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On the function approximation by trigonometric polynomials and the properties of families of function classes over harmonic intervals

The article is devoted to research on approximation theory. When approximating functions by trigonometric polynomials, the spectrum is chosen from various sets. In this paper, the spectrum consists of harmonic intervals. Devices, various processes, perception of the senses have a limited range. In the mathematical modeling of numerous practical problems and in the further study of such mathematical models, it is sufficient to find a solution in this range. It is possible to study such models to some extent with the help of harmonic intervals. To prove the main theorem, an auxiliary lemma was proved, and elements of the theory of approximations with respect to harmonic intervals were used. For the constructed families of function classes associated with the best approximations by trigonometric polynomials with a spectrum of harmonic intervals, their relationship with classical Besov spaces is shown.

Keywords: harmonic interval, spectrum, the best approximation of a function by trigonometric polynomials with a spectrum of harmonic intervals, Dirichlet kernel, family of function classes.

Introduction

In recent decades, the penetration of ideas and methods of the approximation theory into various branches of mathematical science has been observed. According to a certain rule, the approximation of a function is understood as the replacement of one function by another, close to the original in one sense or another. In the study of periodic functions, trigonometric polynomials occupy a central position as approximating objects. The fundamental results in this theory were obtained in classical works [1, 2]. Further development of the theory is connected with the works of [3, 4] and with the works of other mathematicians. The results obtained are also described in detail in books [5, 6] and others.

When choosing an approximating functions, the spectrum is essential. The spectrum of approximating functions can have the most diverse configuration and consist of the most diverse sets. For example, the spectrum can be a hyperbolic cross [7, 8] or the spectrum is a ball [9], etc.

Devices, various processes, perception of the human senses have a limited finite range. In the mathematical modeling of numerous practical and applied problems and in the subsequent study of the compiled mathematical models, it is enough to find a solution in this range. The study of such models [10, 11] can be carried out to some extent using harmonic intervals.

Harmonic intervals are defined as sets I_k^N [12] of a special form, where the parameter characterizes the specified limited range to some extent. The definitions of harmonic segments and harmonic intervals were given by E.D. Nursultanov in [13, 14]. These sets built according to a certain rule, and their accompanying elements have found wide application in harmonic analysis.

The lemma and the main theorem are presented in the second section. The theorem is proved using an auxiliary lemma and using the properties of harmonic intervals and the mathematical objects associated with them.

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As an auxiliary problem, families of function classes $\left\{B^r_{p,q,N}\right\}_N$ connected by the best approximations over harmonic intervals are introduced. Section 3 is based on the study of the properties of these families of function classes $\left\{B^r_{p,q,N}\right\}_N$. The constructed families of function classes are related to the classical Besov spaces, and this is shown in the third section.

1 Definitions and auxiliary results

Definition 1. [12] If $k, \nu, d, N \in \mathbb{N}$, $k < N$, then the sets of the following types

$$I_k^{N,d} = \bigcup_{\nu=-d}^{d} \left([-k,k] + 2\nu N\right),$$

$$I_k^N = \bigcup_{\nu=-\infty}^{\infty} \left([-k,k] + 2\nu N\right) = \bigcup_{\nu=-\infty}^{\infty} \left(m + 2\nu N : m \in [-k,k]\right)$$

are called harmonic segment and harmonic interval in $\mathbb{Z}$, respectively.

Let T_k^N be the set of trigonometric polynomials in the harmonic interval, defined by the formula [12]

$$T_k^N = \left\{ \sum_{\nu=-s}^{s} a_\nu \cdot e^{i\nu x} : a_\nu = 0 \;\; if \;\; \nu \notin I_k^N, s \in \mathbb{N} \right\}.$$

We have

$$E_k^N(f)_p = \inf_{t \in T_k^N} \left\| f - t \right\|_p,$$

where $E_k^N(f)_p$ is the best approximation over the harmonic interval I_k^N of the function $f \in L_p[0, 2\pi)$, $1 \leq p \leq \infty$ by trigonometric polynomials from T_k^N of order less than or equal to k [12].

If $f \in L_p[0, 2\pi)$, $1 \leq p \leq \infty$, then we will consider the following sums as partial sums of the Fourier series of the function f over the harmonic segment $I_k^{N,d}$ and the harmonic interval I_k^N, respectively [12]

$$S_k^{N,d}(f) = \sum_{\nu \in I_k^{N,d}} a_\nu \cdot e^{i\nu x}, \quad S_k^N(f) = \sum_{\nu \in I_k^N} a_\nu \cdot e^{i\nu x}.$$

Lemma 1. Let the functions f and g belong to the space $L_{2k}[0, 2\pi)$, where $k \in \mathbb{N}$. If the functions f and g satisfy the condition

$$\int_0^{2\pi} f^k \cdot \bar{g}^k dx = 0, \tag{1}$$

then we have an inequality of the form

$$\left(\int_0^{2\pi} \left(|f|^{2k} + |g|^{2k} \right) dx \right)^{\frac{1}{2k}} \leqslant k \left(\int_0^{2\pi} |f+g|^{2k} dx \right)^{\frac{1}{2k}}.$$

Lemma 2. [14] Let $B = [-k, k]$ be a segment in $\mathbb{Z}$. $k, d, h \in \mathbb{N}$, $k < h$. $\left\{ I_B^{h,d} \right\}_{d=0}^{\infty}$ be a sequence of harmonic segments in $\mathbb{Z}$, converging to a harmonic integral I_B^h, and

$$I_B^h = \bigcup_{\nu=-\infty}^{\infty} \left(B + \nu h\right).$$

If $f \in L_p[0, 2\pi)$, $1 \le p \le \infty$, $\sum_{\nu \in \mathbb{Z}} a_\nu \cdot e^{i\nu x}$ is its Fourier series, then the sequence of partial sums of the Fourier series of the function f over the harmonic segment

$$S_B^{h,d}(f) = \sum_{\nu \in I_B^{h,d}} a_\nu \cdot e^{i\nu x}$$

converges in $L_p[0, 2\pi)$ as $d \to \infty$ to the function

$$S_B^h(f) = \frac{1}{h} \sum_{\nu=0}^{h-1} f\left(x + \frac{2\pi r}{h}\right) D_B\left(\frac{2\pi r}{h}\right),$$

where

$$D_B(x) = \sum_{m \in B} a_m \cdot e^{imx}$$

is the Dirichlet kernel corresponding to the segment B from $\mathbb{Z}$, and its Fourier series will be the function $\sum_{\nu \in I_B^h} a_\nu \cdot e^{i\nu x}$.

Theorem 1. [15] Let $f \in L_p[0, 2\pi)$, $1 < p < \infty$, $m \in \mathbb{N}$, $S_m^N(f)$ be the partial sum of the Fourier series and $E_m^N(f)$ be the best approximation of the function f over the harmonic interval I_m^N, then the following correspondence is fulfilled

$$E_m^N(f)_p \sim \left\| f - S_m^N(f) \right\|_p.$$

Let $1 \le p$, $q \le \infty$, $r > 0$, $f \in L_p[0, 2\pi)$. Let's construct a family of function classes $\left\{B_{p,q.N}^r\right\}_N$ satisfying the condition

$$B_{p,q.N}^r = \left\{ f : \|f\|_{B_{p,q.N}^r} < \infty \right\}, \quad N \in \mathbb{N},$$

where

$$\|f\|_{B_{p,q.N}^r} = \left(\sum_{k=1}^{N} k^{rq-1} \left(E_{k-1}^N(f)_p\right)^q \right)^{\frac{1}{q}}.$$

2 Properties of partial sums of the Fourier series over harmonic intervals

Lemma 3. Let f be a function from the space $L_{2k}[0, 2\pi)$, where $k \in \mathbb{N}$. $\sum_{n \in \mathbb{Z}} a_n \cdot e^{inx}$ is its Fourier series, $d \in \mathbb{N}$, $(m+1)k < d - 1$.

$$I_1 = I_m^d = \bigcup_{\nu=-\infty}^{\infty} \left([0, m] + \nu d\right),$$

$$I_2 = \bigcup_{\nu=-\infty}^{\infty} \left\{ \left[m + 1, \left[\frac{d-1}{k}\right]\right] + \nu d \right\},$$

$$I_{\left[\frac{d-1}{k}\right]}^d = \bigcup_{\nu=-\infty}^{\infty} \left\{ \left[0, \left[\frac{d-1}{k}\right]\right] + \nu d \right\}$$

are harmonic intervals in $\mathbb{Z}$; $S_m^d(f)$ and $S_{\left[\frac{d-1}{k}\right]}^d(f)$ are partial sums of the Fourier series of the function $f(x)$ over harmonic intervals I_m^d and $I_{\left[\frac{d-1}{k}\right]}^d$, respectively, then the inequality holds

$$\left\| S_m^d(f) \right\|_{2k} \le k \left\| S_{\left[\frac{d-1}{k}\right]}^d(f) \right\|_{2k}.$$

Proof. We introduce the following notation

$$u(x) = \sum_{n \in I_1} a_n \cdot e^{inx}, \quad v(x) = \sum_{n \in I_2} a_n \cdot e^{inx}.$$

The functions $u(x)$ and $v(x)$ are partial sums of the Fourier series of the function $f(x)$ over harmonic intervals I_1 and I_2, respectively, and therefore belongs to the space $L_{2k}[0, 2\pi)$.

Let's prove that

$$\int_0^{2\pi} u^k \cdot \bar{v}^k dx = 0$$

or

$$\int_0^{2\pi} \left(\sum_{n \in I_1} a_n \cdot e^{inx} \right)^k \cdot \left(\sum_{n \in I_2} \bar{a}_n \cdot e^{inx} \right)^k dx = 0.$$

Taking into account the values of the integral $\int_o^{2\pi} e^{inx} dx$ when $n = 0$ and $n \neq 0$ we conclude that the last condition will be satisfied if there are no identical numbers among the numbers $n \in I_1$ and $n \in I_2$ when raising the partial sums $\sum_{n \in I_1} a_n \cdot e^{inx}$ and $\sum_{n \in I_2} a_n \cdot e^{inx}$ to the power of k.

Note that when $u(x)$ is raised to the power of k, the numbers n fall into the set, which is a harmonic interval, which we denote by I_{1k} and

$$I_{1k} = \bigcup_{\nu=-\infty}^{\infty} \left([0, mk] + \nu d \right).$$

Indeed, by definition, we have

$$I_1 + I_2 + ... + I_r = \left\{ n_1 + n_2 + ... n_r, \ \ n_i \in I_i, \ \ n = 1, 2, ... r \right\}$$

so

$$I_{1k} = \underbrace{I_1 + ... + I_1}_{k} = \left\{ n_1 + n_2 + ... n_k, \ \ n_i \in I_1, \ \ n = 1, 2, ... k \right\}.$$

Since $n_i \in I_1$, then $n_i = l_i + \nu d$, where $l_i \in [0, m]$, $\nu \in \mathbb{Z}$, $i = 1, ..., k$. Therefore,

$$\sum_{i=1}^{k} n_i = \sum_{i=1}^{k} l_i + \nu d.$$

Thus,

$$\nu d \leqslant \sum_{i=1}^{k} n_i \leqslant mk + \nu d.$$

It means that $\sum_{i=1}^{k} n_i \in I_k$.

Applying the same reasoning, we get that the numbers n, when the partial sum $\bar{v}(x)$ is raised to the power k, fall into the harmonic interval I_{2k}, and

$$I_{2k} = \bigcup_{\nu=-\infty}^{\infty} \left\{ \left[(m+1)k, \ d - 1 \right] + \nu d \right\}.$$

It is obvious that

$$I_{1k} \cap I_{2k} = \varnothing.$$

This equality ensures the fulfilment of the condition (1) for $u(x)$ and $v(x)$. The fulfilment of this condition guarantees the application of Lemma 1, namely

$$\left\{\int_0^{2\pi} \left|u\right|^{2k} dx\right\}^{\frac{1}{2k}} \leqslant \left\{\int_0^{2\pi} \left(\left|u\right|^{2k} + \left|v\right|^{2k}\right) dx\right\}^{\frac{1}{2k}} \leqslant \left\{\int_0^{2\pi} \left|u+v\right|^{2k} dx\right\}^{\frac{1}{2k}},$$

$$\left(\int_0^{2\pi} \left|\sum_{n\in I_m^d} a_n \cdot e^{inx}\right|^{2k} dx\right)^{\frac{1}{2k}} \leqslant k \left(\int_0^{2\pi} \left|\sum_{n\in I^d_{\left[\frac{d-1}{k}\right]}} a_n \cdot e^{inx}\right|^{2k} dx\right)^{\frac{1}{2k}}$$

or

$$\left\|S_m^d(f)\right\|_{2k} \leq k \left\|S^d_{\left[\frac{d-1}{k}\right]}(f)\right\|_{2k}.$$

Lemma 3 is proved.

Theorem 2. Let $f \in L_p[0, 2\pi)$, $1 < p < \infty$, $\sum_{\nu\in\mathbb{Z}} a_\nu \cdot e^{i\nu x}$ be its trigonometric Fourier series, then the following inequality

$$\left\|f - \frac{1}{2N}\sum_{r=0}^{2N-1} f\left(x + \frac{\pi r}{N}\right) D_m\left(\frac{\pi r}{N}\right)\right\|_p \leqslant C\left\|f - S_m(f)\right\|_p, \tag{2}$$

is true, where $D_m(y)$ is Dirichlet kernel corresponding to the segment$[-m; m]$, C is a constant that depends only on the parameter p.

Proof. According to Lemma 2, we have

$$\left\|f - \frac{1}{2N}\sum_{r=0}^{2N-1} f\left(x + \frac{\pi r}{N}\right) D_m\left(\frac{\pi r}{N}\right)\right\|_p = \left\|f - S_m^N(f)\right\|_p =$$

$$= \left\|\sum_{\nu\in\mathbb{Z}\setminus I_m^N} a_\nu \cdot e^{i\nu x}\right\|_p = \left\|\sum_{\nu\in Q^N_{m+1}} a_\nu \cdot e^{i\nu x}\right\|_p = \left\|S_{Q^N_{m+1}}(f)\right\|_p,$$

where Q^N_{m+1} are harmonic intervals in $\mathbb{Z}$, and

$$Q^N_{m+1} = \bigcup_{\nu=-\infty}^{\infty}\left\{[-N,\ -m-1] \cup [m+1,\ N] + 2\nu N\right\}.$$

Then we have

$$\left\|S_{Q^N_{m+1}}\right\|_p = \left\|S_{Q^N_{m+1}}(f - S_m)\right\|_p.$$

Since $S_{Q^N_{m+1}}(f - S_m)$ is a partial sum of the Fourier series of the function

$$f - S_m(f) = \sum_{\nu\in\mathbb{Z}\setminus[-m,\ m]} a_\nu \cdot e^{i\nu x}$$

then, by the theorem [14] on the boundedness of partial sums of Fourier series over the harmonic interval, we obtain the necessary inequality

$$\left\|f - S_m^N(f)\right\|_p = \left\|S_{Q^N_{m+1}}(f - S_m)\right\|_p \leqslant C\left\|f - S_m(f)\right\|_p.$$

Thereby, Theorem 2 is proved.

Note 1. According to Theorem 1 and Lemma 9.3 [16] the relation (2) can be presented in the equivalent form

$$E_m^N(f)_p \leqslant E_m(f)_p,$$

3 Properties of the family of function classes $\left\{B^r_{p,q.N}\right\}_N$

Definition 2. [12] Let two classes of functions A^N and B^N depending on the parameter N be given. We will say that the class of functions A^N is embedded in the class of functions B^N and denote it by $A^N \hookrightarrow B^N$ if the following conditions are satisfied:

1) $A^N \subset B^N$;

2) there is a parameter C such that for any $f \in A^N$ the relation

$$\|f\|_{B^N} \leqslant C\|f\|_{A^N}$$

is true, moreover, the parameter C does not depend on f and N.

Definition 3. [15] Function classes $\left\{A^N\right\}_N$ and $\left\{B^N\right\}_N$, where $N \in \mathbb{N}$, are equivalent

$$\|f\|_{A^N} \sim \|f\|_{B^N},$$

if there are parameters C_1, C_2 such that for any $f \in A^N$ there is a correspondence

$$C_1\|f\|_{B^N} \leq \|f\|_{A^N} \leq C_2\|f\|_{B^N},$$

moreover, the parameters C_1, C_2 do not depend on f and N.

In this case, the families of function classes $\left\{A^N\right\}_N$ and $\left\{B^N\right\}_N$ coincide, namely

$$\left\{A^N\right\}_N = \left\{B^N\right\}_N.$$

Theorem 3 relates families of function classes $\left\{B^r_{p,q.N}\right\}_N$ to classical Besov spaces [17].

Theorem 3. Let $N \in \mathbb{N}$, $1 \leqslant p,\ q \leqslant \infty$, $r > 0$ then the following relationship is performed

$$\bigcap_{N=1}^{\infty} B^r_{p,q,N} = B^r_{p,q}.$$

Proof By definition, we have

$$\|f\|_{\bigcap\limits_{N=1}^{\infty} B^r_{p,q,N}} = \sup_N \|f\|_{B^r_{p,q,N}}$$

Since the following inequality

$$\|f\|_{B^r_{p,q,N}} \leqslant C\|f\|_{B^r_{p,q}}$$

holds for any $N \in \mathbb{N}$ then we obtain the accordance

$$\sup_N \|f\|_{B^r_{p,q,N}} = \|f\|_{\bigcap\limits_{N=1}^{\infty} B^r_{p,q,N}} \leqslant \|f\|_{B^r_{p,q}}.$$

This correspondence follows from the last inequality

$$B^r_{p,q} \hookrightarrow \bigcap_{N=1}^{\infty} B^r_{p,q,N}.$$

From other side, for a partial sum $S_{2^{m}(f)}$, where $m \in \mathbb{N}$, we get the ratio

$$\left\|S_{2^m}(f)\right\|_{B^r_{p,q}} = \left\|S_{2^m}(f)\right\|_{B^r_{p,q,2^m}} \leqslant C(p,q,r)\left\|f\right\|_{B^r_{p,q,2^m}} \leqslant$$

$$\leqslant C(p,q,r)\sup_N \left\|f\right\|_{B^r_{p,q,N}} = C(p,q,r)\left\|f\right\|_{\bigcap\limits_{N=1}^{\infty} B^r_{p,q,N}}.$$

Further, from the last relation, according to the Banach-Steinhaus theorem [18], we obtain the desired inequality

$$\left\|f\right\|_{B^r_{p,q}} \leqslant C(p,q,r)\left\|f\right\|_{\bigcap\limits_{N=1}^{\infty} B^r_{p,q,N}}$$

or

$$\bigcap_{N=1}^{\infty} B^r_{p,q,N} \hookrightarrow B^r_{p,q}.$$

Thus, Theorem 3 is proved.

References

1 Тиман А.Ф. Теория приближения функций действительного переменного / А.Ф. Тиман. — М.: Физматлит, 1960. — 624 с.

2 Кокилашвили В.М. О приближении периодических функций / В.М. Кокилашвили // Тр. Тбилис. мат. ин-та. — 1968. — № 1. — С. 51–81.

3 Степанец А.И. Равномерные приближения тригонометрическими полиномами / А.И. Степанец. — Киев: Наук. думка, 1981. — 340 с.

4 Жук В.В. Аппроксимация периодических функций / В.В. Жук. — Л.: Изд-во Ленингр. ун-та, 1982. — 566 с.

5 Никольский С.М. Избранные труды. Теория приближений / С.М. Никольский. — М.: Наука, 2006. — Т. 1. — 432 с.

6 Иванов А.П. Численные методы / А.П. Иванов, И.В. Олемской. — Ч. II. — СПб.: СПбГУ, 2012. — 80 с.

7 Темляков В.Н. Дискретизация интегральных норм полиномов с гармониками из гиперболического креста по значениям в точках / В.Н. Темляков // Функциональные пространства, теория приближений и смежные вопросы анализа. сб. ст. к 115-летию со дня рождения акад. Сергея Михайловича Никольского // Труды МИАН. — 2021. — 312. — С. 282–293.

8 Турова М.Е. Оценки наилучших приближений функции со спектром из гиперболических крестов / М.Е. Турова // Междунар. науч.-исслед. журн. — 2015. — № 5-1(36). — С. 29–31.

9 Теляковский С.А. Очерки развития математики в СССР / С.А. Теляковский. — Киев: Наук. думка, 1983. — С. 237–251.

10 Yessenbayeva G.A. On the calculation of rectangular plates by the trigonometric series / G.A. Yessenbayeva, F.M. Akhanov, T.Kh. Makazhanova // Bulletin of the Karaganda University. Mathematics Series. — 2019. — № 2(94). — С. 115–120. https://doi.org/10.31489/2019M2/115-120

11 Yessenbayeva G.A. On the calculation of rectangular plates by the collocation method / G.A. Yessenbayeva, D.N. Yesbayeva, N.K. Syzdykova, I.A. Akpanov // Bulletin of the Karaganda University. Mathematics Series. — 2019. — № 1(93). — С. 154–159. https://doi.org/10.31489/2019M1/154-159

12 Есенбаева Г.А. Интерполяция некоторых неравенств и аппроксимация функций тригонометрическими полиномами со спектром из гармонических интервалов / Г.А. Есенбаева, Е.Д. Нурсултанов // Евраз. мат. журн. — 2004. — № 1. — С. 3–20.

13 Нурсултанов Е.Д. Сетевые пространства и преобразование Фурье / Е.Д. Нурсултанов // Докл. РАН (361). — 1998. — № 5. — С. 597–599.

14 Нурсултанов Е.Д. Сетевые пространства и их приложения к задачам гармонического анализа: дис. ... д-ра физ.-мат. наук. 01.01.01 — «Математический анализ» / Е.Д. Нурсултанов — М., 1999. — С. 171–179.

15 Есенбаева Г.А. On the boundedness of the partial sums operator for the Fourier series in the function classes families associated with harmonic intervals / G.A. Yessenbayeva, G.A. Yessenbayeva, A.T. Kasimov, N.K. Syzdykova // Bulletin of the Karaganda University. Mathematics Series. — 2021. — № 3(103). — С. 131–139. https://doi.org/10.31489/2021M3/131-139

16 Смаилов Е.С. Теория приближения и вложения классов функций многих переменных / Е.С. Смаилов, Г.А. Акишев, М.Г. Есмаганбетов. — Караганда: Изд-во Караганд. гос. ун-та, 1986. — 87 с.

17 Никольский С.М. Пространства Бесова / С.М. Никольский // Тр. МИАН. — 2003. — Т. 243. — С. 9–12.

18 Колмогоров А.Н. Элементы теории функций и функционального анализа / А.Н. Колмогоров, С.В. Фомин. — М.: Наука, 1981. — 623 с.

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Функцияларды тригонометриялық көпмүшелер арқылы жуықтау және гармониялық интервалдарға қатысты функциялар кластарының үйірлерінің қасиеттері туралы

Мақала жуықтау теориясы саласындағы зерттеулерге арналған. Тригонометриялық көпмүшелер арқылы функцияларды жуықтау кезінде спектр әртүрлі жиындардан таңдалады. Бұл жұмыста спектр гармоникалық интервалдардан тұрады. Құрылғылар, әртүрлі процестер, сезімдерді қабылдау мүшелері шектеулі ауқымға ие. Көптеген практикалық есептерді математикалық модельдеу кезінде және берілген математикалық модельдерді одан әрі зерттеу кезінде осындай диапазонда шешім табу жеткілікті. Мұндай модельдерді зерттеу белгілі бір дәрежеде гармоникалық интервалдардың көмегімен мүмкін болады. Негізгі теореманы дәлелдеу үшін көмекші лемма дәлелденді және гармоникалық интервалдар бойынша жуықтау теориясының элементтері қолданылды. Гармоникалық интервалдардың спектрі бар тригонометриялық көпмүшеліктермен функцияның ең жақсы жуықтауымен байланысқан функциялар кластарының құрылған үйірі үшін олардың классикалық Бесов кеңістіктері мен байланысы көрсетілген.

Кілт сөздер: гармоникалық интервал, спектр, гармоникалық интервалдардың спектрі бар тригонометриялық көпмүшеліктермен функцияның ең жақсы жуықтауы, Дирихле өзегі, функция кластарының үйірі.

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О приближении функций тригонометрическими полиномами и свойствах семейств классов функций по гармоническим интервалам

Статья посвящена исследованию по теории приближений. При приближении функций тригонометрическими полиномами спектр выбирается из различных множеств. В работе спектр состоит из гармонических интервалов. Приборы, различные процессы, восприятие органов чувств имеют ограниченный диапазон. При математическом моделировании многочисленных практических задач и дальнейшем исследовании таких математических моделей достаточно найти решение в заданном диапазоне. Проведение исследований таких моделей возможно в некоторой степени с помощью гармонических интервалов. Для доказательства основной теоремы была приведена вспомогательная лемма и использовались элементы теории приближений по гармоническим интервалам. Для построенных семейств классов функций, связанных с наилучшими приближениями тригонометрическими полиномами со спектром из гармонических интервалов, показана их связь с классическими пространствами Бесова.

Ключевые слова: гармонический интервал, спектр, наилучшее приближение функции тригонометрическими полиномами со спектром из гармонических интервалов, ядро Дирихле, семейство классов функций.

References

1 Timan, A.F. (1960). *Teoriia priblizheniia funktsii deistvitelnogo peremennogo [Theory of approximation of functions of a real variable].* Moscow: Fizmatlit [in Russian].

2 Kokilashvili, V.M. (1968). O priblizhenii periodicheskikh funktsii [On approximation of periodic functions]. *Trudy Tbilisskogo matematicheskogo instituta — Proceedings of the Tbilisi Mathematical Institute, 1,* 51–81 [in Russian].

3 Stepanets, A.I. (1981). *Ravnomernye priblizheniia trigonometricheskimi polinomami [Uniform approximations by trigonometric polynomials].* Kiev: Naukova dumka [in Russian].

4 Zhuk, V.V. (1982). *Approksimatsiia periodicheskikh funktsii [Approximation of periodic functions].* Leningrad: Izdatelstvo Leningradskogo universiteta [in Russian].

5 Nikolsky, S.M. (2006). *Izbrannye trudy. Teoriia priblizhenii [Selected works. Approximation theory].* Moscow: Nauka [in Russian].

6 Ivanov, A.P., & Olemskoi, I.V. (2012). *Chislennye metody [Numerical methods].* Saint Petersburg: Sankt-Petersburgskii gosudarstvennyi universitet [in Russian].

7 Temlyakov, V.N. (2021). Diskretizatsiia integralnykh norm polinomov s garmonikami iz giperbolicheskogo kresta po znacheniiam v tochkakh [Discretization of integral norms for polynomials with harmonics from a hyperbolic cross by values at points]. *Funktsionalnye prostranstva, teoriia priblizhenii i smezhnye voprosy analiza. Sbornik statei k 115-letiiu so dnia rozhdeniia akademika Sergeia Mikhailovicha Nikolskogo. Trudy MIAN — Functional spaces, approximation theory, and related issues of analysis. Collection of articles dedicated to the 115th anniversary of the birth of academician Sergei Mikhailovich Nikolsky. Works of MIAS, 312*, 282–293 [in Russian].

8 Turova, M.E. (2015). Otsenki nailuchshikh priblizhenii funktsii so spektrom iz giperbolicheskikh krestov [Estimates for the best approximations of a function with spectrum from hyperbolic crosses]. *Mezhdunarodnyi nauchno-issledovatelskii zhurnal — International research journal, 5-1*(36), 29–31 [in Russian].

9 Telyakovsky, S.A. (1983). *Ocherki razvitiia matematiki v SSSR [Essays on the development of mathematics in the USSR].* Kiev: Naukova dumka [in Russian].

10 Yessenbayeva, G.A., Akhanov, F.M., & Makazhanova, T.Kh. (2019). On the calculation of rectangular plates by the trigonometric series. *Bulletin of the Karaganda University. Mathematics Series, 2*(94), 115–120. https://doi.org/10.31489/2019M2/115-120

11 Yessenbayeva, G.A., Yesbayeva, D.N., Syzdykova, N.K., & Akpanov, I.A. (2019). On the calculation of rectangular plates by the collocation method. *Bulletin of the Karaganda University. Mathematics Series, 1*(93), 154–159. https://doi.org/10.31489/2019M1/154-159

12 Yessenbayeva, G.A., & Nursultanov, E.D. (2004). Interpoliatsiia nekotorykh neravenstv i approksimatsiia funktsii trigonometricheskimi polinomami so spektrom iz garmonicheskikh intervalov [Interpolation of some inequalities and approximation of functions by trigonometric polynomials with spectrum from harmonic intervals]. *Evraziiskii matematicheskii zhurnal — Eurasian mathematical journal, 1,* 3–20 [in Russian].

13 Nursultanov, E.D. (1998). Setevye prostranstva i preobrazovanie Fure [Network spaces and the Fourier transform]. *Doklad RAN (361) — RAS report (361), 5*, 597–599 [in Russian].

14 Nursultanov, E.D. (1999). *Setevye prostranstva i ikh prilozheniia k zadacham garmonicheskogo analiza [Network spaces and their applications to problems of harmonic analysis].* Doctor's thesis. Moscow [in Russian].

15 Yessenbayeva, G.A., Yessenbayeva, G.A., Kasimov, A.T., & Syzdykova, N.K. (2021). On the boundedness of the partial sums operator for the Fourier series in the function classes families associated with harmonic intervals. *Bulletin of the Karaganda University. Mathematics Series, 3*(103), 131–139. https://doi.org/10.31489/2021M3/131-139

16 Smailov, E.S., Akishev, G.A., & Esmaganbetov, M.G. (1986). *Teoriia priblizheniia i vlozheniia klassov funktsii mnogikh peremennykh [Theory of approximation and embedding of classes of functions of many variables].* Karaganda: Izdatelstvo Karagandinskogo gosudarstvennogo universiteta [in Russian].

17 Nikolsky, S.M. (2003). Prostranstva Besova [Besov spaces]. *Trudy MIAN — Proceedings of MIAS, 243,* 9–12 [in Russian].

18 Kolmogorov, A.N., & Fomin, S.V. (1981). *Elementy teorii funktsii i funktsionalnogo analiza [Elements of the theory of functions and functional analysis].* Moscow: Nauka [in Russian].