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The property of independence for Jonsson sets

The studies carried out in this article are connected with the description of model-theoretic properties of some, generally speaking, incomplete classes of theories that make a subclass of inductive theories. These theories are well studied both in algebra and in the theory of models. They are called Jonsson's theories. To study these theories there is introduced a new research approach, namely: on the submultitudes of a semantic model of Jonsson's theory there are separated special multitudes that are, firstly, realizations of some existential formula, secondly, the closing of the set gives us the basic set of some existentially closed submodel of the semantic model. Besides, there is developed a technique of studying the central orbital types. It is well known that the perfect Jonsson theory enough comfortable for model-theoretic researches. Practically, in the perfect case, we can say that with the help of semantic method, we can give a specific description of these objects (Jonsson theory and class its existentially closed models). In this article we will give the notion of forking for fragment of fixing Jonsson theory. The nonforking extensions will be the «Mfree» ones. Also we considered for the notion of independence many desirable properties like monotonicity, transitivity, finite basis and symmetry.

Key words: Jonsson's set, forking, algebraic closure, definable closure, central type, orbital type, independence.

Our research interests are connected with the description of model-theoretic properties of some, generally speaking, incomplete classes of theories that make a subclass of inductive theories. These theories are well studied both in algebra and in the theory of models.

As well, we are always dealing with two objects:

1) Jonsson theory [1] and 2) class of its existentially closed models.

It is well known that the perfect Jonsson theory enough comfortable for model-theoretic researches. Practically, in the perfect case, we can say that with the help of semantic method, we can give a specific description of these objects (Jonsson theory and class its existentially closed models).

This allows us to assume that it would be interesting to learn how to allocate in an arbitrary theory its fragment which will Jonsson theory. This approach is not trivial, if only from the fact that any theory set its universal existential consequences, not necessarily Jonsson theories.

On the other hand, for any theory in some special enrichments can always be achieved firstly Jonsson and then its perfect. At least this holds for operations such as skulemization and morlization. In both cases, the class of existentially closed models received Jonsson theories coincides with the class of models of initial theories.

Morlization and skolemization action is applied to the theory under consideration.

This article is invited to the idea of considering a new approach to a a subset of some model, which allows firstly to expand the semantic aspect, and secondly to try to transfer many of the ideas out of technique the of complete theories for Jonsson fragments, which in itself generalizes the considered problems.

We make the following agreements:

1. In this project, we consider only perfect Jonsson theory, complete of existential sentences.
2. In this project, we consider only classes existentially closed models of the theories.
3. In case of the structure, it is assumed that the model of some signature.

Naturally, when we speak of arbitrary signature (language) without the theory, item 1) of the above arrangements is not important.

Let T is Jonsson perfect theory of complete of existential sentences in the language L. We fix its semantic model C, saturated in a very high power κ (in particular κ is much greater than the power of language). We agree that in the future all the considered models $M, N, \dots$ of theory T will be existentially closed substructures high model C power less than κ. All considered subsets $A, B, C, ...$ will be subsets of C power less than κ.

Note one more useful fact, if f is the automorphism of structure C, leaving in place all the elements of the set $A, f \in Aut_A(C)$, then f it obviously transfer to itself each A is definability subset and therefore transforms to itself and all complete types over A, due to saturation of the semantic model C. Conversely, if $\overline{c}, \overline{d} \in C^n$ then $tp(\overline{c}/A) = tp(\overline{d}/A)$ if and only if there exists $f \in Aut_A(C)$ such that $f(\overline{c}) = \overline{d}$.

The saturated model complete n-types over A exactly correspond to orbits n elements under automorphisms fixing A element by element. Since the theory is complete for the existential sentences of language L, it is true for existential types.

Let L is a language, which from that moment supposed countable. Next, let T is Jonsson perfect theory of complete of existential sentences in the language L and its semantic model C. There remains an agreement sets and model of theory T are strictly less power than C.

Let $A \subseteq C$. We fix some $n \geq 1$ and consider the family Def_A^n of all A — definable subsets power over C^n. We identify this definable subset of C^n and defining its formula $\varphi(\overline{x}, \overline{a})$, where $\overline{x}, \overline{a}$, $\overline{a}$ – tuple elements of A (two different formulas may define a subset of, but we consider the formula with an accuracy to equivalence in C the obvious sense).

The following approach to the definition of a relational structure of some signature, is well known. It allows to consider only the predicate signatures. For example, in the case of moralization.

Let's start with a definition of the relational structure of the signature of a Jonsson theory. Defining family of definable subsets of the structure, we follow the terminology and notation of [2, 1], but in [1], all definitions are given for complete theories, we will to work with Jonsson theories and their positive generalizations.

The relational structure $M = \langle M, (B_i)_{i \in I} \rangle$ consists of a (non-empty) set M and subsets $(B_i)_{i \in I}$ of $\bigcup_{n>1} M^n$ and each B_i is a subset of some $M^{n_i}, n_i \geq 1$. Add an additional condition that one of the sets B_i is the diagonal of the set M.

All B_i are called atomic subsets M.

Let $M = \langle M, (B_i)_{i \in I} \rangle$ — relational structure. We introduce the concept of a family of definable subsets of structures M, denoted $Def(M)$. It is the least of the family subsets of $\bigcup_{n>1} M^n$ with the following properties.

For each $i \in I$ the inclusion $B_i \in Def(M)$.

The set $Def(M)$ is closed relatively to finite Boolean combinations, i.e. of inclusions $A, B \subseteq M^n$, $A, B \in \in Def(M) \subseteq M^n$, follow that $A \cup B \in Def(M), A \cap B \in Def(M)$ and $M^n \setminus A \in Def(M)$. The set $Def(M)$ is closed relatively Cartesian product, i.e. of inclusions $A, B \in Def(M)$ follow that $A \times B \in Def(M)$. The set $Def(M)$ is closed relatively to the projection, i.e. if $A \subseteq M^{n+m}$, $A \in Def(M)$ $\pi_n(A) \in Def(M)$, π_n the projection of the set A on M^n, $\pi_n(A) \in Def(M)$. The set $Def(M)$ is closed relatively to specialization, i.e. if $A \in Def(M)$, $A \subseteq M^{n+k}$ and $\overline{m} \in M^n$ then $A(\overline{m}) = \{\overline{b} \in M^k(\overline{m}, \overline{b}) \in A\} \in Def(M)$. The set $Def(M)$ is closed relatively to permutation of coordinates, i.e. if $A \in Def(M)$, σ – a permutation of the set $1, ..., n$ then $\sigma(A) = \{(a_{\sigma(1)}, ..., a_{\sigma(n)}) \mid (a_1, ..., a_n) \in A\} \in Def(A)$. We now say that $S \subseteq M^n$ is the atomic subset if

$$S = \{(a_1, ..., a_n) \in M^n \mid M \models \varphi(a_1, ..., a_n, b_1, ..., b_m)\}$$

for some atomic formula $\varphi(x_1, ..., x_n, x_{n+1}, ..., x_{n+m})$ and some $b \in M^m$. We say that a subset S defined with parameters $\overline{b}$ or defined above $\overline{b}$.

We now say that $D \subseteq M^n$ is definable subset L-structure of M, where there are $b \in M^m$ (here $\overline{b}$ may be empty) and a formula $\varphi(x_1, ..., x_n, x_{n+1}, ..., x_{n+m})$ such that

$$D = \{(a_1, ..., a_n) \in M^n \mid M \models \varphi(a_1, ..., a_n, b_1, ..., b_m)\}.$$

If $\overline{b} \subseteq B$, then we say that D is definable with the parameters of B (or above B) or that D is defined formula with the parameters of B. Clearly definable sets in this sense – not that other, as $Def(M)$ a relational structure $\langle M, (A_i)_{i \in I} \rangle$, which A_i taken as a whole all nuclear definable set.

The family Def_A^n is a Boolean algebra with relative to the usual operations of intersection, union and complement. Full n-type over A the same ultrafilter in this Boolean algebra. The space above the full n-types, denoted $S_n(A)$ is the Stone space corresponding to the Boolean algebra Def_A^n. We introduce in $S_n(A)$ the (normal) topology in which the open base of the set $\langle \varphi(\overline{x}, \overline{a}) \rangle = \{p \in S_n(A) \mid \varphi(\overline{x}, \overline{a}) \in p\}$.

We say that the set X — Σ-defined, if it is definitely some existential formula.

a) The set X is called Jonsson in the theory T if it satisfies the following properties:

X is a – Σ definability subset of C;

$dcl(X)$ is the universe of some existentially-closed submodels of C;

b) The set X is called algebraic Jonsson in the theory T, if it satisfies the following properties:

X is a – definability subset of C;

$acl(X)$ is the universe some existentially-closed submodels of C.

We consider countable language L and Jonsson perfect theory of complete of existential sentences in language L and their semantic models C, in this language and other models (classes existentially closed models of the theories).

If M is model theory of T and φ – a formula language L, then we will use the following notation:

$$\varphi(M) = \{m \in M^n \mid M \models \varphi(m)\}.$$

The set S will call 0–definability if it ϕ-definability (definable without parameters).

The set of all complete types over A the denoted by $S(A)$, i.e. $S(A) = \bigcup_{n\geq 1} S_n(A)$.

Saturated models of Jonsson theory (κ – saturation models power k) are uniquely determined by their power. But they can not exist without a certain set-theoretic assumptions, such as the generalized continuum hypothesis. On the other hand, there are different ways to avoid set-theoretic problems of this sort. For example, assume stable or weaken the concept of the semantic model as in [2]. Therefore, we assume that we got rid of all the issues of the existence of the semantic model.

Further, it is convenient to work within the semantic model C of Jonsson theory, containing all others.

In the future, any set of parameters A considered in the subset C. Model M is a subset of C which is the universe of existentially closed substructure. This means that any $L(M)$ – existential formula $\varphi(x)$, true in C and performed on some element of M. Formula parameters in the future always belongs to C and if we write $\models \varphi(c)$ if $C \models \varphi(c)$.

Lemma 1. Definable set of D is definable over set A, if and only if it is invariant relatively to all automorphisms of the model C, leaving in place each element of A. (Let's call them over automorphisms over A).

It follows that the definable closure $dcl(A)$ of the set A, e.a. the set of all elements of the definable over A, coincides with the set of elements that are invariant relatively to all automorphisms over A.

The element b contained in the finite A is definability set, called algebraic over A. It follows that the element b algebraic over A if and only if it has only a finite number of adjoint over A.

The set $acl(A)$ consisting of all elements algebraic over A, will be called the algebraic closure of the set A.

Forking. We give an axiomatic reference forking.

Let M $\exists$ – saturated existentially closed model power k (k enough big cardinal) of Jonsson theory T ($\exists$ – saturation means the saturation relative to existential types). Let A — the class of all subsets M, P – the class of all $\exists$-types (not necessarily complete), let $JNF \subseteq P \times A$ – a binary relation. We impose JNF the following axiom:

Axiom 1. If $(p, A) \in JNF, f : A \to B$ — automorphism M, then $(f(p), f(A)) \in JNF$.

Axiom 2. If $(p, A) \in JNF, q \subseteq p$, then $(q, A) \in JNF$.

Axiom 3. If $A \subseteq B \subseteq C, p \in S^G(C)$, then $(p, A) \in JNF \Leftrightarrow (p, B) \in JNF$ and $(p \mid^- B, A) \in JNF$.

Axiom 4. If $A \subseteq B, dom(p) \subseteq B, (p, A) \in JNF$, then $\exists q \in S^J(B)$, $p \subseteq q$ and $(q, a) \in JNF$

Axiom 5. There is a cardinal κ such that if $A \subseteq B \subseteq C$, $p \in S^G(C)$, $(p, A) \in JNF$ then $\mid \{q \in S^J(C) : p \subseteq q \text{ and } (q, a) \in JNF\} \mid < \kappa$.

Axiom 6. There is a cardinal ρ such that if $\forall p \in P, \forall A \in A$, if $(p, A) \in JNF$, then $\exists A_1 \subseteq A$, $(\mid A_1 \mid < \rho)$ and $p, A_1 \in JNF$.

Axiom 7. If $p \in S^J(A)$, then $(p, A) \in JNF$.

The classical notion of forking belongs Shelah.

A set of formulas $\{\varphi(\overline{x}, \overline{a_i}) : i < k\}$ are called k — inconsistent for some positive integer k, if every finite subset p of power k is inconsistent, ie. $\models \neg\overline{x}(\varphi(\overline{x}, \overline{a_{i_1}} \wedge ... \wedge \varphi(\overline{x}, \overline{a_{i_k}}))$ for each $i_1 < ... < i_k < k$.

Partial type p divided over a set of relative to $k \in \omega$ if there is a formula $\varphi(\overline{x}, \overline{a})$ and a sequence $\langle \overline{a_i} : i \in \omega \rangle$ such that

1) $p \vdash \varphi(\overline{x}, \overline{a})$;

2) $tp(\overline{a}/A) = tp(\overline{a_i}/A)$ for all i;

3) $\varphi\{(\overline{x}, \overline{a}) : i \in \omega\}$, k – not jointly.

It is also p divided over A if p divided over A relative to some k. In addition, p fork over A to T, if there are formulas $\phi_1(\overline{x}, \overline{a_0}), ..., \phi_n(\overline{x}, \overline{a_n})$ such that:

(i) $p \models \bigvee_{0\leq i\leq n} \varphi_i(\overline{x}, \overline{a_i})$;

(ii) $\phi_i(\overline{x}, \overline{a_i})$ divided over A for any i.

The following result makes it possible to use all features of forking for complete theories in the class above in this report Jonsson theories.

Theorem 1. Let T perfect Jonsson theory of complete for $\exists$ – sentences. Then the following conditions are equivalent:

• the relation JNF satisfies the axioms 1–7 relative to the theory T;

• T^* stable and for all $p \in P$, $A \in A$ $((p, A) \in JNF \Leftrightarrow p$ not fork over $A)$.

Let T is Jonsson theory, $S^J(X)$ is the set of all full n-types over X, joint with T, for all finite n.

We say that Jonsson theory T is $J - \lambda$ - stable, if for any T existentially closed of model A, for any subset A of the set A, $|X| \geq \lambda \Rightarrow| S^J(X) |\leq \lambda$.

Theorem 2. Let T — complete for existential sentences is perfect Jonsson theory, $\lambda \geq \omega$. Then the following conditions are equivalent:

$T - J - \lambda$-stably;

$T - J - \lambda$-stably, where T^* is center of theory T.

Definition 1. Suppose that A⊆B, $p \in S_n(A)$, $q \in S_n(B)$, and p⊆q. If RM(q) < RM(p), we say that q is a forking extension of p and that q forks over A. If RM(q) = RM(p), we say that q is a nonforking extension of p.

Our first goal is to show that nonforking extensions exist.

Theorem 3. (Existence of nonforking extensions) Suppose that $p \in S_n(A)$ and A⊆B.

i) There is $q \in S_n(B)$ a nonforking extension of p.

ii) There are at most $deg_M(p)$ nonforking extensions of p in $S_n(B)$ and if $\mathcal{M}$ is an $\exists - \aleph_0$ – saturated model with A⊆M, there are exactly $deg_M(p)$ nonforking extensions of p in S $_n$(M). iii) There is at most one $q \in S_n(B)$, a nonforking extension of p with $deg_M(p) = deg_M(q)$. In particular, if $deg_M(p)$=1, then p has a unique nonforking extension in $S_n(B)$.

Independence. The nonforking extensions will be the «free» ones.

Forking as in Theorem 1 can be used to give a notion of independence in $J - \omega$-stable theories.

Definition 2. We say that $\overline{a}$ is independent from B over A if tp($\overline{a}$/A) does not fork over A ∪ B. We write a $\overline{a} \perp_A B$.

This notion of independence has many desirable properties.

Lemma 2 (Monotonicity). If a $\overline{a} \perp_A B$ and C⊆B, then a $\overline{a} \perp_A C$.

Lemma 3 (Transitivity). a $\overline{a} \perp_A \overline{b}, \overline{c}$ if and only if a $\overline{a} \perp_A \overline{b}$ and $\overline{a} \perp_{A,\overline{b}} \overline{c}$.

Lemma 4 (Finite Basis). a $\overline{a} \perp_A B$ if and only if a $\overline{a} \perp_A B_0$ for all finite $B \subseteq B_0$.

Lemma 5 (Symmetry). If a $\overline{a} \perp_A \overline{b}$, then $\overline{b} \perp_A \overline{a}$.

Corollary 1. $\overline{a}, \overline{b} \perp_A C$ if and only if $\overline{a} \perp_A C$ and $\overline{b} \perp_{A,\overline{a}} C$.

Symmetry also gives an easy proof that no type forks when it is extended to the algebraic closure.

Corollary 2. For any $\overline{a}, \overline{a} \perp_A acl(A)$.

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А.Р. Ешкеев

Йонсондық жиындар үшін тәуелсіздік қасиеті

Мақалада жүргізілген зерттеулер индуктивті теорияның ішкі класы болатын, жалпы айтканда, толық емес кластар теориясының модельді-теоретикалық қасиеттерін сипаттаумен байланысты. Бұл теориялар алгебрада және модельдер теориясында да кеңінен қарастырылған. Мұндай теориялар йонсондық деп аталады. Осы теорияларды зерттеу үшін жаңа әдіс-тәсілдер енгізілген. Йонсондық теорияның семантикалық модельдер жиынында айрықша жиындар қарастырылды, олар, біріншіден, кейбір экзистенциалдық формулаларды жүзеге асыу болып табылады, екіншіден, жиындардың тұйықталуы бізге семантикалық модельдің экзистенциалды тұйықталуының ішкі моделінің негізгі жиынын береді. Сонымен қатар орталық орбиталдық типтерді зерттеу үшін техника дамиды. Кемел йонсондық теориялар модельді-теоретикалық зерттеу үшін қолайлы екені жақсы белгілі. Практика жүзінде кемелділік жағдайында семантикалық тәсіл көмегімен жоғарыда айтылған нысандардың анықтамаларын бере аламыз. Ягни, олар йонсондық теориялар және оның экзистенционалды-түйық модельдер класы. Біздің ғылыми қызығушылығымыз, жалпы айтқанда, индуктивті теориялардың ішкі кластары болатын теориялардың толық емес кластарын кейбір модельді-теоретикалық қасиеттермен сипаттауға байланысты. Бұл теориялар алгебрада және модельдер теориясында кеңінен зерттелді. Мақалада йонсондық теорияның фрагменті үшін форкинг ұғымын келтірді. Форкинг болмаса, онда кеңейтулер бос болады. Сонымен қатар біз тәуелсіздік ұғымы үшін транзитивтілік, монотондылық, үзіліссіздік және симметрия сияқты көптеген маңызды қасиеттерді қарастырдық.

А.Р. Ешкеев

Свойство независимости для йонсоновских множеств

Исследования, проведенные в статье, связаны с описанием теоретико-модельных свойств некоторых, вообще говоря, неполных классов теорий, которые являются подклассом индуктивных теорий. Эти теории, хорошо изучаемые и в алгебре, и в теории моделей, называются йонсоновскими. Для изучения этих теорий вводится новый подход исследования. А именно, на подмножествах семантической модели йонсоновской теории выделяются особые множества, которые являются, во-первых, реализациями некоторой экзистенциальной формулы, во-вторых, замыкание этих множеств дает нам основное множество некоторой экзистенциально замкнутой подмодели семантической модели. Помимо этого развивается техника для изучения центральных орбитальных типов. Хорошо известно, что совершенные йонсоновские теории достаточно удобны для теоретико-модельных исследований. Практически, в случае совершенности, мы можем утверждать, что с помощью семантического метода дается определенное описание указанных выше объектов (йонсоновской теории и классом ее экзистенциально-замкнутых моделей). В этой статье рассмотрены понятия форкинга для фрагмента фиксируемой йонсоновской теории и независимости, а также многие полезные свойства, такие как транзитивность, монотонность, непрерывность и симметрия.

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Some properties of Morly rank over Jonsson sets

This article introduced and discussed the concepts of minimal Jonsson sets and respectively strongly minimal Jonsson sets. On this basis, it introduces the concept of the independence of special subsets of existentially closed submodel of the semantic model. The notion of independence leads to the concept of basis and then we have an analogue of the Jonsson theorem on uncountable categorical. The concept of strongly minimal, as for sets and so for theories played a decisive role in obtaining results on the description of uncountable - categorical theories. It is well known that Jonsson Theories are a natural subclass of the broad class of theories, as a class of inductive theories. As is known, the basic examples theories of algebra are examples of inductive theories, and they tend to represent an example of incomplete theories. This modern apparatus of Model Theory developed mainly for complete theories, so nowadays technique studying incomplete theories noticeable poorer than for complete theories. Thus, all of the above says that the study of model-theoretic properties Jonsson theories is an actual problem. This article describes the basic properties of the Morley rank over Jonsson subsets of semantic model for some Jonsson theory.

Key words: Jonsson theory, Jonsson set, fragment of Jonsson sets, lattice existential formulas of Jonsson theory.

This article is devoted to the study of the concept Jonsson sets and its application. The concept of Jonsson sets defined in [1] and further results were obtained, which were presented in [2–4].

The concept of strongly minimal, as for sets and so for theories played a decisive role in obtaining results on the description of uncountable – categorical theories in [5].

It is well known that Jonsson Theories are a natural subclass of the broad class of theories, as a class of inductive theories. As is known, the basic examples theories of algebra are examples of inductive theories, and they tend to represent an example of incomplete theories. This modern apparatus of Model Theory developed mainly for complete theories, so nowadays technique studying incomplete theories noticeable poorer than for complete theories.

On the one hand Jonsson conditions – a natural algebraic demands that emerge in the study of a broad class of algebras.

On the other hand natural examples Jonsson theories enough, for example, the theory of Boolean algebras, Abelian groups, fields of fixed characteristics, polygons, and so on. These examples are important, as in algebra, and in different areas of mathematics. As can be seen from the following is a list of the scope of the technique developed for studying Jonsson theories can be quite broad.

Thus, all of the above says that the study of model-theoretic properties Jonsson theories is an actual problem.

From the experience of the study of inductive theory [6], it follows that Jonsson theory, as a subclass of inductive theories are such a part in which there are certain methods of investigation incomplete theories, namely the properties of the transfer method of the first order center of Jonsson theories itself Jonsson theories. This method, and on research in the study Jonsson theories and unrelated to the material in this article, we refer the reader to the following sources [7–10].

As noted above, the basic technique associated with more subtle methods of researches of behavior elements of the model, refers to the prerogative of technology study complete theories. Therefore, even just trying to find a generalization of the standard terms of arsenal full of theories, we may come across either a tautology or a concept, which is technically not justified. Therefore, it was proposed and Jonsson set. Recall the basic definitions of [1], which are connected with these sets.

Let there be given an arbitary language L.

The theory T is called Jonsson if it satisfies the following conditions:

1) Theory T has an infinite models;

2) Theory T is inductive;

3) Theory T admits JEP;

4) Theory T admits AP.

Jonsson theory T is called a perfect theory, if the semantic model is saturated.

Let T — Jonsson perfect theory is full for existential sentences in the language L and its semantic model is .

We say that the set X — Σ-defined, if it is definitely some existential formula.

a) The set X is called Jonsson in the theory T if it satisfies the following properties:

• X is a — Σ definability subset of C;

• $dcl(X)$ is the universe of some existentially-closed submodels of C;

b) The set X is called algebraic Jonsson in the theory T, if it satisfies the following properties:

• X is a – definability subset of C;

• $acl(X)$ is the universe of some existentially-closed submodels of C.

The definition sets Jonsson can see that they are arranged very simply in the sense of Morley rank [1]. It turns out that the elements of the set-theoretic difference (wells) closing and the set have rank 0; they are algebraic. So, this is a case where we can work with the elements even in the case of incomplete.

The second useful moment of this definition of Jonsson set is that we are closing this set just get some existentially closed model. This in turn gives us firstly to identify Jonsson fragment in the set under consideration and in principle any theory.

At the moment, well enough studied are the perfect Jonsson theories. For them, was proved the criterion of perfectly [7] that allowed to obtain many model-theoretic facts about Jonsson theory and its center. There is a full description of how these theories center and classes of models.

If the case study of complete theories we are mainly dealing with two objects, it herself theory of its models in the case study of Jonsson theory we as models consider the class of existentially closed models of the theory, as well as an additional condition is a certain completeness of this theory in a logical sense.

At least, this theory must be existentially complete.

We give a definition Jonsson fragment. We say that all the $\forall\exists$ — investigation of any theory create Jonsson fragment of this theory, if the deductive closure of these $\forall\exists$ — consequences are Jonsson theories.

Due to the fact that this is not always true, it would be interesting to be able to allocate in an arbitrary theory a part that will Jonsson theory. This problem takes place if only because of the fact that any theory morlization us it provides, moreover, the resulting theory is perfect [6].

Another way is the use of the fact that any countable model of inductive theory necessarily isomorphic to invest in some existentially closed model of the theory [6]. Then we consider all $\forall\exists$ – sentences true in this model.

Then in the case Jonsson theory is well known fact that $\forall\exists$ – sentences are true in the existentially closed model form the Jonsson theories. Otherwise, at the moment apart from enriching the signature (in case skolemization morlezation and [6]), we have no way to reach Jonsson theory.

To study the behavior of elements in case wells Jonsson sets, we can always see $\forall\exists$ - sentences is true in the above closures Jonsson set. In By the above, in this case, that consideration of the set of suggestions will Jonsson theory.

Obtained in this case will be called a theory Jonsson fragment corresponding Jonsson set. It is clear that we can carry out research Jonsson fragments about the connection to the original theory that the new formulation of the problem is the study of Jonsson's theory.

The main objective of this article is the following problem:

Recall that Jonsson theory T has a semantic model of high power enough. If this model is saturated, this Jonsson theory is called perfect. Semantic models of perfect Jonsson theory uniquely determined by their power.

Furthermore, since we are dealing with a perfect Jonsson theories, it is convenient for us to work within a large semantic existentially closed model containing all other existentially closed model considered perfect Jonsson theory. We call this model of universal existential area.

It can also be characterized by the following conditions.

1. Each model of this theory is isomorphic to put in $\mathfrak{C}$.

2. Every isomorphism between the two submodels extends to an automorphism of model $\mathfrak{C}$.

We will not consider all the subsets, and only a subset of the Jonsson.

For any Σ – definable subsets of semantic model we have, the following result.

Lemma 1. Σ – definable subset of the semantic model is definable over a set of parameters A semantic model if and only if it is invariant under all automorphisms of the model $\mathfrak{C}$, leaving in place each element of A.

It follows that the definable closure $dcl(A)$ of Jonsson sets A, i.e, the set of all elements, definable over A, coincide with the set elements that are invariant relatively all automorphisms over A.

From Lemma 1 it follows that the element b is algebraic over A if and only if it has only a finite number of conjugates over A.

We define the rank of Morley for existentially definable subsets of the semantic model.

We want to assign to each Σ – is definable subset D of the semantic model ordinal number (or, perhaps, -1 or ∞) – its rank Morley, denoted by MR.

First, we define relation $MR(\mathbb{D}) \geq \alpha$ by recursion on the ordinal α.

Let T is perfect Jonsson theory, C-its semantic model. D is a definable subset of C.

Definition 1:

–$MR(\mathbb{D}) \geq 0$ if and only if $\mathbb{D}$ is empty;

–$MR(\mathbb{D}) \geq \lambda$ if and only if $MR(\mathbb{D}) \geq \alpha$ for all $\alpha < \lambda$ (λ- the limiting ordinal);

–$MR(\mathbb{D}) \geq \lambda$ if and only if $\mathbb{D}$ exists an infinite family $(\mathbb{D}_i)$ disjoint Σ definable subsets, such that

$$MR[(\mathbb{D})_i] \geq \lambda \text{ for all } i.$$

Then Morley rank of class $\mathbb{D}$ is $MR(\mathbb{D}) = sup\{\alpha | MR(\mathbb{D}) \geq \alpha\}$.

Moreover, we assume that $MR(\emptyset) = -1$ and $MR(\mathbb{D}) = \infty$ if 48 / for all α (in the latter case we say that $\mathbb{D}$ has not rank).

Note that Σ – definable class has rank -1 if it is empty; rank 0 if it is finite; rank 1 if it is infinite, but does not contain an infinite family of disjoint infinite Σ – definable classes.

Lemma 2. The relation $MR(\mathbb{D}_1 \cup \mathbb{D}_2) = max(MR(\mathbb{D}_1), MR(\mathbb{D}_2)$ is true.

Definition 2. The degree of Morley $Md(\mathbb{D})$ Jonsson subset $\mathbb{D}$ of semantic nodel having Morley rank α is the maximum length d of its decomposition $\mathbb{D} = \mathbb{D}_1 \cup ... \cup \mathbb{D}_d$ into disjoint existentially definable subsets of rank α.

In the case of rank 0 degree existentially definable subset $\mathbb{D}$ is a simply a number of its elements. If existentially definable subset has not rank, it is not determined the degree of Morley.

Consider Jonsson minimal sets. Then under the structure of the model refers to the signature of model or of the language L under consideration Jonsson theory.

Let $\mathcal{M}$ is the structure, and let $\mathbb{D} \subseteq M^n$ the infinite Σ - defining subset. We say that D is minimal in $\mathcal{M}$ if any for the Σ - defining $Y \subseteq \mathbb{D}$ either Y is finite or $\mathbb{D}/Y$ is finite. If $\phi(\overline{v}, \overline{a})$ is a formula that determines the $\mathbb{D}$, then we can also say that $\phi(\overline{v}, \overline{a})$ is minimal.

We say that $\mathbb{D}$ and ϕ are Jonsson strongly minimal, if ϕ is minimal any existentially closed extensions $\mathcal{N}$ of $\mathcal{M}$.

We say that theory T is Jonsson strongly minimal if $\forall \mathcal{M} \in E_T$, $\mathcal{M}$ is Jonsson strongly minimal. Consider the example of the algebraic closure of a few Jonsson strongly minimal theories.

If K is an algebraically closed field and $A \subseteq K$, then $acl(A)$ is algebraically closed subfield generated.

The following properties of the algebraic closure true for any algebraically Jonsson set $\mathbb{D}$

If $A \subseteq B$, then $acl(A) \subseteq acl(B)$.

If $a \in acl(A)$ then $a \in acl(A_0)$ for some finite $A_0 \subseteq A$.

A more subtle property is true if $\mathbb{D}$ is Jonsson strongly minimal.

Lemma about a replacement. Suppose that $\mathbb{D}$ is a subset of the semantic model of the theory and it Jonsson strongly minimal $A \subseteq \mathbb{D}$ and $a, b \in \mathbb{D}$. If $a \in acl(A \cup \{b\}) \setminus acl(A)$, then $b \in acl(A \cup \{a\})$.

Remark. Jonsson strongly minimal set is existentially definable subset of the semantic model of the theory of rank 1 and degree 1 in the sense of Morley.

Definition 3:

1. Jonsson Theories T is Jonsson totally transcendence, if every existentially definable subset of its semantic model has Morley rank.

2. Theory T is Jonsson ω-stable if the number of existential types is countable for every countable subset A of semantic model.

Theorem. Jonsson theory T is Jonsson totally transcendence, if and only if it is Jonsson ω - stably.

Lemma 3. Let a and b are an arbitary elements of the semantic model. If the element b algebraically over A and a, where A is existentially definable subset of the semantic model, then $MR(b/A) \leq MR(a/A)$.

Corollary 1. Let M — some ω-saturated existentially closed submodel semantic model, φ some $L(M)$ — formula rank α and degree d in Morley. Then we can expand φ on $L(M)$ — formulae $\varphi_1, ..., \varphi_m$ of rank α and degree 1.

Anyway Jonsson strongly minimal set, we can define the concept of independence, which generalizes linear independence in vector spaces and algebraic independence of algebraically closed fields.

We fix $\mathcal{M}$ and D is Jonsson strongly minimal set in $\mathcal{M}$ existentally closed under the model semantic model of Jonsson theory T.

Definition 4. We say that $A \subseteq D$ independently if $\alpha \notin acl(A\{a\})$ for all $a \in A$. If $C \subset D$ we say that A independently over C, if $\alpha \notin acl(C \cup A\{a\})$ for all $a \in A$.

We show that the endless independent sets are sets of indistinguishable elements.

Lemma 4. Suppose T is Jonsson strongly minimal theory and $\phi(v)$ is Jonsson strongly minimal formula with parameters from A, where either $A \neq \emptyset$ or $A \subseteq M_0$ where $\mathcal{M}_0 \models E_T$,$\mathcal{M}_0 \prec_1 \mathcal{M}$ and $\mathcal{M}_0 \prec_1 \mathcal{N}$. If $a_1, ..., a_n \in \phi(M)$ independent over A and $b_1, ..., b_n \in \phi(N)$ are independent over A, then complete existential types $tp^{\mathcal{M}}(\overline{a}/A), tp^{\mathcal{N}}(\overline{b}/A)$ are equal.

Corollary 2. If $\mathcal{M}$,$\mathcal{N} \models T$, and $\phi(v)$ as indicated above, B is an infinite subset $\phi(\mathcal{M})$ independent over A and C is an infinite subset $\phi(\mathcal{N})$ independent over A, then B and C are indistinguishable infinite sets of the same type over A.

Therefore, power is the only way to distinguish independent subset $\mathbb{D}$.

Definition 5. We say that A is the basis for $Y \subseteq \mathbb{D}$ if $A \subseteq Y$ independent and when $acl(A) = acl(Y)$.

It is obvious that any maximal independent subset of Y is a basis for Y. Just as in the vector spaces and algebraically closed fields, any two bases have the same capacity.

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А.Р. Ешкеев

Йонсондық жиындардың ішіндегі Морли рангінің кейбір қасиеттері

Мақалада йонсондық жиындар ұғымы енгізілген және қарастырылған. Осы негізде семантикалық модельдің экзистенционалды тұйық ішкі моделінің тәуелсіз арнайы ішкі жиындары ұғымы енгізілді. Тәуелсіздік ұғымы базис ұғымына алып келеді және әрі қарай біз саналымсыз үзілді-кесілгендігі туралы теореманың йонсондық аналогін қарастырдық. Қатты минималды ұғымы саналымсыз-үзілді-кесілгендігі теориялардың сипаттауының нәтижесін алу үшін теориялар сияқты жиындар үшін де үлкен рөл атқарады. Йонсондық теориялар индуктивті теориялар класы сияқты теорияның кең класы болатын кәдімгі ішкі класты көрсететіні белгілі. Бізге алгебра теориясының негізгі мысалдары белгілі. Олар индуктивті теориялардың мысалдары да бола алады және ереже бойынша олар толық емес теориялардың мысалдары. Сонымен қатар модельдер теориясының қазіргі зерттеу аппараты толық теориялар үшін дамыған, сондықтан бүгінгі таңда толық теорияларға қарағанда, толық емес теорияларды зерттеу техникасы анағұрлым кем дамыған. Яғни, жоғары айтқандардан йонсондық теорияның модельді-теоретикалық қасиеттерін оқу өзекті мәселе болып табылады. Мақалада кейбір йонсондық теориялар үшін йонсондық ішкі жиындардың семантикалық модельдері мен Морли рангінің негізгі қасиеттері сипатталды.

А.Р. Ешкеев

Некоторые свойства ранга Морли над йонсоновскими множествами

В статье введены и рассмотрены понятия «йонсоновские множества» и, сответственно «сильно минимальные йонсоновские множества». На этой основе введено понятие независимости специальных подмножеств экзистенциально замкнутой подмодели семантической модели. Понятие независимости приводит к понятию базиса, и далее мы имеем йонсоновский аналог теоремы о несчетной категоричности. Понятие сильной минимальности, как для множеств, так и для теорий, сыграло решающую роль при получении результата об описании несчетно-категоричных теорий. Хорошо известно, что йонсоновские теории представляют собой естественный подкласс такого широкого класса теорий, как класс индуктивных теорий. Как известно, основные примеры теорий алгебр являются примерами индуктивных теорий, и они, как правило, представляют пример неполных теорий. При этом современный аппарат теории моделей развивался в основном для полных теорий, поэтому на сегодняшний день техника изучения неполных теорий заметно беднее, чем для полных теорий. Таким образом, всё сказанное выше говорит о том, что изучение теоретико-модельных свойств йонсоновских теорий является актуальной задачей. Эта статья описывает основные свойства ранга Морли над йонсоновскими подмножествами семантической модели для некоторых йонсоновских теорий.

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