UDC 510.67

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The similarity of closures of Jonsson sets

This article is devoted to the study model-theoretic properties of special closures of Jonsson sets. That is considered a syntactic similarity of Jonsson theories for universal existential sentences which true in these models. Due to the fact that the fragments of Jonsson sets are Jonsson theories, such an approach for the study of such theories is acceptable. Besides define the certain range of questions not previously are risen in studies such theories and its of their models classes.

Key words: Jonsson sets, the model-theoretic properties, the existentially-closed model.

When considering the model-theoretic properties of Jonsson theories have recently been proposed various new approaches to the description of such theories. The necessity of using such methods is justified by the fact that Jonsson theory in general are incomplete theory, and usually the basic method of model theory does not work here.

The most effective method is a semantic method, i.e. transfer properties of first-order center considered Jonsson theory to the theory itself. This assumes an existential fullness of this theory, and as a rule, as models, we limit our interest in the class of existentially closed models of the theory. The requirement for such a fullness completely natural, otherwise it all comes down to a trivial case. As for the class existentially closed models of the theories, there are two, it seems to us, the following explanations.

Firstly, as a rule, we are working at the moment in the class Jonsson theories and thus by virtue of the perfectness of Jonsson theory its semantic model is existentially closed and, moreover, by the criterion of perfectness of Jonsson theory, all models of its center such ones.

Secondly, unlike the complete theories, we have quality morphisms between models of the theory with different variants of isomorphic embeddings and homomorphisms.

And for this purpose taking into account the type of the completeness of the theory is more suitable class of existentially closed models of the theory.

The main progress of art theory model, in the case of complete theories associated with the technological advances in the area of stable theories, or theories, which allow classification. And that brings us to the introduction of new notions, which are the Jonsson analogues of the notions for complete theories.

The basic technique associated with more subtle methods of investigation of the behavior of model elements, is, as mentioned above, the prerogative of engineering study of complete theories. Therefore, even just trying to find a generalization of standard notions from the arsenal of complete theories, we can encounter a tautology or a notion that is technically justified. In this regard, we believe it is necessary to start with the simpler cases in the general sense of stability and generally some restrictions on the theory and whether the combination of the first and second.

So, even it has been proposed Jonsson set. One of these notions was the notion Jonsson sets, introduced in previous works of the author of this article [1], as well as other analogues were considered theoretical model properties for the of complete theories [2–5].

Recall the basic definitions:

The theory T is called Jonsson, if:

1) the theory T has infinite models;

2) the theory T is inductive;

3) the theory T has the joint embedding property (JEP);

4) the theory T has the amalgam property (AP).

Let L be a countable first-order language.

Let T Jonsson complete theory for the existential sentences in L and its semantic model has C. Suppose X is a subset of the semantic model of T.

We say that the set X is Σ-definable if it is definable some existential formula:

a) The set X is called Jonsson in T if it satisfies the following properties: X is Σ-definable subset of C; $dsl(X)$ is a bearer of a some existential closed submodel of C where $dsl(X)$ is the set of all X-definable elements $a \in C$ such that for some formula $\varphi(x) \in L(X)$, it follows that $\varphi(C) = a$.

b) The set X is called algebraic Jonsson theory of T if it satisfies the following properties: X is a-definability subset of C; acl(X) is a bearer of a some existential closed submodel of C where $acl(X)$ is the union of all finite X-definable subsets of semantic model of C, i.e. there is a formula $\varphi(x) \in L(X)$ that $acl(X) = \{a \in S : a \in \varphi(G) \text{and} \varphi(G)\}$ finite.

It is clear that in the general case $dl() \subseteq l()$ for each subset semantic model of $H \subseteq S$. Sometimes, in the particular case $dcl(X)acl(X)$, i.e. there are examples Jonsson theories when we introduced the notion of Jonsson sets the same with algebraic Jonsson sets these theories. The most striking examples of such theories is this:

1) the linear space;

2) the infinite sets.

On the other hand a classic example of an algebraically closed field of fixed characteristics is an example of what these notions are not the same.

Definition 1. Let X is a Jonsson set and M such existentially closed submodel of semantic model C of theory T that

$$dcl(X) = M.$$

Consider $Th_{\forall\exists}(M) = T_M$.

We call T_M is Jonsson fragment of Jonsson set X. We say that all $\forall\exists$-corollary arbitrary theory create the Jonsson fragment of this theory, if the deductive closure of these $\forall\exists$-corollary are Jonsson theory.

The requirement Jonsson significantly, i.e. this is not always true. But we can always consider $\forall\exists$-corollaries is true into the above-mentioned circuits of Jonsson set. And it will be always Jonsson theory, because set of universal existential closed sentence is true in any existentially closed model arbitrary of Jonsson theory is the Jonsson theory.

By introducing definitions of Jonsson set we can move a lot of properties for Jonsson theories on an arbitrary subsets of the semantic model. We say that two Jonsson sets (equivalent, cosemantically, categorically), if respectively the will (Jonsson equivalent cosemantically, categorical, syntactically similar, semantically similar, etc.) models are obtained at the respective closing of these sets. The most invariant notion is a syntactic similarity of theory, as it save all the properties of the theories. In the case Jonsson sets we define syntactical similarity as follows: Two (algebraic) Jonsson sets are syntactically similar to each other, if the elementary theory are syntactically similar of the appropriate circuits.

If $\forall\exists$-corollaries of the elementary theories will give the Jonsson theory, in this case, we can consider them Jonsson syntactical similarity, i.e. the invariance of our semantic model is defined.

Let us give the following definitions from [6] relating to the notions of similarities, both for the total and for Jonsson theory.

Let T is complete theory, $F(T) = n < \omega F_n(T)$ then, where $F_n(T)$ is Boolean algebra of formula with n-free variables.

Definition 2. Let T_1 and T_2 are complete theories.

We say that T_1 and T_2 be syntactically similar if there is a bijection $f : F(T_1) \to F(T_2)$, such that:

1) restriction f to $F_n(T_1)$ is isomorphism of Boolean algebra $F_n(T_1)$ and $F_n(T_2)$, $n < \omega$;

2) $f(\exists v_{n+1}\varphi) = \exists\varphi_{n+1}(T)$, $n < \omega$;

3)$f(v_1 = v_2) = (v_1 = v_2)$.

Definition 3.

1) The clean triple is call $< A, \Gamma, M >$, where A is not empty, Γ is the group of permutations of A, and M is family of subsets of A, such that $M \in M \Rightarrow g(M) \in M$ for each $g \in \Gamma$.

2) If $< A_1, \Gamma_1, M_1 >$ and $< A_2, \Gamma_2, M_2 >$ are clean the triple and $\psi : A_1 \to A_2$ be bijection, then ψ is isomorphism if:

(i) $\Gamma_2 = \left\{\psi g \psi^{-1} : g \in \Gamma_1\right\{$;

(ii) $M_2 = \{\psi(E) : E \in M_1\}$.

Definition 4. The clean triple $\langle |C|, G, N\rangle$ is called semantic triple of complete theory T, where $|C|$ is media monster models C of T, $G = Aut(C)$, is the class of all subsets $|C|$, each of which is a carrier of the respective elementary submodel of C.

Definition 5. Complete theories T_1 and T_2 are called semantically similar if their semantic triples are isomorphic to each other.

The following definitions are generalizations of the previous definitions.

Let T is arbitrary Jonsson theory, $E(T) = n < \omega E_n(T)$ then, where $E_n(T)$ is lattices $\exists$-formulas with n-free variables, T^* be centres Jonsson theory T, e.c. $T^* = Th(C)$, where C is semantic model Jonsson theory of T in the sense of [1].

Definition 6. Let T_1 and T_2 are Jonsson theories.

We say that T_1 and T_2 be J-syntactically similar if there is bijection $f : E(T_1) \to E(T_2)$, such that:

1) limitation f to $E_n(T_1)$ is isomorphism of lattices $E_n(T_1)$ and $E_n(T_2)$, $n < \omega$;

2) $f(\exists v_{n+1}\varphi) = \exists\varphi_{n+1} f(\varphi), \varphi \in E_{n+1}(T), n < \omega$;

3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 7. The clean triple $\langle C, AutC, SubC\rangle$ called J-semantic triple, where C be semantic model of T, $AutC$ be group are avtomopfizm of C, $SubC$ is the class of all subsets of the carrier of C, who are the carrier of the relevant sub-models of C.

Definition 8. Two Jonsson theories T_1 and T_2 are called J-semantically similar if their J-semantic three are isomorphic as clean triples.

It is understood that the definition 5 is a generalization of the definition 1 and the definition 7 is generalization of the definition 4 in the following sense:

a) determining 5 for each $n < \omega$ instead of Boolean algebra $F_n(T)$ considered lattice $\exists$-formula $E_n(T)$;

b) in determining 7 instead of the monster model of complete theory T considered semantic model Jonsson theory of T, and as N from definition 4 deals $SubC_i$ which the class of all subsets of the carrier C_i, which are the bearers of the relevant submodels C_i, which satisfies M.

Definition 9. Jonsson theories T is called perfect if every semantic model T is a saturated model center of T^*.

The main result for Jonsson theories regarding syntactical similarity is the following result, coupled with the above definitions.

Theorem 1. Let T_1 and T_2 be $\exists$-complete perfect Jonsson theory.

Then the following conditions are equivalent:

1) T_1 and T_2 be J-syntactically similar;

2) T_1^* and T_2^* be syntactically similar in the sense of Definition 1.

Proof. For to prove need the following two facts.

Fact 1. For any Jonsson theory the following conditions are equivalent:

1) T is perfect;

2) T is model complete.

Proof follows from the fact that the perfectness Jonsson theory T is equivalent to a T^* is model companion of T [6].

Fact 2. For any complete theory for $\exists$-sentence Jonsson theory T following conditions are equivalent:

1) T^* is model complete;

2) for each $n < \omega E_n(T)$ is Boolean algebra, where $E_n(T)$ is lattice of $\exists$-formulas with n-free variables.

Proof. $1 \Rightarrow 2$ Let T^* be model complete $\Rightarrow E_n(T^*)$ is Boolean algebra, as T^* be complete theory (elementary theory of the semantic model), but $E_n(T) \subseteq E_n(T^*), T \subseteq T^*$.

We have two cases:

1) T be complete, then $T = T^* \Rightarrow T$ is model complete $\Rightarrow E_n(T)$ is Boolean algebra;

2) as $T \subset T^* \Leftrightarrow T^* = Th(C)$, where C is semantic model of T, then $\forall \varphi \in T \Rightarrow \varphi \in T^*$. So T is complete for $\exists$-offers, then all $\exists$-sentences, output from T, belongs T^* too. The other in T^* $\exists$-sentences are not, so T is complete for $\exists$-sentences and T^* is complete theory. So $E_n(T^*)$-Boolean algebra, then it has a supplement for all φ is $\exists$-sentences. In general, this φ will not be $\exists$-sentences, so if $\varphi \in \Sigma$, then $\neg\varphi \in \Pi$ (Σ-set of $\exists$-sentences, Π-set of $\forall$-sentences), but T^* model complete $\Leftrightarrow \forall \psi \in T, \exists \theta \in T^* : \psi \equiv \theta, \theta \in \Sigma$. But we know, that $\theta \in T^* \Leftrightarrow \theta \in T \Rightarrow$:

1) $1, 0 \in E_n(T)$;

2) $\varphi \in E_n(T) \Rightarrow \neg\varphi \in E_n(T)$;

3) $\forall \varphi \in E_n(T) \neg\neg\varphi = \varphi \Rightarrow E_n(T)$ is Boolean algebra.

2) $\Rightarrow$ 1) $E_n(T)$ is Boolean algebra $\Rightarrow T$ be model complete, but $T \subset T^* = Th(C)$. Let $A \in$ $\in ModT \Rightarrow A$ isomorphically embedded in C, e.c. C is semantic model. And because T be model complete $\Rightarrow$ this embedded is elementary.

Let C is not saturated, then $\exists X \subset C, |X| < |C|, \exists p \in S_1(X)$: is false, that $(C, x)_{x \in X} \models p$ but $p \cup T$-jointly, $p \cup T^*$-jointly, means $\exists m \notin C$: m realizes p, then $\exists M| = T$, that $m \in M$, M is elementary extension of C that power $\Rightarrow \exists$ semantic model of C', which is $|M|^+$ -saturated and elementary extension of M of power $2^{|M|}$. But any two elementary semantic models are equivalent, in particular $C \equiv C'$. We obtain a contradiction, because at C' realized p. Therefore, our assumption that C is not saturated is wrong $\Rightarrow T$ is perfect $\Rightarrow T^*$ is model complete.

We now proceed directly to the proof of the theorem.

We show $1 \Rightarrow 2$. We have that for each $n < \omega E_n(T_1)$ is isomorphic $E_n(T_2)$. Let this isomorphism is realized by f_{1n}. Condition of the theorem and facts 1 and 2 for each $n < \omega E_n(T_1)$ and $E_n(T_2)$ is a Boolean algebra. But because of perfectness T_1 and $T_2 \Rightarrow T/_1$ and T_2^* are model complete due to the fact 1, and because for each $n < \omega$, for any formula $\varphi(\overline{x})$ from $F_n(T_1^*)$ exists a formula $\psi(\overline{x})$ from $E_n(T_1^*)$, that $T_1^*| = \varphi \leftrightarrow \psi$. And view of the fact that the theory T_1 be $\exists$-complete and $E_n(T_1) \subseteq$ $\subseteq E_n(T_1^*)$ (e.c. $T_1 \subseteq T_1^*$), follows that $E_n(T_1) = E_n(T_1^*)$. And view of the fact that the theory T_2 be $\exists$-complete and $E_n(T_1) \subseteq E_n(T_2^*)$ (e.c. $T_1 \subseteq T_2^*$), follows that $E_n(T_2) = E_n(T_2^*)$. For each $n < \omega$, for each $\varphi_1(\overline{x})$ from $F_(T_1^*)$ ask the next map between $F_(T_1^*)$ and $F_(T_2^*)$: $f_{2n}(\varphi_1(\overline{x})) = f_{1n}(\psi_1(\overline{x}))$, where $T_1^*| = \varphi_1 \leftrightarrow \psi_1, \psi_1 \in E_n(T_1)$. It is easy to understand that, in view of the properties f_{2n} is a bijection, defining an isomorphism between $F_n(T_1^*)$ and $F_n(T_2^*)$. Consequently, T_1^* and T_2^* are syntactically similar. Show that $2) \Rightarrow 1)$. Trivial, so $F_n(T_1^*)$ is isomorphic to $F_n(T_2^*)$ for each $n < \omega$, and in view of the theorem and the conditions facts of 1 and 2 of this isomorphism extends to all subalgebras.

From [5], the following result is known.

Proposition 1. If the theory T_1 and T_2 are syntactically similar, then T_1 and T_2 are semantically similar, reverse is not true.

And therefore can be formulated as follows:

Lemma 1. Any two kosemantic Jonsson theory are J-semantically similar.

Proof follows from the definition.

Lemma 2. If two perfect $\exists$-complete Jonsson theory are J-syntactically similary then they are J-semantically similar.

Proof follows from Theorem 1 and Proposition 1.

Using the above results we can apply them to the main result of this article. Since, by definition, two (algebraic) Jonsson sets be syntactically similar to each other, if the elementary theory of the appropriate circuits are syntactically similar, we have that the following theorem is true:

Theorem 2. Let X and Y be Jonsson sets in some existential complete perfect Jonsson theory of T. And $dcl(X) = M, dcl(Y) = N$, and T_1 and T_2, respectively centers T_M and T_N.

Then the following conditions are equivalent:

1) T_M and T_N be syntactically similar,

2) T_1 and T_2 be syntactically similar as complete theory.

Proof this follows from the above results (Theorem 1, Fact 1, Fact 2).

Corollary 1. The result of Theorem 2 is naturally distributed to the case of an algebraically Jonsson sets.

Corollary 2. The result of Lemma 1 that any two cosemantic Jonsson theory of J-semantic similar due to the fact that two Jonsson sets cosemantic if their appropriate circuit is cosemantic, of course this is extends to algebraically Jonsson sets.

In connection with the notion of cosemantic Jonsson sets will naturally be noted that the notion of Jonsson equivalence within Jonsson sets has to be a place and, accordingly, there is a non-trivial new class of problems associated with this notion. We recall that the two models are Jonsson equivalent, if they are both models of the same Jonsson theories.

All undefined here definitions and notions related to the Jonsson theory can be found in [1].

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А.Р.Ешкеев

Йонсондық жиындар тұйықтамалардың ұқсастығы

Мақалада арнайы тұйықталған йонсондық жиындардың теориялық-модельдік қасиеттері зерттелді. Әсіресе берілген модельде ақиқат әмбебап-экзистенционалды сөйлемдер үшін йонсондық теориялардың синтаксистік ұқсастығы қарастырылды. Осыған байланысты йонсондық жиындардың фрагменттері йонсондық теория болады, мұндағы зерттеу әдісі қарастырылатын теориялар үшін өте тиімді. Осыдан басқа, сондай теориялар және модельдер класын зерттеу шеңберінде бұрында қарастырылмаған кейбір сұрақтар қатары анықталды.

А.Р.Ешкеев

Подобие замыканий йонсоновских множеств

Данная статья посвящена исследованию теоретико-модельных свойств специальных замыканий йонсоновских множеств. А именно рассматривается синтаксическое подобие йонсоновских теорий для универсально-экзистенциальных предложений, истинных в данных моделях. В связи с тем, что фрагменты йонсоновских множеств являются йонсоновскими теориями, такой подход для исследования этих теорий является приемлемым. Кроме этого, определяется некоторый ряд вопросов, ранее не рассматриваемый в рамках исследований таких теорий и классов модели.

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