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On a boundary-value problem in a bounded domain for a time-fractional diffusion equation with the Prabhakar fractional derivative

We aim to study a unique solvability of a boundary-value problem for a time-fractional diffusion equation involving the Prabhakar fractional derivative in a Caputo sense in a bounded domain. We use the method of separation of variables and in time-variable, we obtain the Cauchy problem for a fractional differential equation with the Prabhakar derivative. Solution of this Cauchy problem we represent via Mittag-Leffler type function of two variables. Using the new integral representation of this two-variable Mittag-Leffler type function, we obtained the required estimate, which allows us to prove uniform convergence of the infinite series form of the solution for the considered problem.

Keywords: Time-fractional diffusion equation, regularized Prabhakar fractional derivative, Mittag-Leffler type functions.

Introduction and formulation of a problem

Application of Fractional Calculus in mathematical modeling of real-life processes became crucial and appropriate mathematical tools have been developed [1–5].

A number of stochastic models for explaining anomalous diffusion have been introduced in literature (see, for instance, [6–9]).

There are other applications of time-fractional diffusion, for example, in the image denoising model [10].

Let us consider the following time-fractional diffusion equation

$$ {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}u(t,x) - u_{xx}(t,x) = f(t,x) \tag{1} $$

in a domain $\Omega = \{(t,x):\ 0<x<1,\ 0<t<T\}$. Here $f(t,x)$ is a given function and

$$ {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}y(t) = {}^{P}I_{0t}^{\alpha,m-\beta,-\gamma,\delta}\frac{d^m}{dt^m}y(t) $$

represents regularized Prabhakar fractional derivative [11] and

$$ {}^{P}I_{0t}^{\alpha,\beta,\gamma,\delta}y(t) = \int\limits_0^t (t-\xi)^{\beta-1}E_{\alpha,\beta}^{\gamma}\left[\delta(t-\xi)^{\alpha}\right]y(\xi)d\xi,\ t>0 $$

represents Prabhakar fractional integral [12]. We note that above-given definitions are valid for $\alpha,\beta,\gamma,\delta\in\mathbf{C}$ such that $\Re(\alpha)>0$ and $m-1<\Re(\beta)<m$, $m\in\mathbf{N}$.

We formulate a boundary-value problem for Eq.(1) in the particular case $(0<\beta<1)$ as follows:

Problem: To find a solution of Eq.(1) in Ω, satisfying the following conditions:

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- regularity conditions: $u(t,x) \in C(\overline{\Omega})$, $u(\cdot,x) \in C^1_{-1}(0,T)$, $u(t,\cdot) \in C^2(0,1)$;
- initial condition: $u(0,x) = \psi(x)$, $0 \le x \le 1$;
- boundary conditions: $u(t,0) = u(t,1) = 0$, $0 \le t \le T$.

Here the function $\psi(x)$ is a given function such that $\psi(0) = \psi(1) = 0$ and a class of functions C^m_μ is defined as follows:

Definition 1. [13] We say that $f \in C_\mu[a,b]$, if there is a real number $p > \mu$ $(\mu > -1)$, such that $f(x) = (x-a)^p f_1(x)$ with $f_1 \in C[a,b]$. Similarly, we say that $f \in C^m_\mu[a,b]$, if and only if $f^{(m)} \in C_\mu[a,b]$.

We would like to note related works, where the main objects are PDEs involving the above-mentioned Prabhakar fractional derivative or some generalizations.

The following Cauchy problem for the time-fractional diffusion-wave equation

$$\begin{cases} D^{\sigma}_{\nu,\gamma+\nu,-\lambda,0^+} g(x,t) = C g_{xx}(x,t), \ x \in \mathbf{R}, \ t > 0, \\ g(x,0^+) = \delta(x), \\ g_t(x,t)|_{t \to 0^+} = 0, \ \sigma \in \mathbf{R}, \ \gamma > 0, \nu > 0, \ 0 < \gamma+\nu \le 2 \end{cases}$$

was the subject of investigation in [11]. The authors used the Laplace-Fourier transform to find a solution to this problem in an explicit form. The solution was represented via Prabhakar and Wright's functions.

The explicit solution of the Cauchy problem in $t > 0$, $x \in \mathbf{R}$ has been found for the following time-fractional heat equations [14]:

$$D^{\gamma,\mu,\nu}_{\rho,\omega,0^+} u(x,t) = K u_{xx}(x,t)$$

and

$${}^C D^{\gamma,\mu,\nu}_{\rho,\omega,0^+} u(x,t) = K u_{xx}(x,t),$$

where

$$D^{\gamma,\mu,\nu}_{\rho,\omega,0^+} f(t) = E^{-\gamma,\nu}_{\rho,\nu(1-\mu),\omega,0^+} \frac{d}{dt} E^{-\gamma,(1-\nu)}_{\rho,(1-\nu)(1-\mu),\omega,0^+} f(t),$$

$${}^C D^{\gamma,\mu,\nu}_{\rho,\omega,0^+} f(t) = E^{-\gamma}_{\rho,1-\mu,\omega,0^+} \frac{d}{dt} f(t),$$

$\mu \in (0,1)$, $\nu \in [0,1]$, $\gamma, \omega \in \mathbf{R}$, $\rho > 0$,

$$E^{\gamma}_{\rho,\mu,\omega,0^+} f(t) = \int\limits_0^t (t-y)^{\mu-1} E^{\gamma}_{\rho,\mu}[\omega(t-y)^\rho] f(y) dy$$

is the Prabhakar fractional integral [12].

The following PDE involving the Prabhakar derivative

$$D^{\gamma}_{\alpha,\beta,\omega,0^+} u(x,t) = a(x) u_{xx}(x,t) + b(x) u_x(x,t) + c(x) u(x,t) + d(x,t)$$

was investigated together with the appropriate initial conditions [15]. Using the Sumudu transform, the authors have found an approximate solution to the proposed problem.

Authors in [16] studied the following time-fractional heat conduction equation with a heat absorption term in spherical coordinates in the case of central symmetry [17]:

$${}^C D^{\gamma,\mu}_{\rho,\omega,0^+} T(r,t) = a \left(T_{rr}(r,t) + \frac{2}{r} T_r(r,t) \right) - bT(r,t), \ t > 0, \ 0 \le r < R.$$

Imposing initial $T(r,0) = 0$ and boundary $T(R,t) = pt^\beta$ $(\beta > 0)$ conditions and using the Laplace transform, they found exact solutions for this problem.

The distinctive side of the present problem is that we consider the boundary-value problem in a bounded domain and use a method of separation of variables. We will get the solution to the problem in an infinite series form represented by the new Mittag-Leffler type function of two variables. In the next section, we provide the main result (a unique solvability of the problem) and corresponding proof with details.

Main result

We search solution of the problem $u(t,x)$ and the given function $f(t,x)$ as follows

$$u(t,x)=\sum_{n=0}^{\infty}U_n(t)\sin n\pi x, \tag{2}$$

$$f(t,x)=\sum_{n=0}^{\infty}f_n(t)\sin n\pi x, \tag{3}$$

where $U_n(t)$ are unknowns to be found and $f_n(t)$ are the Fourier coefficients of the function $f(t,x)$, given as

$$f_n(t)=2\int_0^1 f(t,x)\sin n\pi x.$$

Substituting (2) and (3) into (1) and considering initial condition, we obtain the following Cauchy problem:

$$\begin{cases} {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}U_n(t)+(n\pi)^2U_n(t)=f_n(t),\\ U_n(0)=\psi_n, \end{cases}$$

where ψ_n are the Fourier coefficients of the given function $\psi(x)$, which are defined as follows

$$\psi_n=2\int_0^1 \psi(x)\sin n\pi x.$$

Let us first present some statements, required for the further stages. The first statement is devoted to finding an explicit solution to the Cauchy problem for a fractional differential equation with the regularized Prabhakar derivative.

Lemma 1. Let $\alpha,\beta\in\mathbf{R}^+$, $\gamma,\delta,a_0,a_1,...,a_{m-1}\in\mathbf{R}$, $m=[\beta]+1$, $m-1\le\beta<m$. If $f(t)\in C_\mu^m$, then for any real number λ the following Cauchy problem

$$\begin{cases} {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}y(t)-\lambda y(t)=f(t),\\ y^k(0)=a_k,\ \ k=0,1,...,m-1 \end{cases} \tag{4}$$

has a solution represented by

$$\begin{gathered} y(t)=\sum_{k=0}^{m-1}\tfrac{a_kx^k}{k!}+\sum_{k=0}^{m-1}a_kx^{\beta+k}\Gamma(\gamma)E_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta+k+1,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\ \begin{matrix}|\lambda t^\beta\\ |\delta t^\alpha\end{matrix}\right)+\\ +\Gamma(\gamma)\int_0^t(t-z)^{\beta-1}E_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\ \begin{matrix}|\lambda(t-z)^\beta\\ |\delta(t-z)^\alpha\end{matrix}\right)f(z)dz. \end{gathered} \tag{5}$$

Here $E_2(\cdot)$ is the Mittag-Leffler type function in two variables represented as

$$E_2\left(\begin{matrix}\gamma_1,\alpha_1,\beta_1;\gamma_2,\alpha_2 & |x\\ \delta_1,\alpha_3,\beta_2;\delta_2,\alpha_4;\delta_3,\beta_3 & |y\end{matrix}\right)= \\ =\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(\gamma_1)_{\alpha_1 i+\beta_1 j}(\gamma_2)_{\alpha_2 i}}{\Gamma(\delta_1+\alpha_3 i+\beta_2 j)}\frac{x^i}{\Gamma(\delta_2+\alpha_4 i)}\frac{y^j}{\Gamma(\delta_3+\beta_3 j)}. \tag{6}$$

This function for the first time was mentioned in the work [18], but not studied at all.

Proof. In [19], the solution of the Cauchy problem (4) is represented in the following infinite series form:

$$y(t)=\sum_{k=0}^{m-1}\frac{a_k x^k}{k!}+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m-1}a_k\frac{((1+i)\gamma)_j}{j!}\frac{\lambda^{(i+1)}\delta^j t^{\alpha j+(i+1)\beta+k}}{\Gamma(\alpha j+(i+1)\beta+k+1)}+ \\ +\int_0^t\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{((1+i)\gamma)_j}{j!}\frac{\lambda^i\delta^j(t-z)^{\alpha j+(i+1)\beta-1}}{\Gamma(\alpha j+(i+1)\beta)}f_n(z)dz. \tag{7}$$

The double series in this formulae can be represented by the function defined in (6). Considering the well-known definition of the Pochhammer symbol, namely,

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}$$

one can easily deduce (5) from (7) using (6) in the following particular case:

$$\gamma_1=\gamma,\ \alpha_1=\gamma,\ \beta_1=1,\ \gamma_2=1,\ \alpha_2=0,\ \delta_1=\beta+k+1,\ \alpha_3=\beta,$$

$$\beta_2=\alpha,\ \delta_2=\gamma,\ \alpha_4=\gamma,\ \delta_3=1,\ \beta_3=1.$$

The next statements are related to the estimation of the function (6), which is crucial for the proof of the uniform convergence of infinite series. First, we present an integral representation of the function (6) via known functions.

Lemma 2. Let $\Re(\delta_1)>\Re(\gamma_1)>0$. If $\alpha_3=\alpha_1$ and $\beta_2=\beta_1$, then the following integral representation holds true:

$$E_2\left(\begin{matrix}\gamma_1,\alpha_1,\beta_1;\gamma_2,\alpha_2 & |x\\ \delta_1,\alpha_3,\beta_2;\delta_2,\alpha_4;\delta_3,\beta_3 & |y\end{matrix}\right)= \\ =\frac{1}{\Gamma(\gamma_1)\Gamma(\delta_1-\gamma_1)}\int_0^1\xi^{\gamma_1-1}(1-\xi)^{\delta_1-\gamma_1-1}E^{\gamma_2,\alpha_2}_{\alpha_4,\delta_2}\left(x\xi^{\alpha_1}\right)E_{\beta_3,\delta_3}\left(y\xi^{\beta_1}\right)d\xi. \tag{8}$$

Here $E_{\beta_3,\delta_3}(z)$ is two-parameter Mittag-Leffler function and

$$E^{\gamma_2,\alpha_2}_{\alpha_4,\delta_2}(z)=\sum_{m=0}^{\infty}\frac{(\gamma_2)_{\alpha_2 m}z^m}{\Gamma(\alpha_4 m+\delta_2)}.$$

Proof. On the right-hand side of (8) we use the series form of functions $E^{k,p}_{m,n}(z)$ and $E_{m,n}(z)$ and will integrate term-by-term:

$$\frac{1}{\Gamma(\gamma_1)\Gamma(\delta_1-\gamma_1)}\int_0^1\xi^{\gamma_1-1}(1-\xi)^{\delta_1-\gamma_1-1}\sum_{i=0}^{\infty}\frac{(\gamma_2)_{\alpha_2 i}\left(x\xi^{\alpha_1}\right)^i}{\Gamma(\alpha_4 i+\delta_2)}\sum_{j=0}^{\infty}\frac{\left(y\xi^{\beta_1}\right)^j}{\Gamma(\beta_3 j+\delta_3)}d\xi=$$

$$= \frac{1}{\Gamma(\gamma_1)\Gamma(\delta_1-\gamma_1)} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{(\gamma_2)_{\alpha_2 i} x^i}{\Gamma(\alpha_4 i+\delta_2)} \frac{y^j}{\Gamma(\beta_3 j+\delta_3)} \int_0^1 \xi^{\alpha_1 i+\beta_1 j+\gamma_1-1}(1-\xi)^{\delta_1-\gamma_1-1} d\xi.$$

Using the definition of Beta-function and after some simplifications, we deduce the left-hand side of (8).

In particular, if $\gamma=\beta$ and $\alpha=1$, we have

$$E_2\left(\begin{matrix}\gamma,\gamma,1;1,0 & |\lambda x^{\beta}\\ \beta+k+1,\beta,\alpha;\gamma,\gamma;1,1 & |\delta x^{\alpha}\end{matrix}\right)=$$

$$=\frac{1}{\Gamma(\gamma)\Gamma(\beta+k+1-\gamma)}\int_0^1 \xi^{\gamma-1}(1-\xi)^{\beta+k-\gamma} E^{1,0}_{\gamma,\gamma}\left(\lambda x^{\beta}\xi^{\gamma}\right) E_{1,1}\left(\delta x^{\alpha}\xi\right) d\xi.$$

It is known that

$$E^{1,0}_{\gamma,\gamma}(z)=E_{\gamma,\gamma}(z), \;\; E_{1,1}(z)=e^{z}.$$

Hence, considering the fact that if $\lambda<0$, $\delta\leq 0$, then

$$|E_{\gamma,\gamma}(\lambda x^{\beta}\xi^{\gamma})|\leq \frac{C_1}{1+|\lambda x^{\beta}\xi^{\gamma}|}, \;\; |e^{\delta x^{\alpha}\xi}|\leq C_2, \;\; (C_1,\, C_2\in \mathbf{R}^{+}),$$

one can get the following:

$$\left|E_2\left(\begin{matrix}\gamma,\gamma,1;1,0 & |\lambda x^{\beta}\\ \beta+k+1,\beta,\alpha;\gamma,\gamma;1,1 & |\delta x^{\alpha}\end{matrix}\right)\right|\leq$$

$$\leq \frac{1}{\Gamma(\gamma)\Gamma(\beta+k+1-\gamma)}\int_0^1 \xi^{\gamma-1}(1-\xi)^{\beta+k-\gamma}\frac{C_1C_2}{1+|\lambda x^{\beta}\xi^{\gamma}|}d\xi\leq$$

$$\leq \frac{C_1C_2}{\Gamma(\gamma)\Gamma(\beta+k+1-\gamma)}\int_0^1 \xi^{\gamma-1}(1-\xi)^{\beta+k-\gamma}d\xi=\frac{C_1C_2}{(\beta+k-\gamma)\Gamma(\beta+k)}=C,$$

where C is any positive real number.

Based on Lemma 1, we explicitly find $U_n(t)$ as follows

$$U_n(t)=\psi_n\left[1+t^{\beta}\Gamma(\gamma)E_2\left(\begin{matrix}\gamma,\gamma,1;1,0 & |-(n\pi)^2t^{\beta}\\ \beta+1,\beta,\alpha;\gamma,\gamma;1,1 & |\delta t^{\alpha}\end{matrix}\right)\right]+$$
$$+\Gamma(\gamma)\int_0^t (t-z)^{\beta-1}E_2\left(\begin{matrix}\gamma,\gamma,1;1,0 & |-(n\pi)^2(t-z)^{\beta}\\ \beta,\beta,\alpha;\gamma,\gamma;1,1 & |\delta(t-z)^{\alpha}\end{matrix}\right)f_n(z)dz.$$

Since, in our case $\lambda=-(n\pi)^2$ and assuming that $\delta\leq 0$, we can easily get when $\gamma=\beta$, $\alpha=1$ the following estimates:

$$|u(t,x)|\leq \sum_{n=0}^{\infty}\left[C_1|\psi_n|+C_2|f_n(t)|\right]\leq \bar{C}_1\left\|\psi(x)\right\|_2^2+\bar{C}_2\left\|f(t,x)\right\|_2^2.$$

This will be enough for the uniform convergence of the series (2), but for the infinite series corresponding to the function $u_{xx}(t,x)$ we need to impose more conditions to the given functions. Namely,

$$|u_{xx}(t,x)|\leq \bar{C}_3\left\|\psi''(x)\right\|_2^2+\bar{C}_4\left\|\frac{\partial^2 f(t,x)}{\partial x^2}\right\|_2^2.$$

The following statement is valid:

Theorem 1. If $\psi(x)\in C^1[0,1], \psi''(x)\in L_2(0,1)$ and $f(\cdot,x)\in C^1_{-1}[0,T]$, $f_x(t,\cdot)\in C[0,1],\; f_{xx}(t,\cdot)\in L_2(0,1)$, then there exists a unique solution of the problem represented as (2).

Conclusion

In the bounded domain, we have considered a boundary problem for a sub-diffusion equation involving regularized Prabhakar fractional order derivative. Presenting the solution of the corresponding Cauchy problem via a two-variable Mittag-Leffler type function and using its new integral representation, we have proved a unique solvability of the formulated boundary problem. We note that the same approach can be done for the fractional wave equation. Moreover, various inverse problems can be studied by applying obtained results.

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Прабхакар бөлшек туындысы бар уақыт-бөлшек диффузия теңдеуі үшін шектелген облыстағы шекаралық есеп бойынша

Зерттеудің мақсаты шектелген облыста Капуто мағынасындағы Прабхакар бөлшек туындысын қамтитын уақыттық-бөлшек диффузиялық теңдеу үшін шекаралық есептің бірегей шешімін зерттеу. Айнымалыларды бөлу әдісі қолданылған және уақыт айнымалысында Прабхакар туындысы бар бөлшек дифференциалдық теңдеу үшін Коши есебі алынған. Осы Коши есебінің шешімі екі айнымалы Миттаг-Леффлер типті функциясы арқылы берілген. Осы екі айнымалы Миттаг-Леффлер типті функцияның жаңа интегралды көрінісін пайдалана отырып, қарастырылып отырған есептің шешімінің шексіз қатар түрінің біркелкі жинақтылығын дәлелдеуге мүмкіндік беретін қажетті баға алынған.

Кілт сөздер: уақыт-бөлшек диффузия теңдеуі, регуляризацияланған Прабхакар бөлшек туындысы, Миттаг-Леффлер типті функциялар.

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Об одной краевой задаче в ограниченной области для уравнения диффузии дробного времени с дробной производной Прабхакара

Нашей целью является изучение однозначной разрешимости краевой задачи для уравнения диффузии с дробным временем, включающего дробную производную Прабхакара по Капуто в ограниченной области. Воспользуемся методом разделения переменных и в переменной по времени получим задачу Коши для уравнения дробного дифференциала с производной Прабхакара. Решение этой задачи Коши представим через функцию типа Миттаг-Леффлера от двух переменных. Используя новое интегральное представление этой функции типа Миттаг-Леффлера с двумя переменными, мы получили требуемую оценку, которая позволяет доказать равномерную сходимость решения в виде бесконечного ряда для рассматриваемой задачи.

Ключевые слова: уравнение диффузии с дробным временем, регуляризованная дробная производная Прабхакара, функции типа Миттаг-Леффлера.