; If T is complete for $\exists$ – sentences, then all $\exists$ - sentences output from T belongs to T^*. The others in T^* is not $\exists$ - sentences, because $E_n(T^*)$ is Boolean algebra, then it is additions for any φ – $\exists$ – sentences. In generality case, this φ will be not $\exists$ – sentences, because if $\varphi \in \Sigma$, then $\neg\varphi \in \Pi$ (Σ is a set of $\exists$ – sentences, Π is a set of $\forall$ – sentences), but T^* is model complete $\Leftrightarrow \forall \psi \in T, \exists \theta \in T^* : \psi \equiv \theta, \theta \in \Sigma$. But we known that $\theta \in T^* \Leftrightarrow \theta \in T$ $\Rightarrow$ 1) $1,0 \in E_n(T)$; 2) $\varphi \in E_n(T) \Rightarrow \neg\varphi \in E_n(T)$; 3) $\forall \varphi \in E_n(T)$ $\neg\neg\varphi = \varphi \Rightarrow E_n(T)$ is Boolean algebra.

2)$\Rightarrow$1) $E_n(T)$ is Boolean algebra $\Rightarrow T$ is model complete,but $T \subset T^* = Th(C)$. Let $A \in ModT \Rightarrow A$ is isomorphic introduce to C, because C is semantical model. Due to the fact, that T is model complete $\Rightarrow$ is embedding elementary.

Let C is not saturated, then $\exists X \subset C, |X| < |C|, \exists p \in S_1(X)$: is not true, that $(C, x)_{x \in X} \models p$, but $p \cup T$ is jointly, so $\exists m \notin C$: m realize in p, then $\exists M \models T^*$, that $m \in M$, M is the elementary extension of C that power $\Rightarrow \exists$ semantical model C', which $|M|^+$ is saturated and the elementary extension of M power 2^M. But any two semantical models are elementary equivalent between itself, in particular $C \equiv C'$. We give a contradiction, because C' realize in p. Consequently, that C is not saturated, is not true, $\Rightarrow T$ is perfect, $\Rightarrow T^*$ is model complete.

Now show directly to proof statement of the theorem.

We show 1)$\Rightarrow$2). We have that for each $n < \omega$ $E_n(T_1)$ is isomorphic $E_n(T_2)$. Let this isomorphism carried out by f_{1n}. By condition theorem and facts 1 and 2 for each $n < \omega$ $E_n(T_1)$ and $E_n(T_2)$ are Boolean algebra. But by condition perfectness T_1 and $T_2 \Rightarrow T_1^*$ and T_2^* are model complete due to the fact 1, because for each $n < \omega$, for any formulae $\varphi(\overline{x})$ from $F_n(T_1^*)$ there is a formula $\psi(\overline{x})$ from $E_n(T_1^*)$ so that $T_1^* \models \varphi \leftrightarrow \psi$. Due to the that the theory T_2 is $\exists$ - complete and $E_n(T_2) \subseteq E_n(T_2^*)$ (as $T_2 \subseteq T_2^*$), follows that $E_n(T_2) = E_n(T_2^*)$. For each $n < \omega$, for each $\varphi_1(\overline{x})$ from $F_n(T_1^*)$ we ask the following maps between $F_n(T_1^*)$ and $F_n(T_2^*)$: $f_{2n}(\varphi_1(\overline{x})) = f_{1n}(\varphi_1(\overline{x}))$, where in $T_1^* \models \psi_1 \leftrightarrow \varphi_1$, $\varphi_1 \in E_n(T_1)$. Easily understood that by virtue if the properties f_{1n} and the above f_{2n} is bijection specifying isomorphism between $F_n(T_1^*)$ and $F_n(T_2^*)$. Consequently, T_1^* and T_2^* are syntactically similar. We show 2)⇒1) is trivial, because $F_n(T_1^*)$ isomorphic $F_n(T_2^*)$ for each $n < \omega$, and by condition theorem and facts 1 and 2 this isomorphism extends to all subalgebras.

From we known the following results.

Proposition. If theory T_1 and T_2 are syntactically similar, then T_1 and T_2 T_1 and T_2 are semantically similar, the reverse is not true.

In this regard can be formulated as follows:

Lemma 1. Any two cosemantic fragments J is semantically similar.

Proof follows from the definition.

Lemma 2. If two perfect ∃ – complete Jonsson theories J syntactically similar, then they J semantically similar.

Proof follows from the theorem 1 and proposition 1.

All are uncertain definitions and concepts related with Jonsson theories can be found in [5].

References

1 *Yeshkeyev A.R.* Jonsson sets and some of their model-theoretic properties: Abstracts Book // International Congress of Mathematicians, August, 13-21. — 2014. Seoul, Korea. — P. 8.

2 *Yeshkeyev A.R.* On Jonsson sets and some their properties // Bull. of Symbolic Logic. — 2015. — Vol. 21. – No. 1. — P. 99, 100.

3 *Yeshkeyev A.R.* Properties of central type for fragments of Jonsson sets // Bull. of Symbolic Logic. — 2016. — Vol. 22. — No. 3. — P. 429, 430.

4 *Mustafin T.G.* On similarities of complete theories // Logic Colloquium '90: proceedings of the Annual European Summer Meeting of the Association for Symbolic Logic. — held in Helsinki, Finland. — July, 15-22. — 1990. — P. 259–265.

5 *Ешкеев А.Р., Касыметова М.Т.* Йонсоновские теории и их классы моделей. — Караганда: Изд-во КарГУ, 2016. — С. 346.

6 *Мустафин Т.Г.* Обобщенные условия Йонсона и описание обобщенно-йонсоновских теорий булевых алгебр // Математические труды. — 1998. — Т. 1. — № 2. — С. 135–197.

А.Р. Ешкеев

Йонсондық байытылу фрагменттерінің компаньондары

Мақалада кейбір байытудағы экзистенционалды жай йонсондық дөңес теориялар үшін орталық типтердің қасиеттері қарастырылған. Бұл теория класы кең йонсондық теориялардың кластарының ішкі класы болып табылады. Дербес жағдайда йонсондық теорияларға барлық бекітілген сипаттама мен өрістер класын жатқызуға болады. Автор келесі тақырыпқа қатысты жалпы модельдер теориясының классикалық проблемаларымен байланысты есептерді қарастырды. Ең бірінші кезекте байытудың мағынасы көрсетілді. Бір орынды предикаттың көмегімен йонсондық ішкі жиындар және йонсондық толықтырулар үшін p-стабильділік ұғымы және ұқсастықтың әр түрлі түрлері қарастырылды, яғни, орталық тип және оның центрінің прототипі үшін p-стабильділіктің сәйкестігі. Сонымен қатар йонсондық байытулардың фрагменттерінің компаньондарының синтаксистік ұқсастығы және олардың центрлерінің синтаксистік ұқсастығы эквиваленттілігі көрсетілген. Берілген байытылуда централдік типтердің қасиеттерін оқу үшін берілген стабильділік ұғымы қолданбалы мағына береді. Екіншіден, берілген байытылуда централдік типтің ұғымының мағынасын атап өткен жөн. Орталық типтің өзіндік идеясы толық емес орталық толықтырылудың көмегімен алынған йонсондық теориялардың қасиеттерін қосымша сипаттауын ұсынады. Қарастырып отырған экзистенционалды тұйық қатты дөңес йонсондық теориялардың йонсондық ішкі жиындарының семантикалық модельдері модельдітеоретикалық сипаттамадағы жақсы қасиеттерге ие. Бұл Морли рангіне қатысты және ол жоғарыда көрсетілген теориялардың синтаксистік және семантикалық ұқсастықтарында сақталады.

Кілт сөздер: йонсондық теория, байытылудың фрагменттері, ұқсастықтың қасиеттері, компаньондардың синтаксистік ұқсастығы, инвариантты қасиет.

А.Р. Ешкеев

Компаньоны фрагментов йонсоновского обогащения

В статье рассмотрены свойства центральных типов для экзистенциально простых сильно йонсоновских выпуклых теорий в некотором расширении. Этот класс теорий является подклассом широкого класса йонсоновских теорий. В частности, к йонсоновским теориям можно отнести класс всех полей фиксированной характеристики. Автором решены задачи, связанные с классическими проблемами общей теории моделей, касающихся следующей тематики. В первую очередь, отметим значения обогащения. С помощью одноместного предиката выделяется йонсоновское подмножество и для йонсоновских пополнений рассмотрены понятия p-стабильности и различные виды подобий. Получены следующие результаты: совпадение p-стабильности для прототипа центрального типа и его центра; эквивалентность синтаксического подобия компаньонов фрагментов йонсоновского обогащения и синтаксического подобия их центров. Понятие стабильности имеет прикладное значение для изучения свойств центральных типов в данном обогащении. Во вторую очередь нужно отметить значение понятия центрального типа в данном обогащении. Сама идея центрального типа предполагает дополнительное описание свойств неполных йонсоновских теорий с помощью центрального пополнения. Йонсоновские подмножества семантической модели рассматриваемой экзистенционально простой сильно выпуклой йонсоновской теории обладают хорошими свойствами теоретико-модельного характера. Это касается ранга Морли и он сохраняется при синтаксическом и семантическом подобиях указанных выше теорий.

Ключевые слова: йонсоновская теория, фрагменты обогащения, свойства подобия, синтаксическое подобие компаньонов, свойство инвариантности.

References

1 Yeshkeyev A.R. *Jonsson sets and some of their model-theoretic properties:* Abstracts Book, International Congress of Mathematicians, August, 13-21, 2014, Seoul, Korea, p. 8.

2 Yeshkeyev A.R. *Bulletin of Symbolic Logic,* 2015, 21, 1, p. 99, 100.

3 Yeshkeyev A.R. *Bulletin of Symbolic Logic,* 2016, 22, 3, p. 429, 430.

4 Mustafin T.G. *Logic Colloquium '90: proceedings of the Annual European Summer Meeting of the Association for Symbolic Logic, held in Helsinki,* Finland, July, 15-22, 1990, p. 259–265.

5 Yeshkeev A.R., Kasymetova M.T. *Jonsson theory and its classes of models,* Karaganda: Published of KSU, 2016, p. 346.

6 Mustafin T. G. *Mathematical works*, 1998, 1, 2, p. 135–197.