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On the boundedness of the partial sums operator for the Fourier series in the function classes families associated with harmonic intervals

The article is devoted to the study of some data from the theory of functions approximation by trigonometric polynomials with a spectrum from special sets called harmonic intervals. Due to the limited perception range of devices, the perception range of the senses of the person himself, when studying a mathematical model it is often enough to find an approximation of the object so that the error (noise, interference, distortion) is outside the interval of perception. Harmonic intervals model problems of this kind to some extent. In the article the main components of the approximation theory of functions by trigonometric polynomials with a spectrum from harmonic intervals are presented, the theorem on estimating the best approximation of a function by trigonometric polynomials through the best approximations of a function by trigonometric polynomials with a spectrum from harmonic intervals is proved. Theorems on the boundedness of the partial sums operator for the Fourier series in the function classes families associated with harmonic intervals are considered; such a theorem for the Lorentz space is generalized and proved. The article is mainly aimed at scientific researchers dealing with practical applications of the approximation theory of functions by trigonometric polynomials with a spectrum from special sets.

Keywords: harmonic interval, trigonometric polynomials with a spectrum from harmonic intervals, best approximation of a function by trigonometric polynomials, partial sums operator of the Fourier series for a given function, interpolation theorem.

Introduction

In approximation theory one of the most relevant problems is the approximation of periodic functions by polynomials with a spectrum from special families of sets. Here we note the works of K.I. Babenko, S.A. Telyakovsky, V.N. Temlyakov [1] and others in the case when the spectrum is a hyperbolic cross; the works of V.I. Yudin, M.I. Dyachenko [2] in the case when the spectrum is a ball, etc.

In the study of many applied problems the question of approximating the mathematical model of the object under study naturally arises. However, due to the limited range of perception («window of perception») of devices, the range of perception of the human senses, when studying a mathematical model it is often enough to find an approximation of the object so that the error (noise, interference, distortion) is outside the interval («window») of perception.

In this paper we consider approximations of functions by trigonometric polynomials with a spectrum from harmonic intervals, which to some extent model problems of this kind.

Note that harmonic intervals are some fractal self-similar sets, the concept of which was introduced by E.D. Nursultanov in [3–5] and, as it turned out, harmonic intervals have an important role in harmonic analysis. Thus, in the works of N.T. Tleukhanova, K.S. Saydakhmetov, D.S. Karimov, such objects as harmonic segments and harmonic intervals were essentially used.

In the studying the problem of the boundedness of the partial sums operator for the Fourier series in the function classes families associated with the best approximations over harmonic intervals the method of real interpolation is used. Among the works devoted to the properties of interpolation spaces, as well as to the methods of interpolation, one should note the works of Y. Berg and J. Lefstrom [6], S.G. Krein, Yu.I. Petunin, E.M. Semenov, Yu.A. Brudny [7], [8], H. Tribel [9], [10].

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Definitions and auxiliary results

Let $k, \nu, N \in \mathbb{N}$, $k < N$. A set of the form

$$I_k^N = \bigcup_{\nu=-\infty}^{\infty} ([-k,k] + 2\nu N) = \bigcup_{\nu=-\infty}^{\infty} (m + 2\nu N : m \in [-k,k])$$

is called a harmonic interval in $\mathbb{Z}$.

We denote by T_k^N the set of trigonometric polynomials of the form

$$T_k^N = \left\{ \sum_{\nu=-s}^{s} a_\nu \cdot e^{i\nu x} : a_\nu = 0 \;\; if \;\; \nu \notin I_k^N, s \in \mathbb{N} \right\}.$$

The value

$$E_k^N(f)_p = \inf_{t \in T_k^N} \|f - t\|_p$$

is called the best approximation over the harmonic interval I_k^N of the function $f \in L_p[0, 2\pi)$, $1 \le p \le \infty$, by trigonometric polynomials from T_k^N of order less than or equal to k.

Let $f \in L_p[0, 2\pi)$, $1 \le p \le \infty$. The partial sum of the Fourier series for the function f over the harmonic interval I_k^N is called the function

$$S_k^N(f) = \sum_{\nu \in I_k^N} a_\nu \cdot e^{i\nu x}.$$

Theorem 1. [11] Let $f \in L_p[0, 2\pi)$, $1 < p < \infty$, $m \in \mathbb{N}$. $S_m^N(f)$ and $E_m^N(f)$ are the partial sum of the Fourier series and the best approximation of the function f over the harmonic interval I_m^N respectively, then we have the following relation

$$E_m^N(f)_p \sim \|f - S_m^N(f)\|_p.$$

Lemma 1. [11] Let $n \in \mathbb{N}$, $1 \le p < q \le \infty$, $1 \le r \le \infty$, then

$$\|T_n\|_{L_{q,r}} \le C n^{\frac{1}{p} - \frac{1}{q}} \|T_n\|_{L_p}. \tag{1}$$

Let $1 \le p$, $q \le \infty$, $r > 0$, $f \in L_p[0, 2\pi)$. The family of function classes $\left\{B_{p,q.N}^r\right\}_N$ is defined by the equality

$$B_{p,q.N}^r = \left\{ f : \|f\|_{B_{p,q.N}^r} < \infty \right\}, \quad N \in \mathbb{N},$$

where

$$\|f\|_{B_{p,q.N}^r} = \left(\sum_{k=1}^{N} k^{rq-1} \left(E_{k-1}^N(f)_p \right)^q \right)^{\frac{1}{q}}.$$

Let two families of function classes $\left\{A^N\right\}_N$ and $\left\{B^N\right\}_N$, $N \in \mathbb{N}$, be given. We assume that the ratio

$$\|f\|_{A^N} \sim \|f\|_{B^N}$$

holds if there are parameters C_1, C_2 such that for any $f \in A^N$ the following inequality

$$C_1 \|f\|_{B^N} \le \|f\|_{A^N} \le C_2 \|f\|_{B^N}$$

is valid, moreover, the parameters C_1, C_2 do not depend on f and N.

Theorem 2. [12] Let $f \in B_{p,q.2^m}^r$, $m \in \mathbb{N}$. Then for $1 \le p$, $q \le \infty$, $r > 0$ we have

$$\|f\|_{B_{p,q.2^m}^r} \sim \left(\sum_{k=1}^{m} 2^{rqk} \left(E_{2^k-1}^{2^m}(f)_p \right)^q \right)^{\frac{1}{q}}.$$

Theorem 3. [12] Let $m \in \mathbb{N}$, $1 \le p, p_0, p_1 \le \infty$, $0 < \theta < 1$, $r_0 > 0$, $r_1 > 0$, $r_0 \ne r_1$, $\frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}$, $r = (1-\theta) \cdot r_0 + \theta \cdot r_1$, then

$$\left(B_{p_0,p_0,2^m}^{r_0}; B_{p_1,p_1,2^m}^{r_1} \right)_{\theta,p} = B_{p,p,2^m}^r.$$

Estimation by the best approximations over harmonic intervals

Theorem 4. Let $f \in L_p[0,2\pi)$, $1 < p < \infty$, $n \in \mathbb{N}$. $\sum\limits_{\nu\in\mathbb{Z}} a_\nu \cdot e^{i\nu x}$ is the trigonometric Fourier series of the function f, then the following inequality holds

$$E_n(f)_p \leq \sum_{j=1}^{\infty} E^{2^j\cdot n}_{(2^j-1)\cdot n}(f)_p.$$

Proof. By Lemma 9.3 [13] we have

$$E_n(f)_p \sim \|f - S_n(f)\|_p,$$

when $1 < p < \infty$ or

$$E_n(f)_p \sim \left\| \sum_{\nu\in Z\setminus[-n,n]} a_\nu \cdot e^{i\nu x} \right\|. \tag{2}$$

By entering the notation of harmonic intervals in $\mathbb{Z}$

$$V_j = \bigcup_{m=-\infty}^{\infty} \left\{ \left[\left(2^j - 1\right) n; \left(2^j + 1\right) n \right] + 2^{j+1} mn \right\} =$$

$$= \bigcup_{m=-\infty}^{\infty} \left\{ [-n; n] + 2^j n \,(2m+1) \right\}, \quad j = 1, 2, ...,$$

we obtain

$$\mathbb{Z} \setminus [-n; n] = \bigcup_{j=1}^{\infty} V_j.$$

Then from (2) we get the relation in this form

$$E_n(f)_p \sim \left\| \sum_{j=1}^{\infty} \sum_{\nu\in V_j} a_\nu \cdot e^{i\nu x} \right\|_p = \left\| \sum_{j=1}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{s=n[2^j(2m+1)-1]}^{n[2^j(2m+1)+1]} a_s \cdot e^{isx} \right\|_p. \tag{3}$$

We denote by W_j, $j = 1, 2, ...$, the following sets

$$W_j = \mathbb{Z} \setminus V_j, \tag{4}$$

where

$$W_j = \bigcup_{m=-\infty}^{\infty} \left\{ \left[-\left(2^j - 1\right) n; \left(2^j - 1\right) n \right] + 2^{j+1} mn \right\}$$

or

$$W_j = I^{2^j n}_{(2^j-1)n}.$$

We note that the sets W_j, $j = 1, 2, ...$ are also harmonic intervals in $\mathbb{Z}$ as complements of the harmonic intervals V_j, $j = 1, 2, ...$ in $\mathbb{Z}$. Then, according to Theorem 1, using (4), from (3) we obtain the required inequality

$$E_n(f)_p \sim \left\| \sum_{j=1}^{\infty} \sum_{\nu\in\mathbb{Z}\setminus W_j} a_\nu \cdot e^{i\nu x} \right\|_p = \left\| \sum_{j=1}^{\infty} \left(f - S_{W_j}(f)\right) \right\|_p \leq$$

$$\leq \sum_{j=1}^{\infty} \left\| \left(f - S_{W_j}(f)\right) \right\|_p \sim \sum_{j=1}^{\infty} E_{W_j}(f)_p,$$

$$E_n(f)_p \leq \sum_{j=1}^{\infty} E^{2^j\cdot n}_{(2^j-1)\cdot n}(f)_p.$$

The theorem is proved.

Theorems on the boundedness of the partial sums operator for the Fourier series of a function f in the function classes families $\left\{B^{r}_{p,q.N}\right\}_N$

Theorem 5. [11] Let $N \in \mathbb{N}$, $1 \le p < q \le \infty$, $1 \le r \le \infty$, $\beta > 0$, $\alpha - \beta = \frac{1}{p} - \frac{1}{q}$. $B^{\beta}_{q,r}$ is the Besov space [14], then the partial sums operator for the trigonometric Fourier series of the function f

$$S_N\left(f(x)\right) = \sum_{k=-N}^{N} \widehat{f}(k)e^{ikx}$$

such that

$$S_N \,:\, B^{\alpha}_{p,r,N} \to B^{\beta}_{q,r}$$

is bounded, that is, there is the inequality

$$\|S_N(f)\|_{B^{\beta}_{q,r}} \le C\,\|f\|_{B^{\alpha}_{p,r,N}},$$

where the parameter C do not depend on f and N.

Corollary 1. [11] Let $N \in \mathbb{N}$, $1 \le p < q \le \infty$, $1 \le r \le \infty$, $\beta > 0$, $\alpha - \beta = \frac{1}{p} - \frac{1}{q}$, then the partial sums operator for the trigonometric Fourier series of the function f

$$S_N \,:\, B^{\alpha}_{p,r,N} \to B^{\beta}_{q,r,N}$$

is bounded, that is, the following inequality

$$\|S_N(f)\|_{B^{\beta}_{q,r,N}} \le C\,\|f\|_{B^{\alpha}_{p,r,N}},$$

where the parameter C do not depend on f and N.

Theorem 6. [11] Let $m \in \mathbb{N}$, $1 \le p < q \le \infty$, $\alpha = \frac{1}{p} - \frac{1}{q}$, then the partial sums operator for the trigonometric Fourier series of the function f

$$S_{2^m} \,:\, B^{\alpha}_{p,q,2^m} \to L_q$$

is bounded, that is, we have the following inequality

$$\|S_{2^m}(f)\|_{L_q} \le C\,\|f\|_{B^{\alpha}_{p,q,2^m}},$$

where the parameter C do not depend on f and m.

Remark 1. Theorem 6 can be formulated in a more general form.

Let $N \in \mathbb{N}$, $1 \le p < q \le \infty$, $\alpha = \frac{1}{p} - \frac{1}{q}$, then the partial sums operator for the trigonometric Fourier series of the function f

$$S_N \,:\, B^{\alpha}_{p,q,N} \to L_q$$

is bounded, that is, there is the inequality of the form

$$\|S_N(f)\|_{L_q} \le C\,\|f\|_{B^{\alpha}_{p,q,N}},$$

where the parameter C do not depend on f and N.

We generalize Theorem 6 to Lorentz spaces.

Theorem 7. Let $N \in \mathbb{N}$, $1 \le p < q \le \infty$, $1 \le r \le \infty$, $\alpha = \frac{1}{p} - \frac{1}{q}$, then the partial sums operator for the trigonometric Fourier series of the function f

$$S_N \,:\, B^{\alpha}_{p,r,N} \to L_{q,r}$$

is bounded, that is, this inequality holds

$$\|S_N(f)\|_{L_{q,r}} \le C\,\|f\|_{B^{\alpha}_{p,r,N}},$$

where the parameter C do not depend on f and N.

Proof. We estimate the norm of the partial sum operator in the Lorentz space

$$\|S_N(f)\|_{L_{q,r}} \leq \sum_{k=1}^{[\log_2 N]} \left\| \sum_{n=-2^{k-1}}^{2^k-1} a_n e^{inx} \right\|_{L_{q,r}} = \sum_{k=1}^{[\log_2 N]} \|\triangle_k \left(S_N(f)\right)\|_{L_{q,r}}. \tag{5}$$

Applying the inequality of different metrics (1), we transform the relation (5) as follows

$$\|S_N(f)\|_{L_{q,r}} \leq C \sum_{k=1}^{[\log_2 N]} 2^{k\left(\frac{1}{p}-\frac{1}{q}\right)} \|\triangle_k \left(S_N(f)\right)\|_{L_p} = C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} \|\triangle_k \left(S_N(f)\right)\|_{L_p}. \tag{6}$$

Taking into account that $\triangle_k \left(S_N(f)\right)$ is a partial sum of the function $\sum\limits_{n\in\mathbb{Z}\setminus I^N_{2^{k-1}-1}} a_n e^{inx}$ and using the M. Riesz theorem [15], Theorem 1 and Theorem 2, we reduce relation (6) to the form

$$\|S_N(f)\|_{L_{q,r}} \leq C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} \|\triangle_k \left(S_N(f)\right)\|_{L_p} \leq$$

$$\leq C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} \left\| \sum_{n\in\mathbb{Z}\setminus I^N_{2^{k-1}-1}} a_n e^{inx} \right\|_{L_p} = C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} \left\| f - S^N_{2^{k-1}-1}(f) \right\|_{L_p} \leq$$

$$\leq C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} E^N_{2^{k-1}-1}(f)_p = C \cdot 2^{\alpha} \sum_{k=1}^{[\log_2 N]} 2^{\alpha(k-1)} E^N_{2^{k-1}-1}(f)_p \leq$$

$$\leq C \sum_{k=1}^{[\log_2 N]} 2^{\alpha k} E^N_{2^k-1}(f)_p \sim C\|f\|_{B^{\alpha}_{p,1,N}}.$$

$$\Rightarrow \ \|S_N(f)\|_{L_{q,r}} \leq C\|f\|_{B^{\alpha}_{p,1,N}}. \tag{7}$$

We take pairs (α_0, α_1), (q_0, q_1), (r_0, r_1) that satisfy the following conditions

$$\alpha_0 < \alpha < \alpha_1, \ \ q_0 < q < q_1, \ \ r_0 < r < r_1,$$

$$\alpha_0 = \frac{1}{p} - \frac{1}{q_0}, \ \ \alpha_1 = \frac{1}{p} - \frac{1}{q_1}.$$

Taking into account the relation (7), we obtain the following

$$S_N \ : \ B^{\alpha_0}_{p,1,N} \to L_{q_0,r_0},$$

$$S_N \ : \ B^{\alpha_1}_{p,1,N} \to L_{q_1,r_1}$$

then, by the interpolation theorem [6], we have

$$S_N \ : \ \left(B^{\alpha_0}_{p,1,N}; B^{\alpha_1}_{p,1,N}\right)_{\theta,r} \to \left(L_{q_0,r_0}; L_{q_1,r_1}\right)_{\theta,r}. \tag{8}$$

Using Theorem 3, we receive that this relation holds

$$\left(B^{\alpha_0}_{p,1,N}; B^{\alpha_1}_{p,1,N}\right)_{\theta,r} = B^{\alpha_\theta}_{p,r,N},$$

where

$$\alpha_\theta = (1-\theta)\cdot\alpha_0 + \theta\cdot\alpha_1, \ \ \frac{1}{r} = \frac{1-\theta}{r_0} + \frac{\theta}{r_1}, \ \ 0 < \theta < 1.$$

It follows from the theorem on the interpolation of Lorentz spaces [6] that

$$\left(L_{q_0,r_0}; L_{q_1,r_1}\right)_{\theta,r} = L_{q_\theta,r},$$

where

$$\frac{1}{q_\theta} = \frac{1-\theta}{q_0} + \frac{\theta}{q_1}, \;\; \frac{1}{r} = \frac{1-\theta}{r_0} + \frac{\theta}{r_1}.$$

Since there is a dependency

$$\alpha_\theta = (1-\theta)\cdot\alpha_0 + \theta\cdot\alpha_1 = (1-\theta)\left(\frac{1}{p} - \frac{1}{q_0}\right) + \theta\left(\frac{1}{p} - \frac{1}{q_1}\right) = \frac{1}{p} - \frac{1}{q_\theta},$$

then there is $\theta \in (0;1)$ such that

$$\alpha_\theta = \alpha, \;\; q_\theta = q.$$

As a result, from (8) we obtain the required relation

$$S_N \;:\; B^{\alpha}_{p,r,N} \;\to\; L_{q,r},$$

and

$$\|S_N(f)\|_{L_{q,r}} \leq \; C\,\|f\|_{B^{\alpha}_{p,r,N}}\,.$$

where the parameter C do not depend on f and N.

The theorem is proved.

Remark 2. In Theorems 5, 7 and Remark 1, the operator $S_N(f)$ can be replaced by the operator $S_n(f)$, where $0 \leq n \leq N$. Indeed, from M. Riesz's theorem we have

$$\|S_n(f)\|_{L_p} \leq C\,\|S_N(f)\|_{L_p}\,,$$

where the parameter C do not depend on f and N.

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Гармоникалық интервалдармен байланысты функциялар кластары үйіріндегі Фурье қатарының дербес қосындылары операторының шенелгендігі туралы

Мақала гармоникалық интервалдар деп аталатын арнайы жиынтықтар спектрі бар тригонометрикалық полиномдар функцияларын жуықтау теориясының кейбір деректерін зерттеуге арналған. Математикалық модельді зерттеу кезінде құрылғылардың қабылдау ауқымы, адамның сезім мүшелерінің қабылдау ауқымы шектеулі болғандықтан қателік (шу, кедергі, бұрмалау) қабылдау интервалынан тыс болатындай етіп қажетті объектінің жуықтамасын табу көбінесе жеткілікті болады. Гармоникалық интервалдар осындай типтегі мәселелерді белгілі бір деңгейде модельдейді. Мақалада гармоникалық интервалдар деп аталатын арнайы жиынтықтар спектрі бар тригонометрикалық полиномдар функцияларын жуықтау теориясының негізгі компоненттері келтірілген, гармоникалық интервалдар деп аталатын арнайы жиынтықтар спектрі бар тригонометрикалық полиномдар функциясын ең жақсы жуықтау арқылы тригонометрикалық полиномдар функциясын ең жақсы жуықтауды бағалау туралы теоремасы дәлелденді. Гармоникалық интервалдармен байланысты функциялар кластары үйіріндегі Фурье қатарының дербес қосындылары операторының шенелгендігі туралы теоремалар келтірілген, мұндай теорема Лоренц кеңістігі үшін жалпыландырылған және дәлелденген. Негізінен мақала арнайы жиынтықтар спектрі бар тригонометрикалық полиномдар функцияларын жуықтау теориясының практикалық қолдануымен айналысатын ғылыми зерттеушілерге арналған.

Кілт сөздер: гармоникалық интервал, гармоникалық интервалдар спектрі бар тригонометрикалық полиномдар, тригонометрикалық полиномдар функциясын ең жақсы жуықтау, белгіленген функция үшін Фурье қатарының дербес қосындылары операторы, интерполяциялық теорема.

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Об ограниченности оператора частичных сумм ряда Фурье в семействах классов функций, связанных с гармоническими интервалами

Статья посвящена исследованию некоторых данных теории приближения функций тригонометрическими полиномами со спектром из специальных множеств, называемых гармоническими интервалами. В силу ограниченности диапазона восприятия приборов, диапазона восприятия органов чувств самого человека при исследовании математической модели часто достаточно найти приближение искомого объекта так, чтобы погрешность (шумы, помехи, искажения) оказалась вне интервала восприятия. Гармонические интервалы в некоторой степени моделируют задачи такого рода. В статье представлены основные компоненты теории приближения функций тригонометрическими полиномами со спектром из гармонических интервалов, доказана теорема об оценке наилучшего приближения функции тригонометрическими полиномами через наилучшие приближения функции тригонометрическими полиномами со спектром из гармонических интервалов. Приведены теоремы об ограниченности оператора частичных сумм ряда Фурье в семействах классов функций, связанных с гармоническими интервалами, обобщена и доказана такая теорема для пространства Лоренца. Статья ориентирована, в основном, на научных исследователей, занимающихся практическими приложениями теории приближений функций тригонометрическими полиномами со спектром из специальных множеств.

Ключевые слова: гармонический интервал, тригонометрические полиномы со спектром из гармонических интервалов, наилучшее приближение функции тригонометрическими полиномами, оператор частичных сумм ряда Фурье для заданной функции, интерполяционная теорема.

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