DOI 10.31489/2022M1/118-126

UDC 517.925, 519.21

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Construction of the differential equations system of the program motion in Lagrangian variables in the presence of random perturbations

The classification of inverse problems of dynamics in the class of ordinary differential equations is given in the Galiullin's monograph. The problem studied in this paper belongs to the main inverse problem of dynamics, but already in the class of second-order stochastic differential equations of the Ito type. Stochastic equations of the Lagrangian structure are constructed according to the given properties of motion under the assumption that the random perturbing forces belong to the class of processes with independent increments. The problem is solved as follows: First, a second-order Ito differential equation is constructed so that the properties of motion are the integral manifold of the constructed stochastic equation. At this stage, the quasi-inversion method, Erugin's method and Ito's rule of stochastic differentiation of a complex function are used. Then, by applying the constructed Ito equation, an equivalent stochastic equation of the Lagrangian structure is constructed. The necessary and sufficient conditions for the solvability of the problem of constructing the stochastic equation of the Lagrangian structure are illustrated by the example of the problem of constructing the Lagrange function from a motion property of an artificial Earth satellite under the action of gravitational forces and aerodynamic forces.

Keywords: stochastic differential equation, stochastic basic inverse problem, stochastic equation of Lagrangian structure, integral manifold, quasi-inversion method.

Introduction. Problem statement

At present, the theory of inverse problems of dynamics has been developed fully in the class of ordinary differential equations (ODE) [1–9]. This theory originates from the fundamental Erugin's work [10], in which a set of ODEs with a given integral curve is constructed. A generalization of methods for solving inverse problems of dynamics to the class of Ito stochastic differential equations is given in [11–18].

Using the given set

$$\Lambda(t) : \lambda(x, \dot{x}, t) = 0, \lambda \in R^m, \ x \in R^n, \tag{1}$$

it is required to construct stochastic equations of the Lagrangian structure

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_\nu}\right) - \frac{\partial L}{\partial x_\nu} = \sigma'_{\nu j}(x, \dot{x}, t)\dot{\xi}^j, \ \left(\nu = \overline{1, n}, \ j = \overline{1, r}\right), \tag{2}$$

so that the set $\Lambda(t)$ (1) is an integral manifold of equation (2).

Here $\{\xi_1(t, \omega), \dots, \xi_r(t, \omega)\}$ is a system of random processes with independent increments, which, following [19], can be represented as a sum of Wiener processes and Poisson processes: $\xi = \xi_0 + \int c(y)P^0(t, dy)$. ξ_0 is a Wiener process. P^0 is a Poisson process. $P^0(t, dy)$ is a number of process P^0 jumps in the interval $[0, t]$ that fall on the set dy. $c(y)$ is the vector function mapping space R^{2n} to value space R^r of process $\xi(t)$ for any t.

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The assigned problem was considered in the class of ordinary differential equations in [20]. The stochastic Helmholtz problem (the problems of constructing stochastic equations of the Lagrangian structure equivalent to the given second-order stochastic Ito equation) was considered in [21]. In [22, 23] the above problem was considered under the assumption that the system $\{\xi_1(t,\omega),...,\xi_r(t,\omega)\}$ is a system of independent Wiener processes which are a particular case of processes with independent increments.

The scheme for solving the problem is as follows: First, the second-order Ito differential equation

$$\ddot{x} = f(x,\dot{x},t) + \sigma(x,\dot{x},t)\dot{\xi} \tag{3}$$

is constructed so that the given properties of motion are the integral manifold of the constructed stochastic equation (3). At this stage, the quasi-inversion method [3], Erugin's method [10], and Ito's rule of stochastic differentiation of a complex function in the case of processes with independent increments [19] are used. Then, using the constructed Ito equation, an equivalent stochastic equation of the Lagrangian structure is constructed.

1 Construction of Ito equation by the given properties of motion (1)

Previously, the equation of perturbed motion

$$\dot{\lambda} = \frac{\partial\lambda}{\partial t} + \frac{\partial\lambda}{\partial x}\dot{x} + \frac{\partial\lambda}{\partial\dot{x}}f + S_1 + S_2 + S_3 + \frac{\partial\lambda}{\partial\dot{x}}\sigma\dot{\xi} \tag{4}$$

is compiled according to the Ito stochastic differentiation rule, here $S_1 = \dfrac{1}{2}\dfrac{\partial^2\lambda}{\partial\dot{x}^2} : \sigma\sigma^T$, and by $\dfrac{\partial^2\lambda}{\partial\dot{x}^2} : D$, $D = \sigma\sigma^T$, following [19], we mean a vector whose elements are the traces of the products of the matrices of the second derivatives of the corresponding elements $\lambda_\mu(x,\dot{x},t)$ of the vector $\lambda(x,\dot{x},t)$ with respect to the components $\dot{x}$ on the matrix D $\dfrac{\partial^2\lambda}{\partial\dot{x}^2} : D = \left[tr\left(\dfrac{\partial^2\lambda_1}{\partial\dot{x}^2}D\right), \ldots, tr\left(\dfrac{\partial^2\lambda_m}{\partial\dot{x}^2}D\right)\right]^T$; $S_2 = \int\left\{\lambda(x,\dot{x}+\sigma c(y),t) - \lambda(x,\dot{x},t) + \dfrac{\partial\lambda}{\partial\dot{x}}\sigma c(y)\right\}dy$; $S_3 = \int[\lambda(x,\dot{x}+\sigma c(y),t) - \lambda(x,\dot{x},t)]P^0(t,dy)$. Following Eerugin's method [1], we introduce arbitrary vector-function A and matrix B with the properties $A(0,x,\dot{x},t) \equiv 0$, $B(0,x,\dot{x},t) \equiv 0$ such that

$$\dot{\lambda} = A(\lambda,x,\dot{x},t) + B(\lambda,x,\dot{x},t)\dot{\xi}. \tag{5}$$

Comparing equations (4) and (5), we obtain the relations

$$\begin{cases} \dfrac{\partial\lambda}{\partial\dot{x}}f = A - \dfrac{\partial\lambda}{\partial t} - \dfrac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3, \\ \dfrac{\partial\lambda}{\partial\dot{x}}\sigma = B. \end{cases} \tag{6}$$

To determine the required functions f and σ from equalities (6) we need the following statement:

Lemma 1 [4, p. 12–13]. The set of all solutions of a linear system

$$Hv = g, H = (h_{\mu k}),\ v = (v_k),\ g = (g_\mu),\ \mu = \overline{1,m},\ k = \overline{1,n},\ m \le n, \tag{7}$$

here H is the matrix of rank m, is determined by the expression

$$\nu = \alpha\nu^T + \nu^v. \tag{8}$$

Here α is scalar,

$$\nu^T = [HC] = [h_1 ... h_m c_{m+1} ... c_{n-1}] = \begin{vmatrix} e_1 & ... & e_n \\ h_{11} & ... & h_{1n} \\ ... & ... & ... \\ h_{m1} & ... & h_{mn} \\ c_{m+1,1} & ... & c_{m+1,n} \\ ... & ... & ... \\ c_{n-1,1} & ... & c_{n-1,n} \end{vmatrix}$$

is the cross product of vectors $h_\mu = (h_{\mu k})$ and arbitrary vectors $c_\rho = (c_{\rho k})$, $\rho = \overline{m+1, n-1}$; e_k are unit vectors of space R^n, $\nu^T = (\nu_k^T)$

$$\nu_k^T = \begin{vmatrix} 0 & \dots & 1 & \dots & 0 \\ h_{11} & \dots & h_{1k} & \dots & h_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ h_{m1} & \dots & h_{mk} & \dots & h_{mn} \\ c_{m+1,1} & \dots & c_{m+1,n} & \dots & c_{m+1,n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{n-1,1} & \dots & c_{n-1,k} & \dots & c_{n-1,n} \end{vmatrix}, \ \nu^v = H^+ g,$$

$H^+ = H^T(HH^T)^{-1}$, H^T is the matrix transposed to H.

By Lemma 1, using (7), (8) we define the vector function f and the matrix σ in the form

$$f = s_1 \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ \left(A - \frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3\right) \tag{9}$$

$$\sigma_i = s_{2i} \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ B_i, \tag{10}$$

here $\sigma_i = (\sigma_{1i},\ \sigma_{2i},\ \dots,\ \sigma_{ni})^T$ is the i-th column of the matrix $\sigma = (\sigma_{\nu j})$, $\left(\nu = \overline{1,n},\ \ j = \overline{1,r}\right)$; $B_i = (B_{1i}, B_{2i}, \dots, B_{mi})^T$ is the i-th column of the matrix $B = (B_{\mu j})$, $\left(\mu = \overline{1,m},\ j = \overline{1,r}\right)$, s_1, s_2 are the arbitrary scalars.

Consequently, it follows from (9), (10) that the set of the second-order Ito differential equations containing a given integral manifold (1) has the form

$$\ddot{x} = s_1 \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ \left(A - \frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\dot{x} - S_1 - S_2 - S_3\right) +$$

$$+ \left(s_{21} \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ B_1, \dots\ s_{2r} \left[\frac{\partial\lambda}{\partial\dot{x}} C\right] + \left(\frac{\partial\lambda}{\partial\dot{x}}\right)^+ B_r\right) \dot{\xi}.$$

2 Construction of the Lagrangian structure equation (2) according to the Ito equation (3)

We expand the expression $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_\nu}\right)$ according to the Ito stochastic differentiation rule in the case of processes with independent increments [19]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_\nu}\right) = \frac{\partial^2 L}{\partial \dot{x}_\nu \partial t} + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial x_k}\dot{x}_k + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial \dot{x}_k}\ddot{x}_k + \tilde{S}_{1\nu} + \tilde{S}_{2\nu} + \tilde{S}_{3\nu}, \tag{11}$$

here $\tilde{S}_{1\nu} = \frac{1}{2}\frac{\partial^3 L}{\partial \dot{x}_\nu \partial \dot{x}_i \partial \dot{x}_k}\sigma_{ij}\sigma_{kj}$, $\tilde{S}_{2\nu} = \int \left\{\frac{\partial L(x, \dot{x} + \sigma c(y), t)}{\partial \dot{x}_\nu} - \frac{\partial L(x, \dot{x}, t)}{\partial \dot{x}_\nu}\right\} dy$,
$\tilde{S}_{3\nu} = \int [\frac{\partial L(x, \dot{x} + \sigma c(y), t)}{\partial \dot{x}_\nu} - \frac{\partial L(x, \dot{x}, t)}{\partial \dot{x}_\nu}] P^0(t, y)$.

Therefore, equation (2), taking into account (11), can be written in the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_\nu}\right) - \frac{\partial L}{\partial x_\nu} - \sigma'_{\nu j}(x, \dot{x}, t)\dot{\xi}^j = \frac{\partial^2 L}{\partial \dot{x}_\nu \partial t} + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial x_k}\dot{x}_k + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial \dot{x}_k}\ddot{x}_k +$$

$$+ \tilde{S}_{1\nu} + \tilde{S}_{2\nu} + \tilde{S}_{3\nu} - \frac{\partial L}{\partial x_\nu} - \sigma'_{\nu j}(x, \dot{x}, t)\dot{\xi}^j. \tag{12}$$

Or, taking into account (12) and equation (3), we have

$$\frac{\partial^2 L}{\partial \dot{x}_\nu \partial t} + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial x_k}\dot{x}_k + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial \dot{x}_k}\ddot{x}_k + \tilde{S}_{1\nu} + \tilde{S}_{2\nu} + \tilde{S}_{3\nu} - \frac{\partial L}{\partial x_\nu} - \sigma'_{\nu j}(x, \dot{x}, t)\dot{\xi}^j \equiv$$

$$\equiv \ddot{x}_\nu - f_\nu(x,\dot{x},t) + \sigma_{\nu j}(x,\dot{x},t)\dot{\xi}^j. \tag{13}$$

Relation (13) implies the equalities

$$\frac{\partial^2 L}{\partial \dot{x}_\nu \partial \dot{x}_k} = \delta_\nu^k;\ \frac{\partial^2 L}{\partial \dot{x}_\nu \partial t} + \frac{\partial^2 L}{\partial \dot{x}_\nu \partial x_k}\dot{x}_k + \tilde{S}_{1\nu} + \tilde{S}_{2\nu} + \tilde{S}_{3\nu} - \frac{\partial L}{\partial x_\nu} = f_\nu,\ \sigma'_{\nu j}(x,\dot{x},t) = \sigma_{\nu j}. \tag{14}$$

Thus, the following theorem is proven.

Theorem 1. To construct the stochastic equation of the Lagrangian structure (2) by the given set (1), so that set (1) is the integral manifold of the constructed equation, it is necessary and sufficient to satisfy conditions (14).

3 An example

Let us consider the stochastic problem of constructing a Lagrange function for a given property of motion by the example of the motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces [24].

Consider the properties of motion in the following form:

$$\Delta(t):\ \lambda = \theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2 = 0,\ \lambda \in R^1. \tag{15}$$

Then the perturbed motion equation (4) takes the form

$$\dot{\lambda} = 2\theta\dot{\theta} + 2\alpha_1\dot{\theta}\ddot{\theta} + S_1 + S_2 + S_3 = 2\theta\dot{\theta} + 2\alpha_1\dot{\theta}f + S_1 + S_2 + S_3 + 2\alpha_1\dot{\theta}\sigma\dot{\xi}, \tag{16}$$

here $S_1 = \alpha_1\sigma^2$, $S_2 = \int \left\{2\alpha_1\sigma c(y)[4\dot{\theta} + \sigma c(y)]\right\} dy$, $S_3 = \int \left\{2\alpha_1\sigma c(y)[4\dot{\theta} + \sigma c(y)]\right\} P^0(t,dy)$.

Let us introduce Erugin's functions $a = a(\lambda,\ \theta,\ \dot{\theta},\ t)$, $b = b(\lambda,\theta,\dot{\theta},t)$ with the property $a(0,\theta,\dot{\theta},t) = b(0,\theta,\dot{\theta},t) \equiv 0$ and such that the relation

$$\dot{\lambda} = a\lambda(\theta,\dot{\theta},t) + b\lambda(\theta,\dot{\theta},t)\dot{\xi} \tag{17}$$

takes place. From relations (16), (17) it follows that the set of equations (3), in our example is having the form

$$\ddot{\theta} = f(\theta,\dot{\theta},t) + \sigma(\theta,\dot{\theta},t)\dot{\xi},$$

possesses the integral manifold (15) if f and σ have, respectively, the forms

$$f = \frac{a(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) - 2\theta\dot{\theta} - S_1 - S_2 - S_3}{2\alpha_1\dot{\theta}}, \quad \sigma = \frac{b(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2)}{2\alpha_1\dot{\theta}}. \tag{18}$$

The equation of motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces, following [24], can be written in the form

$$\ddot{\theta} = \tilde{f}(\theta,\dot{\theta}) + \tilde{\sigma}(\theta,\dot{\theta})\dot{\xi}, \tag{19}$$

here θ is a pitch angle, functions $\tilde{f}$, $\tilde{\sigma}$ have the forms

$$\tilde{f} = Ql\sin 2\theta - Q[g(\theta) + \eta\dot{\theta}],\ \tilde{\sigma} = Q\delta[g(\theta) + \eta\dot{\theta}]. \tag{20}$$

Let us construct the Lagrangian using equation (19). In equation (19), we take into account relations (18). These relations give the integrality of the given set (15). It follows from the equalities $f = \tilde{f}$, $\sigma = \tilde{\sigma}$ that four parameters Q,δ,η,l, determining the dynamics of the satellite motion (20), must satisfy the following relations

$$\begin{cases} a(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) - 2\theta\dot{\theta} - S_1 - S_2 - S_3 = 2\alpha_1\dot{\theta}\left\{Ql\sin 2\theta - Q[g(\theta) + \eta\dot{\theta}]\right\}, \\ b(\theta^2 + \alpha_1\dot{\theta}^2 + \alpha_2) = 2\alpha_1\dot{\theta}Q\delta[g(\theta) + \eta\dot{\theta}]. \end{cases}$$

Then, by definition from [25], (19) admits an indirect analytical representation in terms of the stochastic Lagrangian equation if there exists a function h such that the identity

$$d\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} - \sigma'(\theta,\dot{\theta},t)\dot{\xi} \equiv h[\ddot{\theta} - f - \sigma\dot{\xi}]$$

takes place.

Let us find the function $h = h(t)$ so that the necessary and sufficient Helmholtz conditions [25, p.107] for the existence of the Lagrangian are satisfied for the scalar equation $l_1(\theta, \dot{\theta}, t)\ddot{\theta} + l_2(\theta, \dot{\theta}, t) = 0$:

$$\frac{\partial l_2}{\partial \dot{\theta}} = \frac{\partial l_1}{\partial t} + \dot{\theta}\frac{\partial l_1}{\partial \theta}.$$

In particular, a function $h = e^{-Q\eta t}$ satisfies this condition. Substituting h in (19), we obtain

$$e^{-Q\eta t}[\ddot{\theta} - f - \sigma\dot{\xi}] = \frac{\partial^2 L}{\partial \dot{\theta}^2}\ddot{\theta} + \frac{\partial^2 L}{\partial \dot{\theta}\partial \theta}\dot{\theta} + \frac{\partial^2 L}{\partial \theta \partial t} - \frac{\partial L}{\partial t}\sigma'\dot{\xi}.$$

Thus, the required Lagrangian is constructed in the form

$$L = e^{-Q\eta t}[\frac{1}{2}\dot{\theta}^2 - Q(\frac{1}{2}l\cos 2\theta + G)], \qquad \text{here} \quad G = \int g(\theta)d\theta,$$

which provides a representation of the equation (19) in the form of the Lagrangian structure equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = e^{-Q\eta t}\sigma(\theta, \dot{\theta})\dot{\xi}.$$

Acknowledgements

This research has been funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP 09258966).

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Кездейсоқ түрткілер болғанда Лагранждың айнымалыларындағы бағдарламалық қозғалыстың дифференциалдық теңдеулер жүйесін құру

Мақалада Ито типті екінші ретті стохастикалық дифференциалдық теңдеулер класындағы динамиканың негізгі (А.С. Галиуллин жіктемесі бойынша) кері есептерінің нұсқаларының бірі қарастырылған. Ито типіндегі стохастикалық дифференциалдық теңдеулер класында Лагранж теңдеулері берілген қозғалыс қасиеттеріне сәйкес құрылады. Бұл жағдайда күштің кездейсоқ түрткілері тәуелсіз өсімшелі үдерістер класынан деп болжалынады. Алдымен есепті шешу үшін квазиқайтару әдісі бойынша қозғалыстың берілген қасиеттеріне сәйкес Еругин әдісімен және тәуелсіз өсімшелі үдерістер жағдайында күрделі функцияның стохастикалық дифференциалдау формуласымен екінші ретті Ито дифференциалдық теңдеуі берілген қозғалыс қасиеттері салынған стохастикалық теңдеудің интегралдық көпбейнесі болатындай етіп құрылады. Екінші кезеңде, алынған Ито теңдеуіне сәйкес, оған эквивалентті Лагранж құрылымының стохастикалық теңдеулері құрылады. Осылайша, тәуелсіз өсімшелі үдерістер класында кездейсоқ түрткілер болған кезде берілген қозғалыс қасиеттерінен Лагранж құрылымының теңдеуін құру мәселесінің шешімділігі үшін қажетті және жеткілікті шарттары алынған. Алынған нәтижелер Жердің жасанды серігінің тартылыс күштері мен аэродинамикалық күштердің әсерінен берілген қозғалыс қасиетіне сәйкес Лагранж функциясын құрудың стохастикалық есебінің мысалында көрсетілген.

Кілт сөздер: Ито типті стохастикалық дифференциалдық теңдеуі, стохастикалық негізгі кері есебі, Лагранж құрылымының стохастикалық теңдеуі, интегралдық көпбейне, квазиқайтару әдісі.

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Построение системы дифференциальных уравнений программного движения в лагранжевых переменных при наличии случайных возмущений

В статье рассмотрен один из вариантов основной (по классификации А.С. Галиуллина) обратной задачи динамики в классе стохастических дифференциальных уравнений второго порядка типа Ито. Построены уравнения Лагранжа по заданным свойствам движения в классе стохастических дифференциальных уравнений типа Ито. При этом случайные возмущающие силы предполагаются из класса процессов с независимыми приращениями. Для решения поставленной задачи на первом этапе по заданным свойствам движения методом квазиобращения в сочетании с методом Еругина и в силу стохастического дифференцирования сложной функции в случае процессов с независимыми приращениями построено дифференциальное уравнение Ито второго порядка так, чтобы заданные свойства движения являлись интегральным многообразием построенного стохастического уравнения. И, далее, на втором этапе по построенному уравнению Ито строятся эквивалентные ему стохастические уравнения лагранжевой структуры. Таким образом, получены необходимые и достаточные условия разрешимости задачи построения уравнения лагранжевой структуры по заданным свойствам движения при наличии случайных возмущений из класса процессов с независимыми приращениями. Полученные результаты проиллюстрированы на примере стохастической задачи построения функции Лагранжа по заданному свойству движения искусственного спутника Земли под действием сил тяготения и аэродинамических сил.

Ключевые слова: стохастическое дифференциальное уравнение Ито, стохастическая основная обратная задача, стохастическое уравнение лагранжевой структуры, интегральное многообразие, метод квазиобращения.

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