DOI 10.31489/2021M3/96-104
UDC 515.1

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"Dedicated to Professor Filippo CAMMAROTO for his 70th Birthday"

Applications of operations on generalized topological spaces

In this paper, γ_μ-open sets and γ_μ-closed sets in a GTS (X, μ) have been studied, where γ_μ is an operation from μ to $\mathcal{P}(X)$. In general, collection of γ_μ-open sets is smaller than the collection of μ-open sets. The condition under which both are same are also established here. Some properties of such sets have been discussed. Some closure as operators are also defined and their properties are discussed. The relation between similar types of closure operators on the GTS (X, μ) has been established. The condition under which the newly defined closure like operator is a Kuratowski closure operator is given. We have also defined a generalized type of closed sets termed as γ_μ-generalized closed set with the help of this newly defined closure operator and discussed some basic properties of such sets. As an application, we have introduced some weak separation axioms and discussed some of their properties. Finally, we have shown some preservation theorems of such generalized concepts.

Keywords: operation, μ-open set, γ_μ-open set, $\gamma_\mu g$-closed set.

Introduction

In 1979, Kasahara [1] introduced the notion of an operation on a topological space and introduced the concept of an α-closed graph of a function. After then Janković defined [2] the concept of α-closed sets and investigated some properties of functions with α-closed graphs. In 1991 Ogata [3] introduced the notion of γ-open sets to investigate some new separation axioms of a topological space. Recently, Krishnan et al. [4] and Van An et al. [5] investigated the notion of operations on the family of all semi-open sets and pre-open sets.

In this paper our aim is to study an operation based on open like sets, where the operation is defined on a collection of generalized open sets instead of a topology. The family of open sets plays an important role in topology. For this, different open like sets or weakly open sets have been introduced by mathematicians to study different weak forms of continuous functions and covering properties of topological spaces. But the most common properties of these open like sets or weakly open sets are that they are closed under arbitrary union and contain the empty set. Observing these, Császár introduced the concept of generalized open sets. We now recall some notions defined in [6]. Let X be a non-empty set. A subcollection $\mu \subseteq \mathcal{P}(X)$ (where $\mathcal{P}(X)$ denotes the power set of X) is called a generalized topology [6], (briefly, GT) if $\varnothing \in \mu$ and any union of elements of μ belongs to μ. A set X with a GT μ on the set X is called a generalized topological space (briefly, GTS) and is denoted by (X, μ). If for a GTS (X, μ) $X \in \mu$, then (X, μ) is known as a strong GTS. The elements of μ are called μ-open sets and μ-closed sets are their complements. The μ-closure of a set $A \subseteq X$ is denoted by $c_\mu(A)$ and defined as the smallest μ-closed set containing A which is equivalent to the intersection of all μ-closed sets containing A. It is also known from [7, 8] that for a GTS (X, μ), $A \subseteq X$ and $x \in X$, $x \in c_\mu(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in \mu$ containing x. We use the symbol $i_\mu(A)$ to mean the μ-interior of A and it is defined as the union of all μ-open sets contained in A i.e., the largest μ-open set contained in A (see [6, 7]). We observe that $x \in i_\mu(A)$ if and only if there exists some μ-open set U containing x such that $U \subseteq A$ and $A \subseteq X$ is μ-open (resp. μ-closed) if and only if $A = i_\mu(A)$ (resp. $A = c_\mu(A)$). It is well known that i_μ and c_μ both are monotonic and idempotent. For any subset A of a GTS (X, μ), $i_\mu(X \setminus A) = X \setminus c_\mu(A)$ holds.

Császár continued to try to find a more general structure from general topology, generalized topology, and minimal structure. In 2010, he introduced the notion of weak structures [9] and proved that it can replace the

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already defined structures in some cases. A sub-collection $w \subseteqq \mathcal{P}(X)$ is said to be a weak structure on X if and only if it contains the empty set. Its properties have been investigated intensively in [10–13]. In Section 2 we have introduced the concept of a type of generalized open sets termed as γ_μ-open sets, the class of which is smaller than that of generalized open sets, by an operator defined on a GT. We have then studied some properties of such sets in detail. In section 3 we have defined a new type of generalized closed sets and studied some separation properties with the help of the idea developed in Section 2.

γ_μ-open sets and operations

Definition 2.1. [14] Let (X, μ) be a GTS. An operation γ_μ on a generalized topology μ is a mapping from μ to $\mathcal{P}(X)$ (where $\mathcal{P}(X)$ is the power set of X) with $G \subseteqq \gamma_\mu(G)$ for each $G \in \mu$. This operation is denoted by $\gamma_\mu : \mu \to \mathcal{P}(X)$. Note that $\gamma_\mu(A)$ and A^{γ_μ} are two different notation for the same set.

Definition 2.2. [14] Let (X, μ) be a GTS and γ_μ an operation on μ. A subset G of a GTS (X, μ) is called γ_μ-open if for each point x of G, there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq G$.

A subset of a GTS (X, μ) is called γ_μ-closed if its complement is γ_μ-open in (X, μ). We shall use the symbol γ_μ to mean the collection of all γ_μ-open sets of the GTS (X, μ).

Remark 2.3. (a) We observe that every γ_μ-open set is a μ-open set i.e., $\gamma_\mu \subseteqq \mu$. Let $G \in \gamma_\mu$. If $G = \varnothing$ then $G \in \mu$. If $G \neq \varnothing$, let $x \in G$. Then there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq G$. Thus for each $x \in G$ there exists a μ-open set U containing x such that $x \in U \subseteqq G$. Thus x is a μ-interior point of G i.e., $x \in i_\mu(G)$ i.e., $G \subseteqq i_\mu(G)$ proving G to be a μ-open set.

(b) We note that γ_μ is a GT on X i.e., $\varnothing \in \gamma_\mu$ and arbitrary unions of γ_μ-open sets are also γ_μ-open. For let $\{G_\alpha : \alpha \in I\}$ be a family of γ_μ-open subsets of X. We shall show that $\cup\{G_\alpha : \alpha \in I\}$ is also a γ_μ-open set. In fact, let $x \in \cup\{G_\alpha : \alpha \in I\}$. Then $x \in G_{\alpha_0}$ for some $\alpha_0 \in I$. Thus by γ_μ-openness of G_{α_0}, there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq G_{\alpha_0} \subseteqq \cup\{G_\alpha : \alpha \in I\}$.

Example 2.4. (a) Let $X = \{1, 2, 3\}$ and $\mu = \{\varnothing, \{1, 2\}, \{1, 3\}, X\}$. Then μ is a GT on X. Consider the mapping $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by $\gamma_\mu(A) = c_\mu(A)$ for each subset A of X. It can be easily checked that $\{1, 2\}$ is a μ-open set but not a γ_μ-open set.
(b) Let $X = \{1, 2, 3\}$ and $\mu = \{\varnothing, \{1\}, \{1, 2\}, \{2, 3\}, X\}$. Then (X, μ) is a GTS. Now $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A, \text{ if } 1 \in A \\ \{2, 3\}, \text{otherwise} \end{cases}$$

is an operation. It can be easily checked that $\{1, 2\}$ and $\{2, 3\}$ are two γ_μ-open sets but their intersection $\{2\}$ is not so.

Definition 2.5. A GTS (X, μ) is said to be a γ_μ-regular space if for each point x of X and each μ-open set V containing x, there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq V$.

Theorem 2.6. Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on a GTS X. Then (X, μ) is a γ_μ-regular space if and only if $\mu = \gamma_\mu$.

Proof. Let (X, μ) be a γ_μ-regular space. In view of Remark 2.3 it is sufficient to show that $\mu \subseteqq \gamma_\mu$. Let G be a μ-open set of X. If $G = \varnothing$, then $G \in \gamma_\mu$. Thus we may assume that $G \neq \varnothing$. Since (X, μ) is γ_μ-regular, then G is a γ_μ-open set. Therefore, we have $\mu \subseteqq \gamma_\mu$.

Conversely, let $x \in X$ and V be a μ-open set containing x. Then V is a γ_μ-open set containing x (as $\mu = \gamma_\mu$). Thus by definition of γ_μ-open sets, there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq V$. Hence (X, μ) is a γ_μ-regular space.

Theorem 2.7. A GTS (X, μ) is a γ_μ-regular space if and only if for each point $x \in X$ and every μ-open set U containing x, there exists a γ_μ-open set W containing x such that $W \subseteqq U$.

Proof. First let us assume that (X, μ) be a γ_μ-regular space. Let $x \in X$ and U be a μ-open set containing x. Then by Definition 2.5, there exists a μ-open set W containing x such that $W \subseteqq \gamma_\mu(W) \subseteqq U$. Thus by Theorem 2.6, W is a γ_μ-open set. Hence there exists a γ_μ-open set W such that $x \in W \subseteqq U$.

Conversely, suppose that for each point $x \in X$ and every μ-open set U containing x there exists a γ_μ-open set W containing x such that $W \subseteqq U$. In view of Theorem 2.6 and Remark 2.3(a) it is now sufficient to show that $\mu \subseteqq \gamma_\mu$. Let $U \in \mu$ and $x \in U$. Then by the given condition there exists a γ_μ-open set W_x containing x such that $W_x \subseteqq U$. Thus $U = \cup\{W_x : x \in U \text{ and } W_x \text{ is } \gamma_\mu\text{-open}\}$. Thus by Remark 2.3(b), U is γ_μ-open.

Definition 2.8. Let (X, μ) be a GTS. An operation $\gamma_\mu : \mu \to \mathcal{P}(X)$ is said to be regular if for each point $x \in X$ and any two μ-open sets U and V of X containing x there exists a μ-open set W containing x such that $\gamma_\mu(W) \subseteqq \gamma_\mu(U) \cap \gamma_\mu(V)$.

Theorem 2.9. Let $\gamma_\mu : \mu \to \mathcal{P}(X)$ be a regular operation. Then the intersection of two γ_μ-open sets is also a γ_μ-open set. Furthermore, γ_μ is a topology if $X \in \mu$.

Proof. Let G and H be two γ_μ-open sets in a GTS (X, μ). We shall show that $G \cap H$ is also a γ_μ-open set. If $G \cap H = \varnothing$ then the proof is done. Let $x \in G \cap H$. Then by Definition 2.2, there exist two μ-open sets U and V with $x \in U \cap V$ such that $\gamma_\mu(U) \subseteqq G$ and $\gamma_\mu(V) \subseteqq H$. Since $\gamma_\mu : \mu \to \mathcal{P}(X)$ is a regular operation, there exists a μ-open set W containing x such that $\gamma_\mu(W) \subseteqq \gamma_\mu(U) \cap \gamma_\mu(V) \subseteqq G \cap H$. Thus by Definition 2.2, $G \cap H$ is γ_μ-open.
If $X \in \mu$, then for each $x \in X$, there exists a μ-open set X (as $X \in \mu$) containing x such that $X \subseteqq \gamma_\mu(X) \subseteqq X$. Thus X is a γ_μ-open set. It follows from Remark 2.3(b) that arbitrary union of γ_μ-open sets is a γ_μ-open set. Thus γ_μ is a topology on X.

Example 2.10. (a) Let $X = \{1, 2, 3\}$ and $\mu = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by $\gamma_\mu(A) = c_\mu(A)$ is an operation on the GTS (X, μ) where μ is not strong. It can be easily checked the X is not a γ_μ-open set. We note that $\gamma_\mu : \mu \to \mathcal{P}(X)$ is a regular operation.
(b) Let $X = \{1, 2, 3\}$, $\mu = \{\varnothing, X, \{2\}, \{1, 3\}, \{2, 3\}\}$. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A \cup \{1\}, & \text{if } A \text{ is any singleton subset of } X \\ A, & \text{otherwise} \end{cases}$$

is an operation on the GTS (X, μ). We note that γ_μ is not a regular operation. It can be checked easily that γ_μ is not a topology on X.

We now define the following two types of closure operators : one follows from the GT γ_μ on X and the second one is defined in the sense of Jankovič.

Definition 2.11. Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation.

(a) It follows from Remark 2.3(b) that γ_μ is a GT. Thus the γ_μ-closure of a set A is denoted by $c_{\gamma_\mu}(A)$ and is defined as $c_{\gamma_\mu}(A) = \cap\{F : F \text{ is a } \gamma_\mu\text{-closed set and } A \subseteqq F\}$.

(b) γ_μ^*-closure of A is denoted by γ_μ-$c(A)$ and defined by γ_μ-$c(A) = \{x : A \cap \gamma_\mu(U) \neq \varnothing$ for every μ-open set U containing $x\}$.

A subset $A(\subseteqq X)$ is called γ_μ^*-closed if γ_μ-$c(A) = A$.

Proposition 2.12. Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. For each $x \in X$, $x \in c_{\gamma_\mu}(A)$ if and only if $V \cap A \neq \varnothing$ for any $V \in \gamma_\mu$ with $x \in V$.

Proof. The proof follows from the fact that γ_μ is a GT on X (by Remark 2.3(b)) and the fact that for any GT μ on X, $x \in c_\mu(A)$ [7, 8] if and only if $U \cap A \neq \varnothing$ for each μ-open set U containing x.

Remark 2.13. It can be checked easily that for any subset A of a GTS (X, μ), $A \subseteqq c_\mu(A) \subseteqq \gamma_\mu$-$c(A) \subseteqq c_{\gamma_\mu}(A)$.

Definition 2.14. An operation $\gamma_\mu : \mu \to \mathcal{P}(X)$ is said to be μ-open if for each point x of X and for every μ-open set U containing x there exists a γ_μ-open set V containing x such that $V \subseteqq \gamma_\mu(U)$.

The next theorem gives the relation between the three types of closure operators.

Theorem 2.15. Let (X, μ) be a GTS, $\gamma_\mu : \mu \to \mathcal{P}(X)$ an operation and A a subset of X.

(i) The subset γ_μ-$c(A)$ is μ-closed in (X, μ).

(ii) If (X, μ) is γ_μ-regular, then γ_μ-$c(A) = c_\mu(A)$.

(iii) If γ_μ is μ-open, then γ_μ-$c(A) = c_{\gamma_\mu}(A)$ and γ_μ-$c[\gamma_\mu$-$c(A)] = \gamma_\mu$-$c(A)$.

Proof. (i) We shall only show that $c_\mu[\gamma_\mu$-$c(A)] \subseteqq \gamma_\mu$-$c(A)$. Let $x \in c_\mu[\gamma_\mu$-$c(A)]$ and U be any μ-open set in X containing x. Then $U \cap \gamma_\mu$-$c(A) \neq \varnothing$. Let $y \in U \cap \gamma_\mu$-$c(A)$. Then $y \in U$ and $y \in \gamma_\mu$-$c(A)$. Thus $\gamma_\mu(U) \cap A \neq \varnothing$ i.e., $x \in \gamma_\mu$-$c(A)$ (by Definition 2.11).

(ii) In view of Remark 2.13 we need only to show that in a γ_μ-regular GTS (X, μ), γ_μ-$c(A) \subseteqq c_\mu(A)$. Let $x \in \gamma_\mu$-$c(A)$ and G be any μ-open set containing x. Then there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq G$ (as (X, μ) is γ_μ-regular). Since $x \in \gamma_\mu$-$c(A)$ we have $\gamma_\mu(U) \cap A \neq \varnothing$ and hence $G \cap A \neq \varnothing$. Thus it follows that $x \in c_\mu(A)$.

(iii) Suppose that $x \notin \gamma_\mu$-$c(A)$. Then there exists a μ-open set U containing x such that $\gamma_\mu(U) \cap A = \varnothing$. Since γ_μ is μ-open, for the μ-open set U containing x, there exists a γ_μ-open set V containing x such that $V \subseteqq \gamma_\mu(U)$. Hence $V \cap A = \varnothing$. This shows that $x \notin c_{\gamma_\mu}(A)$. Thus $c_{\gamma_\mu}(A) \subseteqq \gamma_\mu$-$c(A)$. Also from Remark 2.13, γ_μ-$c(A) \subseteqq c_{\gamma_\mu}(A)$. Thus we have γ_μ-$c(A) = c_{\gamma_\mu}(A)$. Hence γ_μ-$c[\gamma_\mu$-$c(A)] = c_{\gamma_\mu}[c_{\gamma_\mu}(A)] = c_{\gamma_\mu}(A)$ (as γ_μ is a GT on X and c_{γ_μ} is idempotent) $= \gamma_\mu$-$c(A)$.

Example 2.16. (a) Let $X = \{1, 2, 3\}$, $\mu = \{\varnothing, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, X\}$. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A \cup \{3\}, & \text{if } A \neq \{1\} \\ A, & \text{otherwise} \end{cases}$$

is an operation. It can be easily checked that $c_\mu(\{3\}) = \{3\} \neq \gamma_\mu$-$c(\{3\}) = \{2,3\}$ and thus from Theorem 2.15 it follows that (X,μ) is not γ_μ-regular.

(b) Let $X = \{1,2,3,4\}$, $\mu = \{\varnothing, \{1,2\}, \{2,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, X\}$. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A, \text{ if } 1 \in A \\ A \cup \{1\}, \text{if } \ 1 \notin A \end{cases}$$

is an operation. It can be checked that γ_μ-$c(\{2\}) = \{2,3,4\}$ but γ_μ-$c[\gamma_\mu$-$c(\{2\})] = X \neq \gamma_\mu$-$c(\{2\})$. Thus it follows from Theorem 2.15 that γ_μ is not μ-open.

Theorem 2.17. Let μ be a GT on a set X and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. For any subset A of X the followings are equivalent :

(i) A is γ_μ-open in (X,μ).

(ii) $X \setminus A$ is γ_μ^*-closed in (X,μ).

(iii) $c_{\gamma_\mu}(X \setminus A) = X \setminus A$ holds.

(iv) $X \setminus A$ is γ_μ-closed in (X,μ).

Proof. (i) $\Rightarrow$ (ii): Let $x \notin X \setminus A$. Then $x \in A$. Thus there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq A$ i.e., $\gamma_\mu(U) \cap (X \setminus A) = \varnothing$. This shows that $x \notin \gamma_\mu$-$c(X \setminus A)$. Thus it follows that γ_μ-$c(X \setminus A) \subseteqq X \setminus A$.

(ii) $\Rightarrow$ (iii): We have to show that $c_{\gamma_\mu}(X \setminus A) \subseteqq X \setminus A$. Let $x \notin X \setminus A$. It then follows from (ii) that there exists a μ-open set U containing x such that $\gamma_\mu(U) \cap (X \setminus A) = \varnothing$. Then A is a γ_μ-open set containing x. Therefore $A \cap (X \setminus A) = \varnothing$ and hence $x \notin c_{\gamma_\mu}(X \setminus A)$.

(iii) $\Rightarrow$ (iv): We shall show that A is γ_μ-open. Let $x \in A$. Then by Proposition 2.12 and (iii), there exists a γ_μ-open set U containing x such that $U \cap (X \setminus A) = \varnothing$. Since U is γ_μ-open and $x \in U$, there exists a μ-open set V containing x such that $\gamma_\mu(V) \subseteqq U$. Thus we have $x \in \gamma_\mu(V) \subseteqq U \subseteqq A$ and hence A is γ_μ-open.

(iv) $\Rightarrow$ (i) : The proof follows from the definition.

Theorem 2.18. Let (X,μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. If γ_μ is regular, then γ_μ-$c(A \cup B) = \gamma_\mu$-$c(A) \cup \gamma_\mu$-$c(B)$ for any two subsets A and B of X.

Proof. Let $x \notin \gamma_\mu$-$c(A) \cup \gamma_\mu$-$c(B)$. Then $x \notin \gamma_\mu$-$c(A)$ and $x \notin \gamma_\mu$-$c(B)$. Hence there exist two μ-open sets U and V containing x such that $\gamma_\mu(U) \cap A = \gamma_\mu(V) \cap B = \varnothing$. Since γ_μ is regular, there exists a μ-open set W containing x such that $\gamma_\mu(W) \subseteqq \gamma_\mu(U) \cap \gamma_\mu(V)$. Therefore, we have $(A \cup B) \cap \gamma_\mu(W) \subseteqq (A \cup B) \cap [\gamma_\mu(U) \cap \gamma_\mu(V)] \subseteqq$ $\subseteqq [A \cap \gamma_\mu(U)] \cup [B \cap \gamma_\mu(V)] = \varnothing$. Hence $x \notin \gamma_\mu$-$c(A \cup B)$. Therefore, we obtain γ_μ-$c(A \cup B) \subseteqq \gamma_\mu$-$c(A) \cup \gamma_\mu$-$c(B)$.

Corollary 2.19. Let μ be a GT on a set X and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. If γ_μ is regular and μ-open, then the mapping defined by $\psi(A) = \gamma_\mu$-$c(A)$ for $A \subseteqq X$ is a Kuratowski closure operator.

Proof. This follows from Theorem 2.15, Theorem 2.18 and Definition 2.11.

γ_μ-generalized closed sets and γ_μ-T_i spaces ($i = 0, 1/2, 1, 2$)

Definition 3.1. Let $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. A subset A of a GTS (X,μ) is said to be γ_μ-generalized closed (briefly $\gamma_\mu g$-closed) if γ_μ-$c(A) \subseteqq U$ whenever $A \subseteqq U$ and U is γ_μ-open.

The complement of a $\gamma_\mu g$-closed set is called a $\gamma_\mu g$-open set.

We observe that every γ_μ^*-closed set is $\gamma_\mu g$-closed. The converse is false as shown in the next example.

Example 3.2. Consider $X = \{1,2,3\}$, $\mu = \{\varnothing, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A \cup \{2\}, \text{ if } A \neq \{1\} \\ A, \text{otherwise} \end{cases}$$

is an operation. It can be checked easily that $\{1,3\}$ is $\gamma_\mu g$-closed but not γ_μ-closed.

The following theorem gives the characterizations of $\gamma_\mu g$-closed sets.

Theorem 3.3. Let $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation. Then for any $A \subseteqq X$, the following are equivalent:

(i) A is $\gamma_\mu g$-closed.

(ii) For each $x \in \gamma_\mu$-$c(A)$, $c_{\gamma_\mu}(\{x\}) \cap A \neq \varnothing$.

(iii) γ_μ-$c(A) \subseteqq Ker_{\gamma_\mu}(A)$ (where $Ker_{\gamma_\mu}(A) = \cap\{V : A \subseteqq V$ and V is γ_μ-open$\}$ see [15] for detail).

Proof. (i) $\Rightarrow$ (ii) : Suppose that A be a $\gamma_\mu g$-closed subset and also suppose that there exists a point $x \in \gamma_\mu$-$c(A)$ for which $c_{\gamma_\mu}(\{x\}) \cap A = \varnothing$. Then $c_{\gamma_\mu}(\{x\})$ is γ_μ-closed (by Remark 2.3(b) and Definition 2.11(a)). Put $U = X \setminus c_{\gamma_\mu}(\{x\})$. Then $A \subseteqq U$ and $x \notin U$ with U a γ_μ-open set in (X,μ). Since A is $\gamma_\mu g$-closed, γ_μ-$c(A) \subseteqq U$. Thus $x \notin \gamma_\mu$-$c(A)$ which is a contradiction.

(ii) $\Rightarrow$ (iii) : Let $x \in \gamma_\mu$-$c(A)$. We have only to show that $x \in Ker_{\gamma_\mu}(A)$. By (ii) there exists a point $z \in A$ such that $z \in c_{\gamma_\mu}(\{x\})$. Let U be any γ_μ-open subset of X such that $A \subseteqq U$. Since $z \in U$ and $z \in c_{\gamma_\mu}(\{x\})$, by Proposition 2.12 we have $U \cap \{x\} \neq \varnothing$ i.e., $x \in U$. Thus $x \in Ker_{\gamma_\mu}(A)$.

(iii) $\Rightarrow$ (i) : Let $A \subseteqq U$, where U be any γ_μ-open set. Let $x \in \gamma_\mu$-$c(A)$. It then follows from (iii) that $x \in Ker_{\gamma_\mu}(A)$. Thus $x \in U$ i.e., γ_μ-$c(A) \subseteqq U$.

Theorem 3.4. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where (X, μ) is a GTS. For each point x of X, $\{x\}$ is a γ_μ-closed set or $X \setminus \{x\}$ is a $\gamma_\mu g$-closed set in (X, μ).

Proof. Let $\{x\}$ be not a γ_μ-closed set. Then the complement $X \setminus \{x\}$ is not a γ_μ-open set. Let U be any γ_μ-open set with $X \setminus \{x\} \subseteqq U$. Then U must be equal to X. Thus γ_μ-$c(X \setminus \{x\}) \subseteqq U$. Thus $X \setminus \{x\}$ is $\gamma_\mu g$-closed.

Proposition 3.5. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation and A be a subset of a GTS (X, μ). If A is $\gamma_\mu g$-closed, then γ_μ-$c(A) \setminus A$ does not contain any non-empty γ_μ-closed set. If the operation $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ is μ-open, then the converse part is also true.

Proof. If possible, let F be any γ_μ-closed set contained in γ_μ-$c(A) \setminus A$. Then $A \subseteqq X \setminus F$ where $X \setminus F$ is a γ_μ-open set. Thus γ_μ-$c(A) \subseteqq X \setminus F$ (as A is $\gamma_\mu g$-closed). Thus $F \subseteqq X \setminus \gamma_\mu$-$c(A)$. Also $F \subseteqq \gamma_\mu$-$c(A)$. Thus $F \subseteqq \gamma_\mu$-$c(A) \cap (X \setminus \gamma_\mu$-$c(A)) = \varnothing$, which is a contradiction. Thus $F = \varnothing$.

Conversely, let $A \subseteqq U$ where U be any γ_μ-open set. Since the operation γ_μ is μ-open, by Theorem 2.15 γ_μ-$c(A)$ is γ_μ-closed. Thus γ_μ-$c(A) \cap (X \setminus U) = F$ (say) is a γ_μ-closed set (by Remark 2.3(b) and Definition 2.11(a)). Since $X \setminus U \subseteqq X \setminus A$, $F \subseteqq \gamma_\mu$-$c(A) \setminus A$. Thus by the assumption it follows that $F = \varnothing$ and hence we have γ_μ-$c(A) \subseteqq U$.

Definition 3.6. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where μ is a GT on X. Then (X, μ) is said to be a γ_μ-$T_{1/2}$ space if every $\gamma_\mu g$-closed set is a γ_μ-closed set.

The next theorem characterizes a γ_μ-$T_{1/2}$ GTS.

Theorem 3.7. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where μ is a GT on X. Then (X, μ) is γ_μ-$T_{1/2}$ if and only if for each $x \in X$, $\{x\}$ is either γ_μ-open or γ_μ-closed.

Proof. Suppose that that (X, μ) is γ_μ-$T_{1/2}$ and $\{x\}$ is not γ_μ-closed. Then by Theorem 3.4, $X \setminus \{x\}$ is $\gamma_\mu g$-closed. Since (X, μ) is γ_μ-$T_{1/2}$, $X \setminus \{x\}$ is γ_μ-closed. Thus $\{x\}$ is γ_μ-open.

Conversely, let F be a $\gamma_\mu g$-closed set in (X, μ). By Theorem 2.17, it is sufficient to show that γ_μ-$c(F) \subseteqq F$. If possible, let there exist a point $x \in \gamma_\mu$-$c(F) \setminus F$. Then by the given condition $\{x\}$ is either γ_μ-open or γ_μ-closed.
Case -1 : $\{x\}$ is γ_μ-closed : For this case we have a γ_μ-closed set $\{x\}$ such that $\{x\} \subseteqq \gamma_\mu$-$c(F) \setminus F$. This is contrary to Proposition 3.5.
Case -2 : $\{x\}$ is γ_μ-open : Then by Remark 2.13, $x \in c_{\gamma_\mu}(F)$. Thus $\{x\} \cap F \neq \varnothing$. This is a contradiction. Thus we have γ_μ-$c(F) \subseteqq F$.

Definition 3.8. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where μ is a GT on X. Then (X, μ) is said to be
(a) γ_μ-T_0 if for each pair of distinct points $x, y \in X$, there exists a μ-open set G such that either $x \in G$ and $y \notin \gamma_\mu(G)$, or $y \in G$ and $x \notin \gamma_\mu(G)$.
(b) γ_μ-T_1 if for each pair of distinct points $x, y \in X$, there exist μ-open sets G and H containing x and y, respectively, such that either $y \notin \gamma_\mu(G)$ and $x \notin \gamma_\mu(H)$.
(c) γ_μ-T_2 if for each pair of distinct points $x, y \in X$, there exist μ-open sets G and H containing x and y, respectively, such that $\gamma_\mu(G) \cap \gamma_\mu(H) = \varnothing$.

A γ_μ-T_1 GTS is characterized by the following theorem.

Theorem 3.9. Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where μ is a GT on X. Then the following are equivalent:

(i) (X, μ) is γ_μ-T_1.

(ii) For each $x \in X$, $\{x\}$ is a γ_μ^*-closed set.

(iii) For each pair of distinct points $x, y \in X$ there exist γ_μ-open sets U and V containing x and y, respectively, such that either $y \notin U$ and $x \notin V$.

Proof. (i) $\Rightarrow$ (ii) : Let $x \in X$. We shall show that $\{x\}$ is γ_μ^*-closed. Let $y \notin \{x\}$. Then by (i) there exists a μ-open set U_y such that $y \in U_y$, $x \notin \gamma_\mu(U_y)$. Thus $\gamma_\mu(U_y) \cap \{x\} = \varnothing$. Thus $y \notin \gamma_\mu$-$c(\{x\})$. Thus $\{x\}$ is γ_μ^*-closed.

(ii) $\Rightarrow$ (iii) Let x and y be two points of X with $x \neq y$. Then by (ii) $\{x\}$ and $\{y\}$ are two γ_μ-closed sets and hence by Theorem 2.17, $X \setminus \{y\}$ and $X \setminus \{x\}$ are two γ_μ-open sets containing x and y, respectively, such that $x \in (X \setminus \{y\})$ and $y \in (X \setminus \{x\})$.

(iii) $\Rightarrow$ (i) : Obvious.

Let $\gamma_\mu : \mu \rightarrow \mathcal{P}(X)$ be an operation, where μ is a GT on X. Then it follows from Definitions 3.6 and 3.8 that γ_μ-$T_2 \Rightarrow \gamma_\mu$-$T_1 \Rightarrow \gamma_\mu$-$T_{1/2} \Rightarrow \gamma_\mu$-$T_0$. None of the implications are reversible as shown in the next example.

Example 3.10. (a) Let $X = \{1,2,3\}$ and $\mu = \mathcal{P}(X)$. Then μ is a GT on X. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A \cup \{2\}, \text{ if } A = \{1\} \\ A \cup \{3\}, \text{ if } A = \{2\} \\ A \cup \{1\}, \text{ if } A = \{3\} \\ A, \text{otherwise} \end{cases}$$

is an operation. It can be checked that (X,μ) is γ_μ-T_1 but not a γ_μ-T_2 space.

(b) Let $X = \{1,2,3\}$ and $\mu = \mathcal{P}(X)$. Then μ is a GT on X. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A \cup \{3\}, \text{ if } A \neq \{1\} \\ A, \text{otherwise} \end{cases}$$

is an operation. It can be checked that (X,μ) is γ_μ-$T_{1/2}$ but not a γ_μ-T_1 space.

(c) Let $X = \{1,2,3\}$ and $\mu = \{\varnothing, \{1\}, \{1,2\}, \{1,3\}, X\}$. Then μ is a GT on X. Then $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A, \text{ if } A \neq \{1\} \\ \{1,2\}, \text{otherwise} \end{cases}$$

is an operation. It can be checked that (X,μ) is γ_μ-T_0 but not a γ_μ-$T_{1/2}$ space.

Throughout the rest of the paper (X,μ) and (Y,λ) will denote GTS's and $\gamma_\mu : \mu \to \mathcal{P}(X)$ and $\beta_\lambda : \lambda \to \mathcal{P}(Y)$ will denote two operations on μ and λ respectively.

Definition 3.11. A function $f : (X,\mu) \to (Y,\lambda)$ is said to be (γ,β)-continuous if for each $x \in X$ and each λ-open set V with $f(x) \in V$ there exists a μ-open set U containing x such that $f(\gamma_\mu(U)) \subseteqq \beta_\lambda(V)$.

Theorem 3.12. A (γ,β)-continuous mapping $f : (X,\mu) \to (Y,\lambda)$ satisfies the following properties:

(i) $f(\gamma_\mu\text{-}c(A)) \subseteqq \beta_\lambda\text{-}c(f(A))$ for every subset A of X.

(ii) $f^{-1}(W)$ is γ_μ-open for every β_λ-open set W of Y, i.e., the inverse image of any β_λ-closed set of (Y,β) is γ_μ-closed in (X,μ).

Proof. (i) Let y be a point of $f(\gamma_\mu\text{-}c(A))$ and V be any λ-open set containing y. Then there exists a point x in X such that $f(x) = y$ and $x \in \gamma_\mu\text{-}c(A)$. Thus by (γ,β)-continuity of f there exists a μ-open set U containing x such that $f(\gamma_\mu(U)) \subseteqq \beta_\lambda(V)$. As $x \in \gamma_\mu\text{-}c(A)$, we have $\gamma_\mu(U) \cap A \neq \varnothing$, and hence $\varnothing \neq f(\gamma_\mu(U) \cap A) \subseteqq$ $\subseteqq f(\gamma_\mu(U)) \cap f(A) \subseteqq \beta_\lambda(V) \cap f(A)$. This shows that $y \in \beta_\lambda\text{-}c(f(A))$.

(ii) Let W be a β_λ-open set in (Y,λ) and x any point of $f^{-1}(W)$. We have to show that $f^{-1}(W)$ is γ_μ-open. There exists a β-open set V containing $f(x)$ such that $\beta_\lambda(V) \subseteqq W$. Thus by (γ,β)-continuity of f, there exists a μ-open set U containing x such that $f(\gamma_\mu(U)) \subseteqq \beta_\lambda(V)$. Thus $\gamma_\mu(U) \subseteqq f^{-1}(\beta_\lambda(V)) \subseteqq f^{-1}(W)$. Thus $f^{-1}(W)$ is γ_μ-open.

Definition 3.13. A function $f : (X,\mu) \to (Y,\lambda)$ is said to be (γ,β)-closed if for any γ_μ-closed set A of X, $f(A)$ is a β_λ-closed set in Y.

Let $id_\mu : \mu \to \mathcal{P}(X)$ be the identity operation, where (X,μ) is a GTS. We note that id_μ-open sets and μ-open sets are identical.

Proposition 3.14. Let $f : (X,\mu) \to (Y,\lambda)$ be a (γ,β)-continuous function and f be a (id,β)-closed mapping. The following properties hold:

(i) For each $\gamma_\mu g$-closed set A of X, $f(A)$ is $\beta_\lambda g$-closed in Y.

(ii) For each $\beta_\lambda g$-closed set B of Y, $f^{-1}(B)$ is $\gamma_\mu g$-closed.

Proof. (i) Let V be any β_λ-open set of (Y,λ) with $f(A) \subseteqq V$. Then by Theorem 3.12 (ii), $f^{-1}(V)$ is a γ_μ-open set. Now as A is a $\gamma_\mu g$-closed set and $A \subseteqq f^{-1}(V)$, we have $\gamma_\mu\text{-}c(A) \subseteqq f^{-1}(V)$, and thus $f(\gamma_\mu\text{-}c(A)) \subseteqq V$. From the assumption and Theorem 2.15(i) it follows that, $f(\gamma_\mu\text{-}c(A))$ is β_λ-closed. Thus by Remark 2.13, we have $\beta_\lambda\text{-}c(f(A)) \subseteqq c_{\beta_\lambda}((f(\gamma_\mu\text{-}c(A)))) = f(\gamma_\mu\text{-}c(A)) \subseteqq V$. This shows that $f(A)$ is $\beta_\lambda g$-closed in Y.

(ii) Let U be any γ_μ-open set of (X,μ) such that $f^{-1}(B)$ is contained in U. Let $F = \gamma_\mu\text{-}c(f^{-1}(B)) \cap (X \setminus U)$. Then F is μ-closed in (X,μ) (by Theorem 2.15(i) and Remark 2.3(a)). Since f is a (id,β)-closed function, $f(F)$ is a β_λ-closed set in (Y,λ). Then by Proposition 3.5 and the relation $f(F) \subseteqq \beta_\lambda\text{-}c(B) \setminus B$, it follows that $f(F) = \varnothing$ and thus $F = \varnothing$. Thus $\gamma_\mu\text{-}c(f^{-1}(B)) \subseteqq U$ i.e., $f^{-1}(B)$ is $\gamma_\mu g$-closed.

Theorem 3.15. Let $f : (X,\mu) \to (Y,\lambda)$ be a (γ,β)-continuous and (id,β)-closed function.

(i) If f is an injective function and (Y,λ) is a β_λ-$T_{1/2}$ space, then (X,μ) is a γ_μ-$T_{1/2}$ space.

(ii) If f is a surjective function and (X,μ) is a γ_μ-$T_{1/2}$ space, then (Y,λ) is a β_λ-$T_{1/2}$ space.

(iii) If f is bijective,, then (X,μ) is a γ_μ-$T_{1/2}$ space if and only if (Y,λ) is a β_λ-$T_{1/2}$ space.

Proof. (i) We need only to show that every $\gamma_\mu g$-closed set is γ_μ-closed. Let A be a $\gamma_\mu g$-closed set of (X,μ). It then follows from Proposition 3.14(i) that $f(A)$ is $\beta_\lambda g$-closed and thus $f(A)$ is β_λ-closed (as (Y,λ) is β_λ-$T_{1/2}$). Now by Theorem 3.12(ii), $f^{-1}(f(A))$ is γ_μ-closed (as f is (γ,β)-continuous) i.e., A is γ_μ-closed.

(ii) Let B be a $\beta_\lambda g$-closed set of (Y,λ). We have to show that B is a β_λ-closed set. By Theorem 3.14(ii), $f^{-1}(B)$ is a $\gamma_\mu g$-closed set in (X,μ). Thus $f^{-1}(B)$ is γ_μ-closed (as (X,μ) is γ_μ-$T_{1/2}$). Thus from the assumption it follows that $B(=ff^{-1}(B))$ is β_λ-closed in (Y,λ). Thus it follows that (Y,λ) is a β_λ-$T_{1/2}$ space.

(iii) The proof follows from (i) and (ii).

Theorem 3.16. Suppose that $f:(X,\mu)\to(Y,\lambda)$ is a (γ,β)-continuous bijection and $f^{-1}:(Y,\lambda)\to(X,\mu)$ is (β,γ)-continuous. Then (X,μ) is a γ_μ-$T_{1/2}$ space if and only if (Y,λ) is a β_λ-$T_{1/2}$ space.

Proof. Let (X,μ) be a γ_μ-$T_{1/2}$ space. In view of Theorem 3.7 it is sufficient to show that any singleton set of (Y,λ) is either β_λ-closed or β_λ-open. Let $\{y\}$ be any subset of (Y,λ). Then, since f is surjective, there exists $x\in X$ such that $f(x)=y$. Then by Theorem 3.7 it follows that $\{x\}$ is γ_μ-closed or γ_μ-open (as (X,μ) is γ_μ-$T_{1/2}$). Then by Theorem 3.12, $\{y\}(=f(\{x\}))$ is β_λ-closed or β_λ-open. Thus (Y,λ) is a β_λ-$T_{1/2}$ space. The proof of the converse is similar.

Proposition 3.17. Let $f:(X,\mu)\to(Y,\lambda)$ be a (γ,β)-continuous injection and (Y,λ) be a β_λ-T_2 (resp. β_λ-T_1) space. Then (X,μ) is a γ_μ-T_2 (resp. γ_μ-T_1) space.

Proof. Let (Y,λ) be a β_λ-T_2 space. Let x,y be any two points of X with $x\neq y$. Then there exist λ-open sets V and W of Y contaning $f(x)$ and $f(y)$ respectively such that $\beta_\lambda(V)\cap\beta_\lambda(W)=\varnothing$. Now by (γ,β)-continuity of f, there exist μ-open sets G and H containing x and y respectively such that $f(\gamma_\mu(G))\subseteqq\beta_\lambda(V)$ and $f(\gamma_\mu(H))\subseteqq\beta_\lambda(W)$. Thus $\gamma_\mu(G)\cap\gamma_\mu(H)=\varnothing$. Thus (X,μ) is a γ_μ-T_2 space.

The proof of the case of β_λ-T_1 can be done similarly.

Lemma 3.18. Let $\gamma_\mu:\mu\to\mathcal{P}(X)$ be a regular, μ-open operation and $X\in\mu$. If (X,μ) is a γ_μ-T_2 GTS, then (X,γ_μ) is a T_2 space.

Proof. We first note that since $\gamma_\mu:\mu\to\mathcal{P}(X)$ is regular and $X\in\mu$, by Theorem 2.9, γ_μ is a topology on X. Let x,y be two distinct points of X. Then there exist μ-open sets U and V containing x and y, respectively, such that $\gamma_\mu(U)\cap\gamma_\mu(V)=\varnothing$. Since γ_μ is μ-open, there exist γ_μ-open sets U^* and V^* containing x and y, respectively, such that $U^*\subseteqq\gamma_\mu(U)$ and $V^*\subseteqq\gamma_\mu(V)$. Thus $U^*\cap V^*=\varnothing$ and (X,γ_μ) is a T_2 space.

Theorem 3.19. Let $\gamma_\mu:\mu\to\mathcal{P}(X)$ be a μ-open regular operation and $\beta_\lambda:\lambda\to\mathcal{P}(Y)$ be a λ-open regular operation such that $X\in\mu$ and $Y\in\lambda$. If $f,g:(X,\mu)\to(Y,\lambda)$ are (γ,β)-continuous and (Y,λ) is β_λ-T_2, then the set $A=\{x\in X: f(x)=g(x)\}$ is γ_μ-closed in (X,μ).

Proof. We observe first by Lemma 3.18 that, γ_μ and β_λ are two topologies on X and Y, respectively. We shall now show that if $f:(X,\mu)\to(Y,\lambda)$ is (γ,β)-continuous, then $f:(X,\gamma_\mu)\to(Y,\beta_\lambda)$ is continuous. Let $x\in X$ and V be any β_λ-open set containing $f(x)$. Then there exists a λ-open set V' such that $f(x)\in V'$ and $\beta_\lambda(V')\subseteqq V$. Since f is (γ,β)-continuous, there exists a μ-open set W such that $x\in W$ and $f(\gamma_\mu(W))\subseteqq\beta_\lambda(V')\subseteqq V$. Then by μ-openness of γ_μ there exists a γ_μ-open set W' containing x such that $W'\subseteqq\gamma_\mu(W)$. Thus $f(W')\subseteqq V$. Thus $f:(X,\gamma_\mu)\to(Y,\beta_\lambda)$ is continuous and similarly $g:(X,\gamma_\mu)\to(Y,\beta_\lambda)$ is continuous. By Lemma 3.18, (Y,β_λ) is a T_2 space. Therefore the set $A=\{x\in X: f(x)=g(x)\}$ is closed in (X,γ_μ) and hence $X\setminus A$ is γ_μ-open. Thus A is γ_μ-closed in (X,μ).

Acknowledgments

The first author acknowledges the financial support from Department of Higher education, Science and technology and Biotechnology, Government of West Bengal, India. Also, the authors are thankful to the referee for some comments to improve the paper.

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Жалпыланған топологиялық кеңістіктерге операцияларды қолдану

Мақалада γ_μ — ашық жиындар және γ_μ-GTS-тегі жабық жиындар (X, μ), мұнда γ_μ - μ-дан $\mathcal{P}(X)$-ға операция зерттелген. Жалпы, γ_μ — ашық жиындар жиынтығы μ-ашық жиындар жиынтығынан аз. Сонымен қатар, авторлар екі жиын бірдей болатынын анықтаған. Мұндай жиындардың кейбір қасиеттері талқыланды. Сондай-ақ, жабу түрінің кейбір операторлары анықталып, олардың қасиеттері анықталды. GTS (X, μ)-да ұқсас жабу операторларының түрлері арасында байланыс орнатылған. Белгілі бір тұйықталу түрінің операторы Куратовскийдің тұйықталу операторы болып табылатын шарт беріледі. Сондай-ақ, γ_μ деп аталатын жабық жиындардың жалпыланған түрі анықталған-жалпыланған жабық жиын, осы жаңадан анықталған жабу операторының көмегімен және осындай жиындардың кейбір негізгі қасиеттері талқыланған. Қосымша ретінде бөлімнің әлсіз аксиомалары енгізіліп, олардың кейбір қасиеттері талқыланды. Соңында осындай жалпыланған ұғымдарды сақтаудың кейбір теоремалары көрсетілген.

Кілт сөздер: операция, μ — ашық жиын, γ_μ — ашық жиын, $\gamma_\mu g$ — жабық жиын.

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Приложения операций над обобщенными топологическими пространствами

В статье изучены γ_μ-открытые и γ_μ-замкнутые множества в GTS (X, μ), где γ_μ — операция из μ в $\mathcal{P}(X)$. В общем случае набор γ_μ-открытых множеств меньше, чем набор μ-открытых множеств. Кроме того, авторами установлено условие, при котором оба множества являются одинаковыми. Обсуждены и некоторые свойства таких множеств. Определены некоторые операторы типа замыкания и их свойства. Установлена связь между аналогичными типами операторов замыкания на GTS (X, μ). Дано условие, при котором по-новому определенный оператор типа замыкания является оператором замыкания Куратовского. Выявлен обобщенный тип замкнутых множеств, названный γ_μ-обобщенным замкнутым множеством, с помощью этого вновь определенного оператора замыкания и обсуждены некоторые основные свойства таких множеств. В качестве приложения авторами введены несколько слабых аксиом отделения и определены некоторые их свойства. Таким образом, показаны некоторые теоремы сохранения таких обобщенных понятий.

Ключевые слова: операция, μ-открытое множество, γ_μ-открытое множество, $\gamma_\mu g$-замкнутое множество.

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