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On the stability of the difference analogue of the boundary value problem for a mixed type equation

This paper considers a difference problem for a mixed-type equation, to which a problem of integral geometry for a family of curves satisfying certain regularity conditions is reduced. These problems are related to numerous applications, including interpretation problem of seismic data, problem of interpretation of X-ray images, problems of computed tomography and technical diagnostics. The study of difference analogues of integral geometry problems has specific difficulties associated with the fact that for finite-difference analogues of partial derivatives, basic relations are performed with a certain shift in the discrete variable. In this regard, many relations obtained in a continuous formulation, when transitioned to a discrete analogue, have a more complex and cumbersome form, which requires additional studies of the resulting terms with a shift. Another important feature of the integral geometry problem is the absence of a theorem for existence of a solution in general case. Consequently, the paper uses the concept of correctness according to A.N. Tikhonov, particularly, it is assumed that there is a solution to the problem of integral geometry and its differential-difference analogue. The stability estimate of the difference analogue of the boundary value problem for a mixed-type equation obtained in this work is vital for understanding the effectiveness of numerical methods for solving problems of geotomography, medical tomography, flaw detection, etc. It also has a great practical significance in solving multidimensional inverse problems of acoustics, seismic exploration.

Keywords: ill-posed problem, boundary value problem, mixed-type equation, stability estimate, difference problem, quadratic form.

Introduction

The research focuses on a difference-differential problem for a mixed-type equation, to which reduces the problem of integral geometry for a family of curves satisfying certain regularity conditions is reduced.

The problems of integral geometry consist of finding a function or a more complex quantity (differential form, tensor field, etc.) defined on a certain variety, through its integrals over a certain family of sub-variety of smaller dimension.

Some inverse problems for kinetic equations widely used in physics and astrophysics are closely related to problems of integral geometry. The problems of integral geometry refer to ill-posed problems of mathematical physics, the foundations of which were laid in the works [1–3]. These problems are associated with numerous applications (problem of computed tomography, inverse problems of acoustics, and seismic exploration).

The need to study differential-difference and finite-difference analogues of integral geometry problems was first expressed and formulated as a new promising direction by Academician M.M. Lavrentiev. Therefore, the study of differential-difference and finite-difference analogues of integral geometry problems is an urgent problem.

M.M. Lavrentiev and V.G. Romanov first showed in the work [4] that a number of inverse problems for hyperbolic equations are reduced to problems of integral geometry. Further, V.G. Romanov obtained uniqueness theorems and estimates of conditional stability to solve integral geometry problems for a fairly general family of curves on a plane invariant with respect to the rotation group [5], as well as for families of curves and hyper surfaces in n-dimensional space invariant with respect to parallel transfers of these objects along some plane [6].

A very general result on uniqueness and stability estimates for a special family of curves was obtained by R.G. Mukhometov. These stability estimates are based on reducing the integral geometry problem to an equivalent boundary value problem for a partial differential equation of mixed type [7].

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Methods and materials

Let D be a bounded simply connected domain in the plane with a smooth boundary Γ:

$$x = \xi(z), \qquad y = \eta(z), \qquad z \in [0, l], \qquad \xi(0) = \xi(l), \qquad \eta(0) = \eta(l),$$

where z is the length of the curve Γ. In the $\overline{D}$ there are smooth curves defined by the equations

$$x = \varphi(x_0, y_0, \theta, s), y = \psi(x_0, y_0, \theta, s), \tag{1}$$

where (x_0, y_0) is a point from which the curve exits at an angle θ, the variable parameter s is the curve length. The set of function definitions φ and ψ is the set, indeed

$$T = \{(x_0, y_0, \theta, s)/(x_0, y_0) \in \overline{D}, \theta \in [0, 2\pi], s \in [0, \tilde{l}(x_0, y_0, \theta)]\},$$

where $\tilde{l}(x_0, y_0, \theta)$ is the length of the part of the curve leaving the point (x_0, y_0) at an angle θ and lying between (x_0, y_0) and the point of intersection of the curve with the boundary.

Let the set of curves (1) be such that it can be regarded as a two-parameter family of curves $K(\gamma, z)$ satisfying the conditions as follows:

a) through any two different points from $\overline{D}$ single curve $K(\gamma, z)$ passes; each curve of the family $K(\gamma, z)$ intersects Γ at points $(\xi(z), \eta(z))$ and $(\xi(\gamma), \eta(\gamma))$, the other points do not lie on Γ; the lengths of all curves are uniformly bounded;

b)$\varphi \in C^3(T), \psi \in C^3(T)$, and all derivatives of these functions are uniformly bounded in T;

c) $\frac{1}{s}\frac{D(\varphi,\psi)}{D(\theta,s)} \geq c_1 > 0$, where c_1 is a constant;

d) $\varphi(x, y, 0, s) = \varphi(x, y, 2\pi, s), \psi(x, y, 0, s) = \psi(x, y, 2\pi, s)$, similar equalities are also valid for derivatives of these functions up to the third order inclusive.

Let $U(x,\, y) \in C^2(\overline{D})$ and

$$V(\gamma, z) = \int\limits_{K(\gamma,z)} U(x,\, y)\rho(x, y, z)ds; \quad \gamma \in [0, l], \quad z \in [0, l]. \tag{2}$$

The problem of integral geometry (2) is to find a function $U(x, y)$ in the domain $\overline{D}$ according to the given curves $K(\gamma, z)$ and functions $V(\gamma, z)$.

If the family $K(\gamma, z)$ satisfies the conditions a)-d), then problem (2) is equivalent to the following boundary value problem

$$\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial x}\frac{\cos\theta}{\rho} + \frac{\partial W}{\partial y}\frac{\sin\theta}{\rho}\right) = 0, \quad (x, y, z) \in \Omega_1, \tag{3}$$

$$W(\xi(\gamma), \eta(\gamma), z) = V(\gamma, z), \qquad V(z, z) = 0, \qquad \gamma, z \in [0, l], \tag{4}$$

where $\rho(x, y, z)$ is a known function, $\Omega_1 = \Omega \backslash \{(\xi(z), \eta(z), z) : z \in [0, l]\}, \Omega = \overline{D} \times [0, l]$.

$K(x, y, z)$ is a part of the curve from the family $K(\gamma, z)$ connecting the points $(x, y) \in \overline{D}$ and $(\xi(z),\, \eta(z))$,

$$W(x,\, y,\, z) = \int\limits_{K(x,\, y,\, z)} U(x, y, z)\rho(x, y, z)ds.$$

$\theta(x, y, z)$ is an angle between the tangent to $K(x, y, z)$ at the point (x, y) and the x axis, the variable parameter s is the curve length.

The functions $W(x, y, z)$ and $\theta(x, y, z)$ have the following differential properties [7]:

Lemma 1. The function $W(x, y, z) \in C(\Omega)$ has continuous derivatives up to and including the second order on the set Ω_1.

Lemma 2. The derivative W_x, W_y, W_z are bounded in Ω_1, and W_{xz}, W_{yz}, W_{xy} in the neighborhood of any point of the form $(\xi(z), \eta(z), z)$ that can have a type singularity $\left[(x - \xi(z))^2 + (y - \eta(z))^2\right]^{-\frac{1}{2}}$.

Lemma 3. The function $\theta(x, y, z)$ is differentiable on the set Ω_1 and the derivative θ_z in the neighborhood of any point of the form $(\xi(z), \eta(z), z)$ has a type singularity $\left[(x - \xi(z))^2 + (y - \eta(z))^2\right]^{-\frac{1}{2}}$.

Assume that the requirements for the family of curves $K(\gamma, z)$ and the plane D necessary to bring problem (2) to problem (3), (4) are met. Let us also assume that any straight line that is parallel to the axis of the abscissa or ordinate can intersect the boundary of the domain D at no more than two points.

Let

$$a_1 = \inf_{(x,y)\in D} \{x\}, \ b_1 = \sup_{(x,y)\in D} \{x\},$$

$$a_2 = \inf_{(x,y)\in D} \{y\}, b_2 = \sup_{(x,y)\in D} \{y\},$$

$$h_j = (b_j - a_j)/N_j, \ j = 1,2; h_3 = l/N_3,$$

where $N_j, j = 1, 2, 3,$ are natural numbers.

Let ε satisfy the condition

$$0 < \varepsilon < \min \{(b_1 - a_1)/3, (b_2 - a_2)/3\},$$

$$D^\varepsilon = \left\{(x,y) \in D : \min_{(\alpha,\beta)\in} \rho((x,y),(\alpha,\beta)) > \varepsilon\right\},$$

$$R_h = \{(x_i, y_j), \ x_i = a_1 + ih_1, y_j = a_2 + jh_2, i = 0, 1, ..., N_1; j = 0, 1, ..., N_2\}.$$

The neighborhood $\aleph(ih_1, jh_2)$ of the point $(a_1 + ih_1, a_2 + jh_2)$ will be called a set consisting of the point itself $(a_1 + ih_1, a_2 + jh_2)$ and four points of the form $(a_1 + (i \pm 1)h_1, a_2 + (j \pm 1)h_2)$.

D_h^ε is a set of all points $(a_1 + ih_1, a_2 + jh_2)$ lying in $D^\varepsilon \cap R_h$ together with its neighborhood $\aleph(ih_1, jh_2)$.

Γ_h^ε - the set of all points $(a_1 + ih_1, a_2 + jh_2) \in D_h^\varepsilon$ such that the intersection (ih_1, jh_2) with the set $(D^\varepsilon \cap R_h)/D_h^\varepsilon$ is nonempty. Then,

$$\Delta_h^\varepsilon = \bigcup_{\substack{\varepsilon \\ h}} \aleph(ih_1, ih_2), \quad D_h = R_h \cap D.$$

Futher, we assume that the coefficients and a solution of problem (3)–(4) have the following properties:

$$W(x,y,z) \in C^3(\Omega^\varepsilon), \theta(x,y,z) \in C^2(\Omega^\varepsilon), \ \Omega^\varepsilon = \overline{D}^\varepsilon \times [0,l],$$

$$\rho(x,y,z) \in C^2(\Omega), \rho(x,y,z) > C^* > 0, \frac{\partial \theta}{\partial z} > \left| \frac{\partial \rho}{\partial z} \cdot \frac{1}{\rho} \right|.$$

We consider the following difference problem (depending on the parameter z): Find a function $\Phi_{i,j}(z)$, that satisfies the equation

$$\Phi_{\overset{0}{x}} \tfrac{A}{C} + \Phi_{\overset{0}{y}} \tfrac{B}{C} = U_{i,j}, \quad (a_1 + ih_1, a_2 + ih_2) \in D_h, \quad z \in [0,l] \tag{5}$$

and the boundary condition

$$\Phi_{i,j}(z) = F_{i,j}(z), \quad (a_1 + ih_1, a_2 + jh_2) \in \Delta_h^\varepsilon, \quad z \in [0,l], \tag{6}$$

here

$$\Phi_{i,j}(z) = (x_i, y_j, z) = (a_1 + ih_1, a_2 + jh_2, z),$$

$$U_{i,j} = U(x_i, y_j) = U(a_1 + ih_1, \quad a_2 + jh_2), \quad i = \overline{0, N_1}, \quad j = \overline{0, N_2},$$

$$\Phi_{\overset{0}{x}} = (F_{i+1,j} - F_{i-1,j})/2h_1, \quad \Phi_{\overset{0}{y}} = (F_{i,j+1} - F_{i,j-1})/2h_2,$$

$$A = \cos \theta_{i,j}(z), \quad B = \sin \theta_{i,j}(z), \quad \theta_{i,j}(z) = \theta(a_1 + ih_1, a_2 + jh_2, z), \quad C = \rho(a_1 + ih_1, a_2 + jh_2, z).$$

Note that in this formulation, information about the solution is given not only on the boundary Γ but also in some its ε neighborhood, which is due to the presence of type features $\left[(x - \xi(z))^2 + (y - \eta(z))^2\right]^{-\frac{1}{2}}$ in derivatives $\theta_z, W_{xz}, W_{yz}, W_{xy}$ in the neighborhood of any point of the form $(\xi(z), \eta(z), z)$ [7].

Results and Discussion

Theorem. Suppose that the solution to problem (5)-(6) exists. Let for all the $(x_i, y_j) \in D_h$ functions

$$\Phi_{i,j}(z) \in C^1[0,l], \quad \Phi_{i,j}(0) = \Phi_{i,j}(l),$$

$$F_{i,j}(z) \in C^1[0,l], \quad F_{i,j}(0) = F_{i,j}(l),$$

and the functions $C = \rho(a_1 + ih_1, a_2 + jh_2, z)$. $\theta_{i,j}(z)$ satisfy the conditions

$$\theta_{i,j}(0) = \theta_{i,j}(l), \frac{\partial\theta}{\partial z} > \left|\frac{\partial\rho}{\partial z} \cdot \frac{1}{\rho}\right|.$$

Then for all $N_j > 9, \ j = 1, 2$ there is an estimate

$$\sum_{D_h^\varepsilon} (\Phi_{\overset{0}{x}}^2 + \Phi_{\overset{0}{y}}^2) h_1 h_2 \le c_3 \int\limits_0^l \sum_{\Delta_h^\varepsilon} \left[F_{\overset{0}{x}}^2 h_1 + F_{\overset{0}{y}}^2 h_2 + \left(\frac{\partial F}{\partial z}\right)^2 (h_1 + h_2) \right] dz, \tag{7}$$

where c_3 is some positive constant that depends on the function $\rho(x, y, z)$ and the curves family $K(\gamma, z)$.

In estimation (7), it is assumed that with a decrease of h_1 and h_2, the parameter ε can also decrease, since c_3 does not depend on ε (the parameter ε was chosen solely to eliminate features that are present in the original continuous problem). Consequently, the smaller the grid is, the narrower the domain may be in which the feature is concentrated.

Proof. Using the methodology proposed in the papers [8], [9] both parts (5) are multiplied by $2C(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}})\frac{\partial}{\partial z}$, the resulting equality is written in the form

$$J_1 + J_2 = 0. \tag{8}$$

Here

$$J_1 = J_2 = C(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}})\frac{\partial}{\partial z}(\Phi_{\overset{0}{x}}\frac{A}{C} + \Phi_{\overset{0}{y}}\frac{B}{C}).$$

Using the differentiation formula of the product of functions, we transform J_1:

$$\begin{gathered} J_1 = \tfrac{\partial}{\partial z}\left[\left(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}}\right)\left(A\Phi_{\overset{0}{x}} + B\Phi_{\overset{0}{y}}\right)\right] + \\ + AB\tfrac{1}{C}\tfrac{\partial C}{\partial z}\Phi_{\overset{0}{x}}{}^2 - \tfrac{1}{C}\tfrac{\partial C}{\partial z}A^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + \\ + \tfrac{1}{C}\tfrac{\partial C}{\partial z}B^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - \tfrac{1}{C}\tfrac{\partial C}{\partial z}AB\Phi_{\overset{0}{y}}{}^2 + \tfrac{\partial\theta}{\partial z}A^2\Phi_{\overset{0}{x}}{}^2 + \\ + AB\Phi_{\overset{0}{x}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) + \tfrac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - A^2\Phi_{\overset{0}{x}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \\ + \tfrac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + B^2\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)\Phi_{\overset{0}{y}} + \\ + \tfrac{\partial\theta}{\partial z}B^2\Phi_{\overset{0}{y}}{}^2 - AB\Phi_{\overset{0}{y}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right). \end{gathered} \tag{9}$$

Opening the brackets in J_2, we have

$$\begin{gathered} J_2 = -AB\Phi_{\overset{0}{x}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \tfrac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + \tfrac{1}{C}\tfrac{\partial C}{\partial z}B^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - \\ - B^2\Phi_{\overset{0}{x}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \tfrac{\partial\theta}{\partial z}B^2\Phi_{\overset{0}{x}} + \tfrac{1}{C}\tfrac{\partial C}{\partial z}AB\Phi_{\overset{0}{x}}{}^2 + \\ + A^2\Phi_{\overset{0}{y}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \tfrac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - \tfrac{1}{C}\tfrac{\partial C}{\partial z}A^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + \\ + AB\Phi_{\overset{0}{y}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \tfrac{\partial\theta}{\partial z}A^2\Phi_{\overset{0}{y}}{}^2 - \tfrac{1}{C}\tfrac{\partial C}{\partial z}AB\Phi_{\overset{0}{y}}{}^2. \end{gathered} \tag{10}$$

Substituting these expressions J_1, J_2 into (8) and denoting $D = \sin 2\theta = 2\sin\theta\cos\theta = 2AB$, $E = \cos 2\theta =$ $= \cos^2\theta - \sin^2\theta = A^2 - B^2$, from (9), (10) we get

$$\begin{gathered} \left(\tfrac{\partial\theta}{\partial z} + \tfrac{1}{C}\tfrac{\partial C}{\partial z}D\right)\Phi_{\overset{0}{x}}{}^2 - 2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}\tfrac{1}{C}\tfrac{\partial C}{\partial z}E + \left(\tfrac{\partial\theta}{\partial z} - \tfrac{1}{C}\tfrac{\partial C}{\partial z}D\right)\Phi_{\overset{0}{y}}{}^2 + \\ + \Phi_{\overset{0}{y}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \Phi_{\overset{0}{x}}\tfrac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \tfrac{\partial}{\partial z}\left[\left(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}}\right)\left(A\Phi_{\overset{0}{x}} + B\Phi_{\overset{0}{y}}\right)\right] = 0. \end{gathered} \tag{11}$$

It is not difficult to notice that

$$\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)=\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{x}},\qquad \frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right)=\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{y}},$$

$$(uv)_{\overset{0}{x}}=u_{\overset{0}{x}}v+uv_{\overset{0}{x}}+\frac{h_1^2}{2}[u_x v_x]_{\bar{x}},$$

where

$$f_x=\tfrac{f_{i+1}-f_i}{h_1},\quad f_{\bar{x}}=\tfrac{f_i-f_{i-1}}{h_1}.$$

Then,

$$\begin{gathered}\Phi_{\overset{0}{y}}\left(\tfrac{\partial\Phi}{\partial z}\right)_{\overset{0}{x}}-\Phi_{\overset{0}{x}}\left(\tfrac{\partial\Phi}{\partial z}\right)_{\overset{0}{y}}=\left[\Phi_{\overset{0}{y}}\left(\tfrac{\partial\Phi}{\partial z}\right)\right]_{\overset{0}{x}}-\left[\Phi_{\overset{0}{x}}\left(\tfrac{\partial\Phi}{\partial z}\right)\right]_{\overset{0}{y}}-\\ -\tfrac{h_1^2}{2}\left[\Phi_{\overset{0}{y}x}\left(\tfrac{\partial\Phi}{\partial z}\right)_x\right]_{\bar{x}}+\tfrac{h_1^2}{2}\left[\Phi_{\overset{0}{x}y}\left(\tfrac{\partial\Phi}{\partial z}\right)_y\right]_{\bar{y}},\end{gathered}$$

from (11) we obtain

$$\begin{gathered}J_3+\tfrac{\partial}{\partial z}\left[\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\left(A\Phi_{\overset{0}{x}}+B\Phi_{\overset{0}{y}}\right)\right]+\left[\Phi_{\overset{0}{y}}\tfrac{\partial\Phi}{\partial z}\right]_{\overset{0}{x}}-\left[\Phi_{\overset{0}{x}}\tfrac{\partial\Phi}{\partial z}\right]_{\overset{0}{y}}-\\ -\tfrac{h_1^2}{2}\left[\Phi_{\overset{0}{y}x}\left(\tfrac{\partial\Phi}{\partial z}\right)_x\right]_{\bar{x}}+\tfrac{h_2^2}{2}\left[\Phi_{\overset{0}{x}y}\left(\tfrac{\partial\Phi}{\partial z}\right)_y\right]_{\bar{y}}=0,\end{gathered}\tag{12}$$

where

$$J_3=\left(\frac{\partial\theta}{\partial z}+\frac{1}{C}\frac{\partial C}{\partial z}D\right)\Phi_{\overset{0}{x}}^{\;2}-2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}\frac{1}{C}\frac{\partial C}{\partial z}E+\left(\frac{\partial\theta}{\partial z}-\frac{1}{C}\frac{\partial C}{\partial z}D\right)\Phi_{\overset{0}{y}}^{\;2}.$$

Considering the expression J_3 as a quadratic form with respect to $\Phi_{\overset{0}{x}}$ and $\Phi_{\overset{0}{y}}$, it is not difficult to ensure that the determinant of this quadratic form is

$$\left(\frac{\partial\theta}{\partial z}\right)^2-\left(\frac{1}{C}\frac{\partial C}{\partial z}\right)^2.$$

Then from the condition

$$\frac{\partial\theta}{\partial z}>\left|\frac{1}{C}\frac{\partial C}{\partial z}\right|$$

the positive definiteness of the quadratic form J_3 follows.

Using the inequality

$$ax^2+2bxy+cy^2\geq\frac{2\left(ac-b^2\right)}{a+c+\sqrt{\left(a-c\right)^2+4b^2}}(x^2+y^2),$$

which is true for a positive-definite quadratic form $ax^2+2bxy+cy^2$, we have

$$J_3\geq\left(\frac{\partial\theta}{\partial z}-\left|\frac{1}{C}\frac{\partial C}{\partial z}\right|\right)\left(\Phi_{\overset{0}{x}}^{\;2}+\Phi_{\overset{0}{y}}^{\;2}\right).\tag{13}$$

Considering that

$$C=\rho(x,y,z),\qquad \rho(x,y,z)>{}^{*}>0,\qquad \left(\frac{\partial\theta}{\partial z}-\left|\frac{1}{C}\frac{\partial C}{\partial z}\right|\right)>0,\tag{14}$$

it is not difficult to make sure that there is such $c_2>0$ and there is an inequality

$$\int\limits_0^l\left(\frac{\partial\theta}{\partial z}-\left|\frac{1}{C}\frac{\partial C}{\partial z}\right|\right)C^2dz\geq\frac{1}{c_2^2}>0.$$

Further, summing over i, j using condition (6) and integrating over z, taking into account the formulas (13), (14) as well as the periodicity of functions $\Phi_{i,j}(z), \theta_{i,j}(z)$ over z and inequality $|ab| \leq (a^2+b^2)/2$, from equality (12) after simple transformations, we obtain an estimate

$$\sum_{D_h^\varepsilon}(\Phi^2_{\overset{0}{x}} + \Phi^2_{\overset{0}{y}})h_1h_2 \leq c_3 \int_0^l \sum_{\Delta_h^\varepsilon}\left[F^2_{\overset{0}{x}}h_1 + F^2_{\overset{0}{y}}h_2 + \left(\frac{\partial F}{\partial z}\right)^2(h_1+h_2)\right]dz,$$

in which c_3 depends on the functions $\rho(x, y, z)$ and the curves family $K(\gamma, z)$. So, the theorem is proved.

Conclusions

The stability estimate of the difference analogue of the boundary value problem for a mixed-type equation obtained in the work can be used to justify the convergence of numerical methods for solving problems of geotomography, medical tomography, flaw detection and is of great practical significance in solving multidimensional inverse problems of acoustics, seismic exploration.

References

1 Lavrentev M.M. Ill-posed problems of mathematical physics and analysis / M.M. Lavrent'ev, V.G. Romanov, S.P. Shishatskii // Translations of Mathematical Monographs. American Mathematical Society, Providence, R.I. — 1986. — Vol. 64.

2 Tikhonov A.N. Solutions of Ill-posed Problems / A.N. Tikhonov, V.Ya. Arsenin // V.H. Winston & Sons, a Division of Scripta Technica, Inc. — 1977.

3 Ivanov V.K. Theory of linear ill-posed problems and its applications / V.K. Ivanov, V.V. Vasin, V.P. Tanana // Berlin, Boston: De Gruyter. — 2013.

4 Лаврентьев М.М. О трех линеаризированных обратных задачах для гиперболических уравнений / М.М. Лаврентьев, В.Г. Романов // Докл. АН СССР. — 1966. — 171. — № 6. — С. 1279–1281.

5 Романов В.Г. О некоторых классах единственности решения задач интегральной геометрии / В.Г. Романов // Математические заметки. — 1974. — 16. — № 4. — С. 657–668.

6 Романов В.Г. О восстановлении функции через интегралы по семейству кривых / В.Г. Романов // Сиб. мат. журн. — 1967. — 8. — № 5. – С. 1206–1208.

7 Мухометов Р.Г. О задаче интегральной геометрии / Р.Г. Мухометов // Математические проблемы геофизики. — Новосибирск, 1975. — 6. — С. 212–252.

8 Кабанихин С.И. Об устойчивости конечно-разностного аналога двумерной задачи интегральной геометрии / С.И. Кабанихин, Г.Б. Баканов // Докл. АН СССР. — 1987. — 292. — № 1. — С. 25–29.

9 Bakanov G. On the stability of a differential-difference analogue of a two-dimensional problem of integral geometry / G. Bakanov // Filomat. — 2018. — 32(3). — P. 933–938.

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Аралас типті теңдеу үшін шекаралық есептің айырымдық аналогының орнықтылығы жайлы

Мақалада кейбір аралас типті теңдеу үшін айырымдық есепке келтірілетін регулярлық шарттарын қанағаттандыратын қисықтар үйірі үшін қойылған интегралдық геометрия есебі қарастырылды. Бұл есептер көптеген қосымшалармен байланысты, оның ішінде сейсмобарлау мәліметтерін интерпретациялау есептері, рентген суреттерін интерпретациялау, компьютерлік томография және техникалық диагностика есептері. Интегралдық геометрия есептерінің айырымдық аналогтарын зерттеудің өзіне тән күрделі тұстары бар, дербес туындылардың шектеулі-айырымдық аналогтары үшін негізгі қатынастар дискретті айнымалы бойынша белгілі бір ығысумен жүргізілуіне байланысты болады. Сондықтан, үзіліссіз қойылымда алынатын көптеген қатынастар дискретті аналогқа ауысқанда анағұрлым күрделі түрге ие болады және ығысу барысында туындайтын қосылғыштарға қатысты қосымша зерттеулерді талап етеді. Бұл есептердің тағы да бір ерекшелігі — жалпы жағдайда шешімнің бар болуы жайлы теорема жоқ. Осыған байланысты А.Н. Тихонов бойынша корректілік ұғымы қолданылады, яғни интегралдық геометрия есебі мен оның дифференциалдық-айырымдық аналогының шешімі бар болады деп жорамалданады. Сонымен қатар, аралас типті теңдеу үшін шекаралық есептің айырымдық аналогының алынған орнықтылық бағасы геотомография, медициналық томография, дефектоскопия және т.б. есептерді сандық әдістермен шешудің тиімділігін түсіну үшін өте маңызды. Сондай-ақ, акустика, сейсмобарлаудың көп өлшемді кері есептерін шешуде де үлкен практикалық мәні бар.

Кілт сөздер: корректілі емес есеп, шекаралық есеп, аралас типті теңдеу, орнықтылық бағасы, айырымдық есеп, квадратты форма.

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Об устойчивости разностного аналога граничной задачи для уравнения смешанного типа

В статье рассмотрена разностная задача для уравнения смешанного типа, к которой сводится задача интегральной геометрии для семейства кривых, удовлетворяющих некоторым условиям регулярности. Эти задачи связаны с многочисленными приложениями, в том числе с задачами интерпретации данных сейсморазведки, интерпретации рентгеновских снимков, компьютерной томографии и задачами технической диагностики. Исследование разностных аналогов задач интегральной геометрии имеет специфические трудности, связанные с тем обстоятельством, что для конечно-разностных аналогов частных производных основные соотношения выполняются с некоторым сдвигом по дискретной переменной. В связи с этим многие соотношения, получаемые в непрерывной постановке, при переходе к дискретному аналогу имеют более сложную и громоздкую форму, что требует дополнительных исследований возникающих слагаемых со сдвигом. Еще одной важной особенностью задачи интегральной геометрии является отсутствие теоремы существования решения в общем случае. В связи с этим в работе использовано понятие корректности по А.Н. Тихонову, а именно, предполагается, что решение задачи интегральной геометрии и ее дифференциально-разностного аналога существует. Полученная авторами оценка устойчивости разностного аналога граничной задачи для уравнения смешанного типа имеет важное значение для понимания эффективности численных методов решения задач геотомографии, медицинской томографии, дефектоскопии и т. д. Кроме того, имеет большое практическое значение при решении многомерных обратных задач акустики, сейсморазведки.

Ключевые слова: некорректная задача, краевая задача, уравнение смешанного типа, оценка устойчивости, разностная задача, квадратичная форма.

References

1 Lavrentev, M.M., Romanov, V.G., & Shishatskii, S.P. (1986). Ill-posed problems of mathematical physics and analysis. *Translations of Mathematical Monographs. American Mathematical Society, Providence,* 64.

2 Tikhonov, A.N., & Arsenin, V.Ya. (1977). *Solutions of Ill-posed Problems.* V.H. Winston & Sons, a Division of Scripta Technica, Inc.

3 Ivanov, V.K., Vasin, V.V., & Tanana, V.P. (2013). *Theory of linear ill-posed problems and its applications.* Berlin, Boston: De Gruyter.

4 Lavrentev, M.M., & Romanov, V.G. (1966). O trekh linearizirovannykh obratnykh zadachakh dlia giperbolicheskikh uravnenii [Three-linearized inverse problems for hyperbolic equations]. *Doklady Akademii nauk SSSR – Reports of the USSR Academy of Sciences, 171*(6), 1279–1281 [in Russian].

5 Romanov, V.G. (1974). O nekotorykh klassakh edinstvennosti resheniia zadach integralnoi geometrii [On some classes of uniqueness of solutions to problems of integral geometry]. *Matematicheskie zametki – Math notes, 16*(4), 657–668 [in Russian].

6 Romanov, V.G. (1967). O vosstanovlenii funktsii cherez integraly po semeistvu krivykh [Reconstruction of a function in terms of integrals over a family of curves]. *Sibirskii matematicheskii zhurnal – Siberian Mathematical Journal, 8*(5), 1206–1208 [in Russian].

7 Mukhometov, R.G. (1975). O zadache integralnoi geometrii [On a problem of inegral geometry]. *Matematicheskie problemy geofiziki – Mathematical Problems of Geophyzics. 6,* 212–252. Novosibirsk [in Russian].

8 Kabanikhin, S.I., & Bakanov, G.B. (1987). Ob ustoichivosti konechno-raznostnogo analoga dvumernoi zadachi integralnoi geometrii [On the stability of a finite-difference analogue of a two-dimensional problem of integral geometry]. *Doklady Akademii nauk SSSR – Reports of the USSR Academy of Sciences, 292*(1), 25–29 [in Russian].

9 Bakanov, G. (2018). On the stability of a differential-difference analogue of a two-dimensional problem of integral geometry. *Filomat, 32*(3), 933–938.