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On the boundary value problem for the vibration and wave processes in two-dimensional environs

In the article research of the third boundary value problem for the two-dimensional equation of free membrane vibrations is presented. The solution of the original differential operator is found in the form of a combination of the linearly independent system of orthonormal eigenfunctions on the given interval. Using the spectral decomposition for sufficiently smooth function, one can obtain the exact analytical representation of the deflection function for investigated problem in two-dimensional environs. The deflection function characterizes a membrane deviation from the equilibrium position.

Key words: boundary value problem, membrane, vibrations, spectrum problem, orthonormal system of functions, deflection function.

Vibration process of a flat homogeneous membrane is described by the equation

$$u_{tt} = a^2(u_{xx} + u_{yy}). \tag{1}$$

The rectangular membrane has sides a and b. This membrane is fixed along side edges and is located in a plane (x, y), where $0 < x < a$, $0 < y < b$, $t > 0$. The vibration of this membrane is caused by using of the initial deflection and the initial velocity [1].

To find the function $u(x, y, t)$, characterizing the deviation from the equilibrium position of the membrane (the deflection), it is necessary to solve the equation of oscillations at the given initial conditions

$$u(x, y, 0) = \varphi(x, y); \tag{2}$$

$$u_t(x, y, 0) = \psi(x, y) \tag{3}$$

and with boundary conditions

$$u_x(0, y, t) - h\, u(0, y, t) = 0, \quad u_x(a, y, t) + h\, u(a, y, t) = 0; \tag{4}$$

$$u_y(x, 0, t) - g\, u(x, 0, t) = 0, \quad u_y(x, b, t) + g\, u(x, b, t) = 0, \tag{5}$$

$h > 0$, $g > 0$ are given constants [2].

The unknown function $u(x, y, t)$ characterizes the deflection of the membrane at the time t. The solution of problem (1) - (5) is founded in the form function, not identically zero.

$$u(x, y, t) = v(x, y) \cdot T(t). \tag{6}$$

We substitute the function expression $u(x, y, t)$ in equation (1) and with dividing of both sides of the equation by $a^2 \cdot v \cdot T$ we obtain

$$\frac{T''}{a^2\, T} = \frac{v_{xx} + v_{yy}}{v}, \tag{7}$$

at the same time we don't lose solutions since $T(t) \neq 0$, $v(x, y) \neq 0$.

The function (6) will be a solution of equation (1) if equality (7) is satisfied identically for all values of the variables $0 < x < a$, $0 < y < b$, $t > 0$.

The right-hand side of equality (7) is a function of variables (x, y) and the left side depends only on variable t.

Therefore, equality in the ratio (7) is achieved when the right and left sides of (7) with changing their arguments remain constant. Let it is equal to σ.

$$\frac{T''(t)}{a^2\,T(t)} = \frac{v_{xx}(x,y) + v_{yy}(x,y)}{v(x,y)} = \sigma. \tag{8}$$

From the ratio (8) we obtain the homogeneous differential equation of second order for the function $T(t)$

$$T'' - \sigma \cdot a^2\,T = 0 \tag{9}$$

and for the function $v(x, y)$ we have the following boundary value problem

$$v_{xx} + v_{yy} - \sigma v = 0; \tag{10}$$

$$v_x(0,y) + h\,v(0,y) = 0, \quad v_x(a,y) + h\,v(a,y) = 0; \tag{11}$$

$$v_y(x,0) - g\,v(x,0) = 0, \quad v_y(x,b) + g\,v(x,b) = 0. \tag{12}$$

As a result, our problem of the eigenvalues is the solving of the homogeneous partial differential equation (10) with the given boundary conditions (11), (12).

The solution of the boundary value problem (10)-(12) will be sought in the form

$$v(x,y) = X(x) \cdot Y(y), \tag{13}$$

where the function $v(x, y) \neq 0$.

Substituting (13) into (10) and dividing the resulting equation on $X \cdot Y \neq 0$, we obtain the following relation

$$\frac{-Y'' + \sigma\,Y}{Y} = \frac{X''}{X}. \tag{14}$$

The right-hand side of equality (14) is a function of variable x, and the left side depends only on variable y. Hence, equality in the ratio (14) is achieved when the right and left sides of (14) with changing their arguments remain constant. Let it is equal to p.

$$\frac{-Y''(y) + \sigma\,Y}{Y} = \frac{X''}{X} = p. \tag{15}$$

From the equality (15) we obtain two homogeneous second-order differential equation

$$X'' - pX = 0, \quad Y'' - qY = 0,$$

where p, q are constants and $q = \sigma - p$.

The boundary conditions for the functions $X(x)$ and $Y(y)$ follow from the boundary conditions (11), (12) for the function $v(x, y)$.

$$v_x(0,y) - hv(0,y) = Y(y)\left[X'(0) - hX(0)\right] = 0 \;\Rightarrow\; X'(0) - hX(0) = 0;$$

$$v_x(a,y) + hv(a,y) = Y(y)\left[X'(a) + hX(a)\right] = 0 \;\Rightarrow\; X'(a) + hX(a) = 0;$$

$$v_y(x,0) - gv(x,0) = X(x)\left[Y'(0) - gY(0)\right] = 0 \;\Rightarrow\; Y'(0) - gY(0) = 0;$$

$$v_y(x,b) + gv(x,b) = X(x)\left[Y'(b) + gY(b)\right] = 0 \;\Rightarrow\; Y'(b) + gY(b) = 0.$$

Thus, we get two spectral eigenvalue problem

$$\begin{cases} X'' - pX = 0; \\ X'(0) - hX(0) = 0; \\ X'(a) + hX(a) = 0; \end{cases} \tag{16}$$

$$\begin{cases} Y'' - qY = 0; \\ Y'(0) - qY(0) = 0; \\ Y'(a) + qY(a) = 0. \end{cases} \tag{17}$$

Note 1. We investigate for what values C the spectral problem

$$\begin{cases} Z'' - CZ = 0; \\ \alpha Z(0) + \beta Z'(0) = 0; \\ \alpha_1 Z(l) + \beta_1 Z'(l) = 0, \end{cases} \tag{18}$$

where $Z = Z(z)$; $0 < z < l$; l, α, α_1, β, β_1 are initially given numbers, has non-trivial solutions.

I. Let $C = 0$, then $Z'' = 0$, $Z(z) = Az + B$, $Z'(z) = A$.

$\begin{cases} \alpha Z(0) + \beta Z'(0) = \alpha B + \beta A = 0; \\ \alpha_1 Z(l) + \beta_1 Z'(l) = \alpha_1 Al + \alpha_1 B + \beta_1 A = 0; \end{cases}$

$\Delta = \begin{vmatrix} \beta & \alpha \\ \beta_1 + \alpha_1 l & \alpha_1 \end{vmatrix} \Rightarrow \Delta = \alpha_1\beta - \alpha(\beta_1 + \alpha_1 l) \neq 0.$

If $\alpha \neq 0$ and $\alpha_1 \neq 0$ simultaneously, then $\Delta \neq 0 \Rightarrow A = B = 0 \Rightarrow Z(z) \equiv 0$.

The particular case: $\alpha = \alpha_1 = 0$.

$\begin{cases} Z'' = 0, \\ Z'(0) = Z'(l) = 0, \end{cases} \begin{cases} Z(z) = A_z + B, \\ Z'(0) = Z'(l) = A = 0 \end{cases} \Rightarrow Z(z) = B \neq 0.$

II. Let $C > 0$, $C = \mu^2$, then

$$Z'' - \mu^2 Z = 0 \Rightarrow \begin{cases} Z(z) = A\mathrm{ch}\mu z + B\mathrm{sh}\mu z; \\ Z'(z) = \mu A\mathrm{sh}\mu z + \mu B\mathrm{ch}\mu z; \end{cases}$$

$$\begin{cases} \alpha Z(0) + \beta Z'(0) = \alpha A + \beta\mu B = 0; \\ \alpha_1 Z(l) + \beta_1 Z'(l) = \alpha_1 A\mathrm{ch}\mu l + \alpha_1 B\mathrm{sh}\mu l + \beta_1\mu A\mathrm{sh}\mu l + \beta_1\mu B\mathrm{ch}\mu l = 0; \end{cases}$$

$$\begin{cases} B = -\frac{\alpha}{\beta\mu}A; \\ A\left(\alpha_1\mathrm{ch}\mu l - \frac{\alpha\alpha_1}{\beta\mu}\mathrm{sh}\mu l + \beta_1\mu\mathrm{sh}\mu l - \frac{\alpha\beta_1}{\beta}\mathrm{ch}\mu l\right) = 0; \end{cases}$$

$$\left(\alpha_1 - \frac{\alpha\beta_1}{\beta}\right)\mathrm{ch}\mu l - \left(\beta_1\mu - \frac{\alpha\alpha_1}{\beta\mu}\right)\mathrm{sh}\mu l \neq 0, \quad \forall \alpha,\ \alpha_1,\ \beta,\ \beta_1,\ l \in R;$$

$A = B = 0 \Rightarrow Z(z) \equiv 0$.

II$'$. $C > 0$, $C = \mu^2$, $Z'' - \mu^2 Z = 0 \Rightarrow \begin{cases} Z(z) = Ae^{\mu z} + Be^{-\mu z}; \\ Z'(z) = \mu Ae^{\mu z} - \mu Be^{-\mu z}; \end{cases}$

$A = B = 0 \Rightarrow Z(z) \equiv 0$,

$$\begin{cases} B = -\frac{\alpha+\beta\mu}{\alpha-\beta\mu}A; \\ A\left(\alpha_1 e^{\mu l} - \frac{\alpha_1(\alpha+\beta\mu)}{\alpha-\beta\mu}e^{-\mu l} + \beta_1\mu e^{\mu l} + \frac{\beta_1\mu(\alpha+\beta\mu)}{\alpha-\beta\mu}e^{-\mu l}\right) = 0; \end{cases}$$

$\mu \neq 0 \Rightarrow e^{\mu l}$ and $e^{-\mu l}$ linearly independent functions, hence their coefficients must be zero, but $(\alpha_1 + \beta_1\mu) - \forall$, $\frac{(\beta_1\mu - \alpha_1)(\alpha+\beta\mu)}{\alpha-\beta\nu} - \forall$,

$$\Rightarrow \begin{cases} \alpha_1+\beta_1\mu=0, \\ \frac{(\beta_1\mu-\alpha_1)(\alpha-\beta\mu)}{\alpha-\beta\mu}=0 \end{cases} \Rightarrow \begin{cases} \mu=-\frac{\alpha_1}{\beta_1}, \\ \frac{\left(-\frac{\beta_1\alpha_1}{\beta_1}-\alpha_1\right)\left(\alpha+\beta\left(-\frac{\alpha_1}{\beta_1}\right)\right)}{\alpha-\beta\left(-\frac{\alpha_1}{\beta_1}\right)}=\frac{-2\alpha_1\left(\alpha-\frac{\beta\alpha_1}{\beta_1}\right)}{\alpha+\frac{\alpha_1\beta}{\beta}}\neq 0 \end{cases}$$

$$\Rightarrow A=B=0\Rightarrow Z(z)\equiv 0.$$

III. Let $C<0$, $C=-\lambda^2$, then
$Z''+\lambda^2Z=0$, $Z(z)=A\cos\lambda z+B\sin\lambda z$, $Z'(z)=-A\lambda\sin\lambda z+B\lambda\cos\lambda z$.

$$\begin{cases} \alpha Z(0)+\beta Z'(0)=\alpha A+\beta\lambda B=0, \\ \alpha_1 Z(l)+\beta_1 Z'(l)=\alpha_1(A\cos\lambda l+B\sin\lambda l)+\beta_1\lambda(-A\sin\lambda l+B\cos\lambda l)=0), \end{cases}$$

$$\begin{cases} B=-\frac{\alpha}{\beta\lambda}A, \\ A\left(\alpha_1\cos\lambda l+\left(-\frac{\alpha\alpha_1}{\beta\lambda}\right)\sin\lambda l-\beta_1\lambda\sin\lambda l-\frac{\alpha\beta_1}{\beta}\cos\lambda l\right)=0, \end{cases}$$

$$\left(\alpha_1-\frac{\alpha\beta_1}{\beta}\right)\cos\lambda l=\left(\frac{\alpha\alpha_1}{\beta\lambda}+\beta_1\lambda\right)\sin\lambda l,$$

$\mathrm{tg}\lambda l=\frac{(\alpha_1\beta-\alpha\beta_1)\lambda}{\alpha\alpha_1+\beta_1\beta\lambda^2}$ is a transcendental equation having roots. $A\neq 0$, $B\neq 0\Rightarrow Z(z)\neq 0$. So, we got that the spectral problem (18) has non-trivial solutions with $C<0$. We denotate $C=-\lambda^2$, then the spectral problem

$$\begin{cases} Z''+\lambda^2Z=0; \\ Z'(0)-hZ(0)=0; \\ Z'(l)+hZ(l)=0, \end{cases} \tag{19}$$

where $0<z<l$ and $h>0$ is the given constant, has non-trivial solutions.

Note 2. We will find the eigenvalues and eigenfunctions of the problem (19).

$$\begin{cases} Z''+\lambda^2Z=0; \\ Z'(0)-hZ(0)=0; \quad 0<z<l, \\ Z'(l)+hZ(l)=0; \end{cases}$$

$$Z(z)=A\cos\lambda z+B\sin\lambda z.$$

$$\begin{cases} Z'(0)-hZ(0)=B\lambda-hA=0; \\ Z'(l)+hZ(l)=-A\lambda\sin\lambda l+B\lambda\cos\lambda l+hA\cos\lambda h+Bh\sin\lambda l=0, \end{cases}$$

$$\begin{cases} B=\frac{h}{\lambda}A; \\ A\left(-\lambda\sin\lambda l+2h\cos\lambda l+\frac{h^2}{\lambda}\sin\lambda l=0\right); \end{cases}$$

$A\neq 0\Rightarrow -\sin\lambda l+2h\cos\lambda l+\frac{h^2}{\lambda}\sin\lambda l=0\Rightarrow \mathrm{ctg}\lambda l=\frac{1}{2}\left(\frac{\lambda}{h}-\frac{h}{\lambda}\right).$

$Z_k(z)=A_k\left(\lambda_k\cos\lambda_k z+h\sin\lambda_k z\right);$ $k=1,\,2...$, where λ_k are the roots of the equation $\mathrm{ctg}\lambda l==\frac{1}{2}\left(\frac{\lambda}{h}-\frac{h}{\lambda}\right)$.

Based on the notes 1 and 2 we have the following solutions of the problems (16), (17)

$$X_k(x)=A_k(\lambda_k\cos\lambda_k x+h\sin\lambda_k x),\ k=1,\,2... \tag{20}$$

$p=-\lambda^2$; λ_k are the roots of the equation $\mathrm{ctg}\lambda a=\frac{1}{2}\left(\frac{\lambda}{h}-\frac{h}{\lambda}\right).$

$$Y_n(x)=B_n(\mu_n\cos\mu_n y+q\sin\mu_n y),\ n=1,\,2... \tag{21}$$

$q=-\mu^2$; μ_n are the roots of the equation $\mathrm{ctg}\mu b=\frac{1}{2}\left(\frac{\mu}{g}-\frac{g}{\mu}\right).$

Thus we have that the eigenvalues [3]

$$\sigma_{k,n} = -\lambda_k^2 - \mu_n^2$$

corresponds to the proper function $v_{k,n}$ from (20), (21). It has the following form

$$v_{k,n} = C_{km}(\lambda_k \cos\lambda_k x + h\sin\lambda_k x)(\mu_n \cos\mu_n y + g\sin\mu_n y), \tag{22}$$

where $C_{k,n}$ are constants. We choose $C_{k,n}$ so that the norm of functions $v_{k,n}$ is equal to one.

$$\int_0^a\int_0^b v_{k,n}^2(x,y)dxdy = C_{k,n}^2\int_0^a(\lambda_k\cos\lambda_k x + h\sin\lambda_k x)^2dx\int_0^b(\mu_n\cos\mu_n y + g\sin\mu_n y)dy = 1. \tag{23}$$

Now we calculate the integrals from the equation (23)

$$\int_0^a(\lambda_k\cos\lambda_k x + h\sin\lambda_k x)^2dx = \frac{1}{2}\int_0^a\left[\lambda_k^2(1+2\lambda_k x) + 2\lambda_k h\sin 2\lambda_k x + h^2(1-\cos 2\lambda_k x)\right]dx =$$

$$= \frac{1}{2}\left[\lambda_k^2\left(a + \frac{1}{2\lambda_k}\sin 2\lambda_k a\right) + h(1-\cos 2\lambda_k a) + h^2\left(a\frac{1}{2\lambda_k}\sin 2\lambda_k a\right)\right].$$

If λ_k are the roots of the equation $\mathrm{ctg}\lambda a = \frac{1}{2}\left(\frac{\lambda}{h} - \frac{h}{\lambda}\right)$ or $\mathrm{tg}\lambda a = \frac{2\lambda h}{\lambda^2 - h^2}$, then

$$\sin 2\lambda_k a = \frac{4\lambda_k h\left(\lambda_k^2 - h^2\right)}{\left(\lambda_k^2 + h^2\right)}, \quad \cos 2\lambda_k a = \frac{\left(\lambda_k^2 - h^2\right)^2 - 4\lambda_k^2 h^2}{\left(\lambda_k^2 + h^2\right)^2}.$$

We substitute the values $\sin 2\lambda_k l$ and $\cos 2\lambda_k l$ from this ratios in the calculated integral. Thus, we obtain

$$\int_0^a(\lambda_k\cos\lambda_k x + h\sin\lambda_k x)^2dx = \frac{1}{2}\left[\lambda_k^2\left(a + \frac{1}{2\lambda_k}\frac{4\lambda_k h(\lambda_k^2 - h^2)}{\left(\lambda_k^2+h^2\right)^2}\right)\right] +$$
$$+h\left(1 - \frac{\left(\lambda_k^2 - h^2\right) - 4\lambda_k^2h^2}{\left(\lambda_k^2+h^2\right)^2}\right) + h^2\left(l - \frac{1}{2\lambda_k}\frac{4\lambda_k h(\lambda_k^2-h^2)}{(\lambda_k^2+h^2)^2}\right) =$$
$$= \frac{1}{2\left(\lambda_k^2+h^2\right)^2}\left[\lambda_k^2 a\left(\lambda_k^2+h^2\right)^2 + 2\lambda_k^2 h\left(\lambda_k^2 - h^2\right) + h\left(\lambda_k^2 + h^2\right) -\right.$$
$$\left. - h\left(\lambda_k^2 - h^2\right)^2 + 4\lambda_k^2 h^3 + h^2 l\left(\lambda_k^2 + h^2\right)^2 - 2h^3\left(\lambda_k^2 - h^2\right)\right] =$$
$$= \frac{1}{2\left(\lambda_k^2+h^2\right)^2}\left[\left(\lambda_k^2+h^2\right)^2 \cdot \left(a\left(\lambda_k^2+h^2\right) + h\right) + h\left(\lambda_k^4 + 2\lambda_k^2h^2 + h^4\right)\right] =$$
$$= \frac{1}{2}\left[a\left(\lambda_k^2+h^2\right) + 2h\right] = \frac{a(\lambda_k^2+h^2)+2h}{2}.$$

$$\int_0^a(\lambda_k\cos\lambda_k x + h\sin\lambda_k x)^2dx = \frac{a(\lambda_k^2+h^2)+2h}{2}. \tag{24}$$

$$\int_0^b(\mu_n\cos\mu_n y + g\sin\mu_n y)^2dy = \frac{b(\mu_n^2+g^2)+2g}{2}. \tag{25}$$

Taking into account (24) and (25) we have from (23)

$$C_{k,n} = \frac{2}{\sqrt{\left[a\left(\lambda_k^2+h^2\right)+2h\right]\cdot\left[b\left(\mu_n^2+g^2\right)+2g\right]}}. \tag{26}$$

As a result, we get the following

$$v_{k,n} = \frac{2}{\sqrt{\left[a\left(\lambda_k^2 + h^2\right) + 2h\right] \cdot \left[b\left(\mu_n^2 + g^2\right) + 2g\right]}} \left(\lambda_k \cos \lambda_k x + h \sin \lambda_k x\right) \left(\mu_n \cos \mu_n y + g \sin \mu_n y\right). \tag{27}$$

Taking into account that $\sigma = p + q$ and $p = -\lambda^2$, $q = -\mu^2$, we derive $\sigma = -\left(\lambda^2 + \mu^2\right)$.

From (9) we have

$$T'' + a^2\left(\lambda^2 + \mu^2\right) T = 0. \tag{28}$$

Eigenvalues $\sigma_{k,n}$ corresponds to the solutions of the equation (28) in the form

$$T_{k,n}(t) = D_{k,n} \cos a\sqrt{-\sigma_{k,n}} t + E_{k,n} \sin a\sqrt{-\sigma_{k,n}} t, \tag{29}$$

where $D_{k,n}$ and $E_{k,n}$ are arbitrary constants.

By (6) we receive that particular solutions of the problem (1)-(5) have the form

$$u_{k,n}(x,y,t) = v_{k,n}(x,y) \cdot T_{k,n}(t) = v_{k,n}(x,y) \cdot \left(D_{k,n} \cos a\sqrt{-\sigma_{k,n}} t + E_{k,n} \sin a\sqrt{-\sigma_{k,n}} t\right),$$

on the principle of superposition the common solution of the problem (1.5) is defined by the formula

$$u(x,y,t) = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \left(D_{k,n} \cos a\sqrt{-\sigma_{k,n}} t + E_{k,n} \sin a\sqrt{-\sigma_{k,n}} t\right) \cdot v_{k,n}(x,y), \tag{30}$$

where the functions $v_{k,n}(x,y)$ are determined by the equality (27).

Now we define the coefficients $D_{k,n}$ and $E_{k,n}$ from the initial conditions (2) and (3) respectively

$$u(x,y,0) = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \left(D_{k,n} v_{k,n}\right) = \varphi(x,y);$$

$$u_t(x,y,0) = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} E_{k,n} a\sqrt{-\sigma_{k,n}} v_{k,n} \psi(x,y). \tag{31}$$

Since $v_{k,n}$ are orthonormal functions then from the relations (31) we find he coefficients $D_{k,n}$ and $E_{k,n}$

$$D_{k,n} = \int_0^a \int_0^b \varphi(x,y) v_{k,n}(x,y) dxdy;$$

$$E_{k,n} = \frac{1}{a\sqrt{-\sigma_{k,n}}} \int_0^a \int_0^b \psi(x,y) v_{k,n}(x,y) dxdy. \tag{32}$$

In the long run, we receive the solution of our problem [4]

$$u(x,y,t) = 2\sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\sqrt{\left[a\left(\lambda_k^2 + h^2\right) + 2h\right] \cdot \left[b\left(\mu_n^2 + g^2\right) + 2g\right]}} \cdot \left(\lambda_k \cos \lambda_k x + h \sin \lambda_k x\right) \times$$

$$\times \left(\mu_n \cos \mu_n y + g \sin \mu_n y\right) \cdot \left(D_{k,n} \cos a\sqrt{-\sigma_{k,n}} t + E_{k,n} \sin a\sqrt{-\sigma_{k,n}} t\right),$$

where λ_k , μ_n are the roots of the equations

$$\mathrm{ctg}\lambda a = \frac{1}{2}\left(\frac{\lambda}{h} - \frac{h}{\lambda}\right), \quad \mathrm{ctg}\mu b = \frac{1}{2}\left(\frac{\mu}{g} - \frac{g}{\mu}\right)$$

respectively and the coefficients $D_{k,n}$ and $E_{k,n}$ are calculated according to the formulas (32).

Thus we have found the function $u(x, y, t)$. This function describes the membrane deflection from the equilibrium position.

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Екіөлшемді ортадағы тербелмелі және толқындық процестер үшін шеттік есеп туралы

Мақалада мембрананың бос тербелісінің екіөлшемді теңдеуі үшін үшінші шеттік есепті зерттеу ұсынылған. Дифференциалды оператордың шешімі берілген кесіндінің ортонормалды меншікті функциясының сызықты тәуелсіз жүйесінде комбинация түрі болып табылады. Жеткілікті тегіс функциялар үшін тепе-теңдіктен мембрананың ауытқуын сипаттайтын спектрлі ыдырауды пайдалану екіөлшемді ортада зерттеліп отырған есептердің майысу функциясы туралы нақты аналитикалық түрді анықтауға мүмкіндік береді.

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О краевой задаче для колебательных и волновых процессов в двумерных средах

В статье представлено исследование третьей краевой задачи для двумерного уравнения свободных колебаний мембраны. Решение исходного дифференциального оператора находится в виде комбинации по линейно независимой системе ортонормированных собственных функций на заданном отрезке. Использование спектрального разложения для достаточно гладкой функции, характеризующей отклонение мембраны от положения равновесия, позволяет определить точное аналитическое представление функции прогиба для исследуемой задачи в двумерных средах.

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