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T.D. Tokmagambetova, N.T. Orumbayeva*

Karaganda University of the name of academician E.A. Buketov, Karaganda, Kazakhstan
(E-mail: tenggesh.tokmagambetova@gmail.com, OrumbayevaN@mail.ru)

On one solution of a periodic boundary value problem for a hyperbolic equations

In a rectangular domain, we consider a boundary value problem periodic in one variable for a system of partial differential equations of hyperbolic type. Introducing a new unknown function, this problem is reduced to an equivalent boundary value problem for an ordinary differential equation with an integral condition. Based on the parametrization method, new approaches to finding an approximate solution to an equivalent problem are proposed and its convergence is proved. This made it possible to establish conditions for the existence of a unique solution of a semiperiodic boundary value problem for a system of second-order hyperbolic equations.

Keywords: boundary value problem, hyperbolic equations, algorithm, parametrization method, approximate solution.

Introduction

Boundary value problems for hyperbolic equations arise when studying the processes of transverse vibrations of a string, longitudinal vibrations of a rod, electrical vibrations in a wire, torsional vibrations of a shaft, gas vibrations, etc. [1–3].

To date, well-known methods are used to solve the problems under consideration, such as the Fourier method, the method of successive approximations, methods of function theory, variational methods, numerical methods, etc. [4–9]. This makes it possible to obtain various solvability conditions for boundary value problems for hyperbolic equations and construct analytical or approximate solutions [10–16].

In [17–20], such problems were solved by introducing functional parameters. Using this method, sufficient conditions were obtained for the correct solvability of nonlocal boundary value problems for systems of hyperbolic equations with a mixed derivative in terms of the initial data, and algorithms for finding their solutions were proposed. Based on the equivalence of the correct solvability of a boundary value problem with data on the characteristics for systems of linear hyperbolic equations and the correct solvability of a two-point boundary value problem for a family of systems of ordinary differential equations, a criterion for the correct solvability of the problem under study is established.

In this paper, we propose an algorithm where, in contrast to works [18–21], there is no need to find the Goursat or Cauchy problem at each step of the algorithm. In addition, when compared with the algorithm proposed in [22–23], this approach is more simplified. But despite this, the approximate solution is more accurate. The main characteristic of this algorithm is the effective verifiability of the conditions for their applicability and the ability to use it to find solutions with a given accuracy. This approach can be applied to problems of the third and fourth orders [24, 25] and obtain verifiable conditions.

On $\Omega = [0, X] \times [0, Y]$ the semiperiodic boundary value problem is considered

$$\frac{\partial^2 z}{\partial x \partial y} = A(x, y)\frac{\partial z}{\partial x} + B(x, y)z + f(x, y), \quad (x, y) \in \Omega, \tag{1}$$

*Corresponding author.
E-mail: OrumbayevaN@mail.ru

$$z(0,y) = \varphi(y), \quad y \in [0,Y], \tag{2}$$

$$z(x,0) = z(x,Y), \quad x \in [0,X], \tag{3}$$

where $(n \times n)$ - matrices $A(x,y), B(x,y)$, n-vector function $f(x,y)$ are continuous on Ω, n-vector function $\varphi(y)$, is continuously differentiable on $[0,Y]$, there is a condition of agreement $\varphi(0) = \varphi(Y)$, $\|z(x,y)\| = \max\limits_{i=\overline{1,n}} |z_i(x,y)|, \quad \|A(x,y)\| = \max\limits_{i=\overline{1,n}} \sum\limits_{j=1}^{n} |a_{ij}(x,y)|$.

Let $C(\Omega, R^n)$ be the spaces of functions $z : \Omega \to R^n$ which are continuous on Ω.

The function $z(x,y) \in C(\Omega, R^n)$, with partial derivatives $\frac{\partial^2 z(x,y)}{\partial x \partial y} \in C(\Omega, R^n)$, $\frac{\partial z(x,y)}{\partial x} \in C(\Omega, R^n)$, $\frac{\partial z(x,y)}{\partial y} \in C(\Omega, R^n)$ is called the classical solution to the problem (1)–(3), if it satisfies the system (1) for all $(x,y) \in \Omega$ and boundary conditions $(2), (3)$.

2 Main results

We introduce the functions $u(x,y) = \frac{\partial z(x,y)}{\partial x}$, to find a solution and the problem (1)–(3) we write as

$$\frac{\partial u}{\partial y} = A(x,y)u + B(x,y)z(x,y) + f(x,y), \quad (x,y) \in \Omega, \tag{4}$$

$$u(x,0) = u(x,Y), \quad x \in [0,X], \tag{5}$$

$$z(x,y) = \varphi(y) + \int\limits_0^x u(\xi,y)d\xi. \tag{6}$$

Problems (1)–(3) and (4)–(6) are equivalent in the sense that if the function $z(x,y)$ is a solution to problem (1)–(3), then the pair $(u(x,y), z(x,y))$ will be a solution to problem (4)–(6), and vice versa, if the pair $\widehat{u}(x,y), \widehat{z}(x,y)$ is a solution to problem (4)–(6), then $\widehat{z}(x,y)$ will be a solution to problem (1)–(3).

To solve problem (4)–(6) we will apply the parameterization method. For the step $h > 0 : Nh = Y$ we partition $[0,Y) = \bigcup\limits_{r=1}^{N} [(r-1)h, rh)$, $N = 1, 2, ...$ In this case Ω is divided into N parts. By $u_r(x,y)$ we denote the restrictions of the functions $u(x,y)$ on $\Omega_r = [0,X] \times [(r-1)h, rh), \quad r = \overline{1,N}$. Then problem (4), (5) will be equivalent to the boundary value problem

$$\frac{\partial u_r}{\partial y} = A(x,y)u_r(x,y) + B(x,y)z_r(x,y) + f(x,y), \quad (x,y) \in \Omega_r, \quad r = \overline{1,N}, \tag{7}$$

$$u_1(x,y) - \lim_{y \to Y-0} u_N(x,y) = 0, \quad x \in [0,X], \tag{8}$$

$$\lim_{y \to sh-0} u_s(x,y) = u_{s+1}(x,y), \quad x \in [0,X], \quad s = \overline{1,N-1}, \tag{9}$$

$$z_r(x,y) = \varphi(y) + \int\limits_0^x u_r(\xi,y)d\xi, \quad (x,y) \in \Omega_r, \quad r = \overline{1,N}. \tag{10}$$

where (9) is the condition for the continuity of functions in the internal partition lines. Problems (1)–(3) and (7)–(10) are equivalent. If $z(x,y)$ - solution of problem (1)–(3), then the system of its restrictions $z(x,[y]) = (z_1(x,y), z_2(x,y), ..., z_N(x,y))'$, $u(x,[y]) = (u_1(x,y), u_2(x,y), ..., u_N(x,y))'$, where $u_r(x,y) = \frac{\partial z_r(x,y)}{\partial x}, r = \overline{1,N}$ will be a solution to problem (7)–(10).

By $\lambda_r(x)$ we denote the function $u_r(x,y)$ for $y=(r-1)h$, i.e. $\lambda_r(x)=u_r(x,(r-1)h)$ and make a replacement $v_r(x,y)=u_r(x,y)-\lambda_r(x), r=\overline{1,N}$. We get an equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial v_r}{\partial y}=A(x,y)v_r(x,y)+A(x,y)\lambda_r(x)+B(x,y)z_r(x,y)+f(x,y),(x,y)\in\Omega_r, r=\overline{1,N}, \tag{11}$$

$$v_r(x,(r-1)h)=0, \quad x\in[0,X], \quad r=\overline{1,N}, \tag{12}$$

$$\lambda_1(x)-\lambda_N(x)-\lim_{y\to Y-0}v_N(x,y)=0, \quad x\in[0,X], \tag{13}$$

$$\lambda_s(x)+\lim_{y\to sh-0}v_s(x,y)-\lambda_{s+1}(x)=0, \quad x\in[0,X], \quad s=\overline{1,N-1}. \tag{14}$$

$$z_r(x,y)=\varphi(y)+\int\limits_0^x v_r(\xi,y)d\xi+\int\limits_0^x\lambda_r(\xi)d\xi, \quad (x,y)\in\Omega_r, \quad r=\overline{1,N}. \tag{15}$$

Problems (7)–(10) and (11)–(15) are equivalent in the sense that if the system of pairs $\{u_r(x,y), z_r(x,y)\}, r=\overline{1,N}$, is a solution to the problem (7)–(10), then the system $\{\lambda_r(x)=u_r(x,(r-1)h), v_r(x,y)=u_r(x,y)-u_r(x,(r-1)h), z_r(x,y)\}$, $r=\overline{1,N}$, is a solution to the problem (11)–(15), and vice versa, if the pair $\{\lambda_r(x),v_r(x,y),z_r(x,y)\}, r=\overline{1,N}$, is a solution to problem (11)–(15) , then $\{\lambda_r(x)+v_r(x,y),z_r(x,y)\}, r=\overline{1,N}$, will be a solution to problem (7)–(10).

Problem (10), (11) at fixed $\lambda_r(x),v_r(x,y),z_r(x,y)$ is a family of Cauchy problems for ordinary differential equations, where $x\in[0,X]$, and is equivalent to the integral equation

$$v_r(x,y)=\int\limits_{(r-1)h}^{y}A(x,\eta)v_r(x,\eta)d\eta+\int\limits_{(r-1)h}^{y}A(x,\eta)d\eta\cdot\lambda_r(x)+\int\limits_{(r-1)h}^{y}[B(x,\eta)z_r(x,\eta)+f(x,\eta)]d\eta. \tag{16}$$

Passing to the limit at $y\to rh-0$ in (16) and substituting into (13), (14) instead of $\lim\limits_{t\to rh-0}v_r(x,y)$, $r=\overline{1,N}$, their corresponding right-hand sides for unknown functions $\lambda_r(x), r=\overline{1,N}$, we get a system of functional equations:

$$\lambda_1(x)-\lambda_N(x)-\int\limits_{(N-1)h}^{Y}A(x,\eta)v_N(x,\eta)d\eta+\int\limits_{(N-1)h}^{Y}A(x,\eta)d\eta\cdot\lambda_N(x)+$$

$$+\int\limits_{(N-1)h}^{Y}[B(x,\eta)z_N(x,\eta)+f(x,\eta)]d\eta=0,$$

$$\lambda_s(x)+\int\limits_{(s-1)h}^{sh}A(x,\eta)v_s(x,\eta)d\eta+\int\limits_{(s-1)h}^{sh}A(x,\eta)d\eta\cdot\lambda_s(x)+$$

$$+\int\limits_{(s-1)h}^{sh}[B(x,\eta)z_s(x,\eta)+f(x,\eta)]d\eta-\lambda_{s+1}(x)=0, \quad s=\overline{1,N-1}.$$

We write the resulting system of equations in the following form

$$Q(x,h)\lambda(x)=-G(x,h,v)-F(x,h,z), \tag{17}$$

where

$$Q(x,h)=$$

$$=\begin{pmatrix} I & 0 & \dots & 0 & -I-\int\limits_{(N-1)h}^{Nh}A(x,\eta)d\eta \\ I+\int\limits_0^h A(x,\eta)d\eta & -I & \dots & 0 & 0 \\ 0 & I+\int\limits_h^{2h}A(x,\eta)d\eta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I+\int\limits_{(N-2)h}^{(N-1)h}A(x,\eta)d\eta & -I \end{pmatrix},$$

$$G(x,h,v)=\begin{pmatrix} -\int\limits_{(N-1)h}^{Nh}A(x,\eta)v_N(x,\eta)d\eta \\ \int\limits_0^h A(x,\eta)v_1(x,\eta)d\eta \\ \int\limits_h^{2h}A(x,\eta)v_2(x,\eta)d\eta \\ \dots \\ \int\limits_{(N-2)h}^{(N-1)h}A(x,\eta)v_{N-1}(x,\eta)d\eta \end{pmatrix},$$

$$F(x,h,z)=\begin{pmatrix} -\int\limits_{(N-1)h}^{Nh}[B(x,\eta)z_N(x,\eta)+f(x,\eta)]d\eta \\ \int\limits_0^h[B(x,\eta)z_1(x,\eta)+f(x,\eta)]d\eta \\ \int\limits_h^{2h}[B(x,\eta)z_2(x,\eta)+f(x,\eta)]d\eta \\ \dots \\ \int\limits_{(N-2)h}^{(N-1)h}[B(x,\eta)z_{N-1}(x,\eta)+f(x,\eta)]d\eta \end{pmatrix}.$$

I is an identity matrix of dimension n.

To find a solution to a system of three functions $\{\lambda_r(x), v_r(x,y), z_r(x,y)\}$, $r=\overline{1,N}$, we have a closed system consisting of equations (17), (16), (15).

Suppose that the matrix $Q(x,h)$ is invertible for all $x\in[0,X]$.

Taking $z_r^{(0)}(x,y)=\varphi(y)$, $r=\overline{1,N}$, as the initial approximation, we find the solution of the boundary value problem (11)–(15) as the limit triple sequences $\{\lambda_r^{(k)}(x), v_r^{(k)}(x,y), z_r^{(k)}(x,y)\}$, $k=1,2,\dots$, determined by the following algorithm:

A) Assuming that $z_r(x,y)=z_r^{(k-1)}(x,y)$, $r=\overline{1,N}$, we find $k-$th approximations $\lambda_r^{(k)}(x), v_r^{(k)}(x,y)$ $r=\overline{1,N}$, as the limit of sequences $\lambda_r^{(k,m)}(x), v_r^{(k,m)}(x,y)$ $r=\overline{1,N}$, $\quad m=0,1,2,\dots$, defined as follows:

$$\lambda_r^{(k,0)}(x)=\lambda_r^{(k-1)}(x),\quad v_r^{(k,0)}(x,y)=v_r^{(k-1)}(x,y),$$

$$\lambda^{(k,m+1)}(x)=-[Q(x,h)]^{-1,}\Big(G(x,h,v^{(k,m)})+F(x,h,z^{(k-1)})\Big),$$

$$v_r^{(k,m+1)}(x,y) = \int\limits_{(r-1)h}^{y} A(x,\eta)v_r^{(k,m)}(x,\eta)d\eta + \int\limits_{(r-1)h}^{y} A(x,\eta)d\eta \cdot \lambda_r^{(k,m+1)}(x)+$$

$$+\int\limits_{(r-1)h}^{y} [B(x,\eta)z_r^{(k-1)}(x,\eta) + f(x,\eta)]d\eta,$$

those. pair system sequence $\{\lambda_r^{(k,m+1)}(x), v_r^{(k,m+1)}(x,y)\}$, for $m \to \infty$ converges to $\{\lambda_r^{(k)}(x), v_r^{(k)}(x,y)\}$, $r = \overline{1,N}$,

B) The functions $z_r^{(k)}(x,y), r = \overline{1,N}$, are determined from the relations

$$z_r^{(k)}(x,y) = \varphi(y) + \int\limits_0^x v_r^{(k)}(\xi,y)d\xi + \int\limits_0^x \lambda_r^{(k)}(\xi)d\xi.$$

The conditions of the following statement ensure the feasibility and convergence of the proposed algorithm, as well as the unique solvability of problem (11)–(15).

Theorem 1. Let $(nN \times nN)$ matrix $Q(x,h)$ be invertible for all $x \in [0,X]$ and the inequalities 1) $\|[Q(x,h)]^{-1}\| \le \gamma(x,h)$; 2) $q(x,h) = h\alpha(x)\Big(1+\gamma(x,h)h\alpha(x)\Big) \le \mu < 1$,

where $\alpha(x) = \sup\limits_{y\in[0,Y]} \|A(x,y)\|$. Then there is a unique solution to problem (11)–(15) and fair assessments

$$a)\ \max_{r=\overline{1,N}} \|\lambda_r^*(x) - \lambda_r^{(1)}(x)\| + \max_{r=\overline{1,N}} \sup_{(x,y)\in\Omega_r} \|v_r^*(x,y) - v_r^{(1)}(x,y)\| \le$$

$$\le \theta(x,h)\beta(x)\exp\left(\int\limits_0^x \theta(\xi,h)\beta(\xi)d\xi\right)\int\limits_0^x \theta(\xi,h)d\xi \max\left\{\max_{y\in[0,Y]}\|\varphi(y)\|, \max_{(x,y)\in\Omega}\|f(x,y)\|\right\},$$

$$b)\ \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^*(x,y) - z_r^{(1)}(x,y)\| \le$$

$$\le \int\limits_0^x \max_{r=\overline{1,N}} \|\lambda^*(\xi) - \lambda^{(1)}(\xi)\|d\xi + \int\limits_0^x \max_{r=\overline{1,N}} \sup_{(x,y)\in\Omega_r} \|v_r^*(\xi,y) - v_r^{(1)}(\xi,y)\|d\xi,$$

where $\beta(x) = \sup\limits_{y\in[0,Y]} \|B(x,y)\|$, $\quad \theta(x,h) = \frac{1}{1-q(x,h)}h\Big(1+\gamma(x,h)+\alpha(x)\gamma(x,h)h\Big)$.

Proof. Under assumptions about the data of the problem, the inequalities take place

$$\|G(x,h,v)\| \le \alpha(x)h \max_{l=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r(x,y)\|,$$

$$\|F_\nu(x,h,\varphi)\| \le h\beta(x) \max_{l=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r(x,y)\| + h \max_{(x,y)\in\Omega} \|f(x,y)\|.$$

The following estimates follow from the algorithm:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,1)}(x)\| \le \gamma(x,h)h(\beta(x)+1)\max\left\{\max_{y\in[0,Y]}\|\varphi(y)\|, \max_{(x,y)\in\Omega}\|f(x,y)\|\right\},$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,1)}(x,y)\| \le h\Big(1+\alpha(x)\gamma(x,h)h\Big)(\beta(x)+1)\max\left\{\max_{y\in[0,Y]}\|\varphi(y)\|, \max_{(x,y)\in\Omega}\|f(x,y)\|\right\}.$$

The following estimates follow from the algorithm:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,2)}(x) - \lambda_r^{(1,1)}(x)\| \leq \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,1)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,2)}(x,y) - v_r^{(1,1)}(x,y)\| \leq q(x,h) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,1)}(x,y)\|.$$

Let's establish the inequality

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,m+2)}(x) - \lambda_r^{(1,m+1)}(x)\| \leq \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+1)}(x,y) - v_r^{(1,m)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+2)}(x,y) - v_r^{(1,m+1)}(x,y)\| \leq$$

$$\leq h\alpha(x)[1+\alpha(x)\gamma(x,h)h] \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+1)}(x,y) - v_r^{(1,m)}(x,y)\| \leq$$

$$\leq q(x,h) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+1)}(x,y) - v_r^{(1,m)}(x,y)\|.$$

By virtue of the inequality $q(x,h) < 1$, the sequences $v_r^{(1,m+2)}(x,y)$ converge uniformly as $(x,y) \in \Omega_r$ to $v_r^{(1)}(x,y)$ and the convergence of the sequence of systems of functions $\lambda_r^{(1,m+2)}(x)$ to functions $\lambda_r^{(1)}(x)$ continuous on $x \in [0,X]$ for all $r = \overline{1,N}$.

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,m+2)}(x) - \lambda_r^{(1,1)}(x)\| \leq$$

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,m+2)}(x) - \lambda_r^{(1,m+1)}(x)\| + ... + \max_{r=\overline{1,N}} \|\lambda_r^{(1,2)}(x) - \lambda_r^{(1,1)}(x)\| \leq$$

$$\leq \sum_{j=0}^{m} [q(x,h)]^j \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,1)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+2)}(x,y) - v_r^{(1,1)}(x,y)\| \leq$$

$$\leq \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+2)}(x,y) - v_r^{(1,m+1)}(x,y)\| + ... + \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,2)}(x,y) - v_r^{(1,1)}(x,y)\| \leq$$

$$\leq \sum_{j=1}^{m+1} [q(x,h)]^j \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,1)}(x,y)\|.$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1,m+2)}(x,y)\| \leq$$

$$\leq \sum_{j=0}^{m+1} [q(x,h)]^j h \Big(1 + \alpha(x)\gamma(x,h)h\Big)(\beta(x)+1) \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\},$$

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1,m+2)}(x)\| \leq \sum_{j=0}^{m+1} [q(x,h)]^j \gamma(x,h)h(\beta(x)+1) \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\}.$$

Passing to the limit as $m \to \infty$ we obtain the estimates:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1)}(x)\| \leq \frac{\gamma(x,h)h(\beta(x)+1)}{1-q(x,h)} \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\},$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1)}(x,y)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} h\Big(1+\alpha(x)\gamma(x,h)h\Big)(\beta(x)+1) \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\},$$

$$\max_{r=\overline{1,N}} \|\lambda_r^{(1)}(x)\| + \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1)}(x,y)\| \leq \theta(x,h)(\beta(x)+1) \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\},$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y)-\varphi(y)\| \leq \int_0^x \max_{r=\overline{1,N}} \|\lambda_r^{(1)}(\xi)\| d\xi + \int_0^x \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(1)}(\xi,y)\| d\xi \leq$$

$$\leq \int_0^x \theta(\xi,h)(\beta(\xi)+1)d\xi \max\left\{ \max_{y\in[0,Y]} \|\varphi(y)\|, \max_{(x,y)\in\Omega} \|f(x,y)\| \right\}.$$

The following estimates follow from the algorithm:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2,1)}(x) - \lambda_r^{(2,0)}(x)\| \leq \gamma(x,h)h\beta(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y)-\varphi(y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\| \leq h\beta(x)[1+\gamma(x,h)h\alpha(x)] \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y)-\varphi(y)\|$$

The following estimates follow from the algorithm:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2,2)}(x) - \lambda_r^{(2,1)}(x)\| \leq \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,2)}(x,y) - v_r^{(2,1)}(x,y)\| \leq q(x,h) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\|$$

Let's establish the inequality

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2,m+2)}(x) - \lambda_r^{(2,m+1)}(x)\| \leq \gamma(x,h)h\alpha(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,m+1)}(x,y) - v_r^{(2,m)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,m+2)}(x,y) - v_r^{(2,m+1)}(x,y)\| \leq q(x,h) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,m+1)}(x,y) - v_r^{(2,m)}(x,y)\|.$$

By virtue of the inequality $q(x,h) < 1$, the sequences $v_r^{(2,m+1)}(x,y)$ converge uniformly as $(x,y) \in \Omega_r$ to $v_r^{(2)}(x,y)$ and the convergence of a sequence of systems of functions $\lambda_r^{(2,m+1)}(x)$ to functions $\lambda_r^{(2)}(x)$ continuous on $x \in [0,X]$ for all $r = \overline{1,N}$.

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2,m+2)}(x) - \lambda_r^{(2,0)}(x)\| \leq$$

$$\leq \sum_{j=0}^{m} [q(x,h)]^j \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\| + \max_{r=\overline{1,N}} \|\lambda_r^{(2,1)}(x) - \lambda_r^{(2,0)}(x)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,m+1)}(x,y) - v_r^{(2,0)}(x,y)\| \leq \sum_{j=0}^{m} [q(x,h)]^j \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\|.$$

Passing to the limit as $m \to \infty$ we obtain the estimates:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2)}(x) - \lambda_r^{(1)}(x)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} \alpha(x)\gamma(x,h)h \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\| + \max_{r=\overline{1,N}} \|\lambda_r^{(2,1)}(x) - \lambda_r^{(2,0)}(x)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} \alpha(x)\gamma(x,h)hh\beta(x)[1+\gamma(x,h)h\alpha(x)] \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y) - \varphi(y)\| +$$

$$+\gamma(x,h)h\beta(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y) - \varphi(y)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} \gamma(x,h)h\beta(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y) - \varphi(y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2)}(x,y) - v_r^{(1)}(x,y)\| \leq \frac{1}{1-q(x,h)} \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2,1)}(x,y) - v_r^{(2,0)}(x,y)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} h\beta(x)[1+\gamma(x,h)h\alpha(x)] \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y) - \varphi(y)\|,$$

$$\max_{r=\overline{1,N}} \|\lambda_r^{(2)}(x) - \lambda_r^{(1)}(x)\| + \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(2)}(x,y) - v_r^{(1)}(x,y)\| \leq$$

$$\leq \theta(x,h)\beta(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(x,y) - \varphi(y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(2)}(x,y) - z_r^{(1)}(x,y)\| \leq \int_0^x \theta(\xi,h)\beta(\xi) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(1)}(\xi,y) - \varphi(y)\| d\xi.$$

At the k -th step, we obtain the estimates:

$$\max_{r=\overline{1,N}} \|\lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x)\| \leq$$

$$\leq \frac{q(x,h)}{1-q(x,h)} \gamma(x,h)h\beta(x) \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(k)}(x,y) - z_r^{(k-1)}(x,y)\|,$$

$$\max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|v_r^{(k+1)}(x,y) - v_r^{(k)}(x,y)\| \leq$$

$$\leq \frac{1}{1-q(x,h)} h\beta(x)[1+\gamma(x,h)h\alpha(x)] \max_{r=\overline{1,N}} \sup_{y\in[0,Y]} \|z_r^{(k)}(x,y) - z_r^{(k-1)}(x,y)\|,$$

$$\max_{r=\overline{1,N}}\|\lambda_r^{(k+1)}(x)-\lambda_r^{(k)}(x)\|+\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|v_r^{(k+1)}(x,y)-v_r^{(k)}(x,y)\|\leq$$

$$\leq\theta(x,h)\beta(x)\int_0^x\Big(\max_{r=\overline{1,N}}\|\lambda_r^{(k)}(\xi)-\lambda_r^{(k-1)}(\xi)\|+\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|v_r^{(k)}(\xi,y)-v_r^{(k-1)}(\xi,y)\|\Big)d\xi,$$

$$\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|z_r^{(k+1)}(x,y)-z_r^{(k)}(x,y)\|\leq$$

$$\leq\int_0^x\Big(\max_{r=\overline{1,N}}\|\lambda_r^{(k+1)}(\xi)-\lambda_r^{(k)}(\xi)\|+\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|v_r^{(k+1)}(\xi,y)-v_r^{(k)}(\xi,y)\|\Big)d\xi.$$

Let's establish the inequalities

$$\max_{r=\overline{1,N}}\|\lambda_r^{(k+1)}(x)-\lambda_r^{(k)}(x)\|+\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|v_r^{(k+1)}(x,y)-v_r^{(k)}(x,y)\|\leq$$

$$\leq\frac{\theta(x,h)\beta(x)}{k!}\Big(\int_0^x\theta(\xi,h)\beta(\xi)d\xi\Big)^k\int_0^x\Big(\max_{r=\overline{1,N}}\|\lambda_r^{(1)}(\xi)\|+\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|v_r^{(1)}(\xi,y)\|\Big)d\xi.$$

$$\max_{r=\overline{1,N}}\|\lambda^{(k+1)}(x)-\lambda^{(1)}(x)\|+\max_{r=\overline{1,N}}\sup_{(x,y)\in\Omega_r}\|v_r^{(k)}(x,y)-v_r^{(1)}(x,y)\|\leq$$

$$\leq\theta(x,h)\beta(x)\sum_{j=0}^{k-1}\frac{1}{j!}\Big(\int_0^x\theta(\xi,h)\beta(\xi)d\xi\Big)^j\int_0^x\theta(\xi,h)d\xi\max\Big\{\max_{y\in[0,Y]}\|\varphi(y)\|,\max_{(x,y)\in\Omega}\|f(x,y)\|\Big\}.$$

$$\max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|z_r^{(k+1)}(x,y)-z_r^{(1)}(x,y)\|\leq$$

$$\leq\int_0^x\max_{r=\overline{1,N}}\|\lambda^{(k+1)}(\xi)-\lambda^{(1)}(\xi)\|d\xi+\int_0^x\max_{r=\overline{1,N}}\sup_{(x,y)\in\Omega_r}\|v_r^{(k+1)}(\xi,y)-v_r^{(1)}(\xi,y)\|d\xi.$$

Passing to the limit as $k\to\infty$, we obtain the estimates of Theorem 1. The uniqueness of the solution of this problem is proved by contradiction. Theorem 1 is proved.

The proof is complete.

If instead of $v_r(x,y)$ we substitute the corresponding right side of equality (16) and repeating this process $\nu(\nu=1,2,...)$ times we get

$$v_r(x,y)=G_{\nu r}(x,y,v_r)+D_{\nu r}(x,y)\lambda_r(\xi)d\xi+F_{\nu r}(x,y,z_r),\tag{18}$$

where

$$G_{\nu r}(x,y,v_r)=\int_{(r-1)h}^{y}A(x,\eta_1)...\int_{(r-1)h}^{\eta_{\nu-2}}A(x,\eta_{\nu-1})\int_{(r-1)h}^{\eta_{\nu-1}}A(x,\eta_\nu)v_r(x,\eta_\nu)d\eta_\nu d\eta_{\nu-1}...d\eta_1,$$

$$D_{\nu r}(x,y)=\sum_{j=0}^{\nu-1}\int_{(r-1)h}^{y}A(x,\eta_1)...\int_{(r-1)h}^{\eta_j}A(x,\eta_{j+1})d\eta_{j+1}...d\eta_1,$$

$$F_{\nu r}(x,y,z_r)=\int\limits_{(r-1)h}^{y}[B(x,\eta_1)z_r(x,\eta_1)+f(x,\eta_1)]d\eta_1+$$

$$+\sum_{j=1}^{\nu-1}\int\limits_{(r-1)h}^{y}A(x,\eta_1)...\int\limits_{(r-1)h}^{\eta_{j-1}}A(x,\eta_j)\int\limits_{(r-1)h}^{\eta_j}[B(x,\eta_{j+1})z_r(x,\eta_{j+1})+f(x,\eta_{j+1}))]d\eta_{j+1}d\eta_j...d\eta_1.$$

Passing to the limit at $y\to rh-0$ in (18) and substituting in (13), (14) instead of $\lim\limits_{t\to rh-0}v_r(x,y)$, $r=\overline{1,N}$, the corresponding right-hand sides for the unknown functions $\lambda_r(x)$, $r=\overline{1,N}$, we obtain the system of functional equations:

$$Q_\nu(x,h)\lambda(x)=-G_\nu(x,h,v)-F(x,h,z), \tag{19}$$

where

$$Q_\nu(x,h)=$$

$$=\begin{pmatrix} I & 0 & \dots & 0 & -I-D_{\nu N}(x,Nh)\\ I+D_{\nu 1}(x,h) & -I & \dots & 0 & 0\\ 0 & I+D_{\nu 2}(x,2h) & \dots & 0 & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & I+D_{\nu(N-1)}(x,(N-1)h) & -I \end{pmatrix},$$

$$G_\nu(x,h,v)=\begin{pmatrix} -G_{\nu N}(x,Nh,v_N)\\ G_{\nu 1}(x,h,v_1)\\ G_{\nu 2}(x,2h,v_2)\\ ...\\ G_{\nu(N-1)}(x,(N-1)h,v_{N-1})\end{pmatrix},\quad F_\nu(x,h,z)=\begin{pmatrix} -F_{\nu N}(x,Nh,z_N)\\ F_{\nu 1}(x,h,z_1)\\ F_{\nu 2}(x,2h,z_2)\\ ...\\ F_{\nu(N-1)}(x,(N-1)h,z_{N-1})\end{pmatrix}.$$

I is an identity matrix of dimension n.

To find a solution to a system of three functions $\{\lambda_r(x),v_r(x,y),z_r(x,y)\}$, $r=\overline{1,N}$, we have a closed system consisting of from equations (19), (18), (15).

Suppose that the matrix $Q_\nu(x,h)$ is invertible for all $x\in[0,X]$.

Taking $z_r^{(0)}(x,y)=\varphi(y)$, $r=\overline{1,N}$, as an initial approximation we find the solution of the boundary value problem (11)–(15) as the limit of the sequence of the system of triplets $\{\lambda_r^{(k)}(x),v_r^{(k)}(x,y),z_r^{(k)}(x,y)\}$, $k=1,2,\dots$, determined by the following algorithm: A) Assuming that $z_r(x,y)=z_r^{(k-1)}(x,y)$, $r=\overline{1,N}$, we find $k-$th approximations $\lambda_r^{(k)}(x),v_r^{(k)}(x,y)$ $r=\overline{1,N}$, as the limit of sequences $\lambda_r^{(k,m)}(x),v_r^{(k,m)}(x,y)$ $r=\overline{1,N}$, $\quad m=0,1,2,\dots$, defined as follows:

$$\lambda_r^{(k,0)}(x)=\lambda_r^{(k-1)}(x),\quad v_r^{(k,0)}(x,y)=v_r^{(k-1)}(x,y),$$

$$\lambda^{(k,m+1)}(x)=-[Q_\nu(x,h)]^{-1}\Big(G_\nu(x,h,v^{(k,m)})+F_\nu(x,h,z^{(k-1)})\Big),$$

$$v_r^{(k,m+1)}(x,y)=G_{\nu r}(x,y,v_r^{(k,m)})+D_{\nu r}(x,y)\lambda_r^{(k,m+1)}(\xi)d\xi+F_{\nu r}(x,y,z_r^{(k-1)}),$$

those pair system sequence $\{\lambda_r^{(k,m+1)}(x),v_r^{(k,m+1)}(x,y)\}$, for $m\to\infty$ converges to $\{\lambda_r^{(k)}(x),v_r^{(k)}(x,y)\}$, $r=\overline{1,N}$.

B) Functions $z_r^{(k)}(x,y),r=\overline{1,N}$, are determined from the relations

$$z_r^{(k)}(x,y)=\varphi(y)+\int\limits_0^x v_r^{(k)}(\xi,y)d\xi+\int\limits_0^x\lambda_r^{(k)}(\xi)d\xi.$$

The conditions of the following statement ensure the feasibility and convergence of the proposed algorithm, as well as the unique solvability of problem (11)–(15).

Theorem 2. Let for some choice of step $h > 0 : Nh = Y$, $N = 1, 2, ...$, and the number of substitutions ν, $\nu \in \mathbb{N}$, matrix $Q_\nu(x,h)$ of dimension $(nN \times nN)$ is invertible for all $x \in [0, X]$ and the inequalities hold:

1) $\|[Q_\nu(x,h)]^{-1}\| \leq \gamma_\nu(x,h)$;

2) $q_\nu(x,h) = \frac{(h\alpha(x))^\nu}{\nu!}\left(1 + \gamma_\nu(x,h)\sum\limits_{j=1}^{\nu}\frac{(h\alpha(x))^j}{j!}\right) \leq \mu < 1$, where $\alpha(x) = \sup\limits_{y\in[0,Y]}\|A(x,y)\|$.

Then there is a unique solution to problem (11)–(15) and fair assessments

$$a)\ \max_{r=\overline{1,N}}\|\lambda^*(x) - \lambda^{(1)}(x)\| + \max_{r=\overline{1,N}}\sup_{(x,y)\in\Omega_r}\|v_r^*(x,y) - v_r^{(1)}(x,y)\| \leq$$

$$\leq \theta_\nu(x,h)\beta(x)\exp\left(\int_0^x \theta_\nu(\xi,h)\beta(\xi)d\xi\right)\int_0^x \theta_\nu(\xi,h)d\xi \max\left\{\max_{y\in[0,Y]}\|\varphi(y)\|, \max_{(x,y)\in\Omega}\|f(x,y)\|\right\},$$

$$b)\ \max_{r=\overline{1,N}}\sup_{y\in[0,Y]}\|z_r^*(x,y) - z_r^{(1)}(x,y)\| \leq$$

$$\leq \int_0^x \max_{r=\overline{1,N}}\|\lambda^*(\xi) - \lambda^{(1)}(\xi)\|d\xi + \int_0^x \max_{r=\overline{1,N}}\sup_{(x,y)\in\Omega_r}\|v_r^*(\xi,y) - v_r^{(1)}(\xi,y)\|d\xi,$$

where $\beta(x) = \sup\limits_{y\in[0,Y]}\|B(x,y)\|$, $\quad \theta_\nu(x,h) = \frac{h}{1-q_\nu(x,h)}\left(1 + \gamma_\nu(x,h)\sum\limits_{j=0}^{\nu}\frac{(h\alpha(x))^j}{j!}\right)$.

The proof of Theorem 2 is similar to the proof of Theorem 1.

By virtue of the equivalence of problems (1)–(3) and (11)–(15), Theorem 1 implies

Theorem 3. Let the conditions of Theorem 1 be satisfied. Then problem (1)–(3) has a unique solution $z^*(x,t)$ and the estimate

$$\max\left\{\max_{(x,y)\in\Omega}\|z^*(x,y)\|, \max_{(x,y)\in\Omega}\left\|\frac{\partial z^*(x,y)}{\partial x}\right\|\right\} \leq$$

$$\leq \max\{1 + XM(h)(\beta+1), M(h)(\beta+1)\}\max\left\{\max_{y\in[0,Y]}\|\varphi(y)\|, \max_{(x,y)\in\Omega}\|f(x,y)\|\right\},$$

where

$$M(h) = \frac{1 - q(h) + [q(h)]^2}{1 - q(h)}\gamma_1(h)hXe^{\gamma_1(h)h\beta X} + \frac{Xh}{1-q(h)}\left(1 + \gamma_1(h)h(\alpha + \beta X)e^{\gamma_1(h)h\beta X}\right).$$

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Т.Д. Токмагамбетова, Н.Т. Орумбаева

Академик Е.А. Бөкетов атындағы Қарағанды университеті, Қарағанды, Қазақстан

Гиперболалық теңдеулер үшін периодтық шеттік есептің бір шешімі туралы

Тікбұрышты облыста гиперболалық типті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бір айнымалыдан тәуелді периодты шекаралық есеп қарастырылды. Авторлар жаңа белгісіз функцияны енгізе отырып, бұл есепті интегралдық шарты бар қарапайым дифференциалдық теңдеу үшін эквивалентті шекаралық есепке келтіреді. Параметрлеу әдісі негізінде эквивалентті есептің жуық шешімін табудың жаңа тәсілдері ұсынылып, оның жинақтылығы дәлелденеді. Бұл екінші ретті гиперболалық теңдеулер жүйесі үшін жартылай периодтық шекаралық есептің бірегей шешімі бар жағдайларды анықтауға мүмкіндік берді.

Кілт сөздер: шеттік есеп, гиперболалық теңдеулер, алгоритм, параметрлеу әдісі, жуық шешім.

Т.Д. Токмагамбетова, Н.Т. Орумбаева

Карагандинский университет имени академика Е.А. Букетова, Караганда, Казахстан

Об одном решении периодической краевой задачи для гиперболического уравнения

В прямоугольной области рассмотрена периодическая по одной переменной краевая задача для системы дифференциальных уравнений в частных производных гиперболического типа. Авторы вводя новую неизвестную функцию, данную задачу сводят к эквивалентной краевой задаче для обыкновенного дифференциального уравнения с интегральным условием. На основе метода параметризации предложены новые подходы нахождения приближенного решения эквивалентной задачи и доказана его сходимость. Это позволило установить условия существования единственного решения полупериодической краевой задачи для системы гиперболических уравнений второго порядка.

Ключевые слова: краевая задача, гиперболические уравнения, алгоритм, метод параметризации, приближенное решение.

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