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A note on well-posedness of source identification elliptic problem in a Banach space

We study the source identification problem for an elliptic differential equation in a Banach space. The exact estimates for the solution of source identification problem in Hölder norms are obtained. In applications, four elliptic source identification problems are investigated. Stability and coercive stability estimates for solution of source identification problems for elliptic equations are obtained.

Keywords: well-posedness, elliptic equations, positivity, coercive stability, source identification, exact estimates, boundary value problem.

Introduction

Several source identification problems for partial differential equations have been extensively investigated by many researchers (see [3,4,8–11,14,15,17–19] and the bibliography herein). Well-posedness of nonclassical boundary value problems for various partial differential and difference equations was established in a number of publications (see [1]-[22] and references therein).

Large number of the source identification problems for an elliptic differential equations can be written as the source identification problem for the second order differential equation

$$\begin{cases} -u''(t) + Au(t) = f(t) + p, & 0 < t < 1, \\ u(0) = u(1), \ u'(0) = u'(1), u(\lambda) = \xi, \lambda \in (0,1) \end{cases} \tag{1}$$

in an arbitrary Banach space E with a positive operator A. Here parameter $p \in E$ and abstract function $u : [0,1] \to E$ are unknown and element $\xi \in D(A)$ and abstract function $f : [0,1] \to E$ are given.

Let $E_1 \subset E$ and $F(E)$ be the Banach space of $E-$valued smooth functions on $[0,1]$. We say that the pair $\{u(t), p\}$ is the solution of the source identification problem (1) in $F(E) \times E_1$ if the following conditions are valid:

(i) $p \in E_1, u''(t) \in F(E), \ Au(t) \in F(E)$,

(ii) $\{u(t), p\}$ is satisfied the equation and all three conditions of (1).

In the present paper, theorem on well-posedness of the source identification problem (1) in Hölder spaces is established. In applications, stability and coercive stability estimates for solution of the four type of source identification problems for elliptic equations are obtained.

Stability and coercive stability estimates

Denote by $C_{01}^{\alpha,\alpha}(E)$ $(0 < \alpha < 1)$, the Banach space obtained by completion of the set of $E-$valued smooth functions $\varphi(t)$ defined on $[0,1]$ with values in E in the norm

$$\|\varphi\|_{C_{01}^{\alpha,\alpha}(E)} = \|\varphi\|_{C(E)} + \sup_{0 \leq t < t+\tau \leq 1} \tau^{-\alpha}(1-t)^{\alpha}(t+\tau)^{\alpha}\|\varphi(t+\tau) - \varphi(t)\|_E,$$

where $C(E)$ is the Banach space of all continuous functions $\varphi(t)$ defined on $[0,1]$ with values in E equipped with the norm

$$\|\varphi\|_{C(E)} = \max_{0 \leq t \leq 1}\|\varphi(t)\|_E.$$

Assume that $v(t)$ is the solution of the nonlocal boundary value problem

$$\begin{cases} -v''(t) + Av(t) = f(t), \ 0 < t < 1, \\ \\ v(0) = v(1), \ v'(0) = v'(1). \end{cases} \tag{2}$$

Then, for the solution of problem (1) we have the following formulas

$$u(t) = v(t) + A^{-1}p, \tag{3}$$

$$p = A\xi - Av(\lambda). \tag{4}$$

Therefore, the following algorithm can be used to find the solution of problem (1):
(1) Find the solution $v(t)$ of nonlocal boundary value problem (2).
(2) Use (4) to obtain the source element p of source identification problem (1).
(3) Applying (3), obtain the solution $u(t)$ of source identification problem (1).

It is known that the operator $B = A^{\frac{1}{2}}$ is the strongly positive operator for any positive operator A. Therefore, the operator $-B$ will be a generator of an analytic semigroup $\exp-tB$ $(t \geq 0)$ with exponentially decreasing norm (see [7]), when $t \to \infty$, i.e. there exist some $M(B) \in [1,+\infty)$, $\alpha(B) \in (0,+\infty)$ such that the following estimates

$$\|\exp(-tB)\|_{E \to E} \leq M(B)\exp(-\alpha(B)t), \tag{5}$$

$$\|tB \ \exp(-tB)\|_{E \to E} \leq M(B)\exp(-\alpha(B)t)(t > 0), \tag{6}$$

$$\|T\|_{E \to E} \leq M(B)\left(1 - \exp(-\alpha(B))\right)^{-1} \tag{7}$$

are satisfied. Here $T = (I - \exp(-B))^{-1}$.

The solution of direct problem (2) is defined by (formula (1.7) [1])

$$v(t) = \tfrac{1}{2}B^{-1}T\exp(-(1-t)B)\int_0^1 \exp(-sB)f(s)ds \tag{8}$$

$$+\tfrac{1}{2}B^{-1}\int_0^t \exp(-(t-s)B)f(s)ds \ + \tfrac{1}{2}B^{-1}\int_t^1 \exp((t-s)B)f(s)ds$$

$$+ \ \tfrac{1}{2}B^{-1}T\exp(-tB)\int_0^1 \exp(-(1-s)B)f(s)ds \ .$$

From (4) and (8), it follows that

$$p = A\xi - \tfrac{1}{2} BT \exp(-(1-\lambda)B) \int\limits_0^1 \exp(-sB)f(s)ds \tag{9}$$

$$-\tfrac{1}{2}B \int\limits_0^\lambda \exp(-(\lambda-s)B)f(s)ds - \tfrac{1}{2}B \int\limits_\lambda^1 \exp((\lambda-s)B)f(s)ds$$

$$-\tfrac{1}{2}BT \exp(-B\lambda) \int\limits_0^1 \exp(-(1-s)B)f(s)ds.$$

Finally, by using formulas (8), (3) and (9), we can obtain $u(t)$.

Now, we formulate result on well-posedness of the source identification problem (1) in the space $C_{01}^{\alpha,\alpha}(E)$.

Theorem 1. Assume that $\xi \in D(A)$ and $f(t) \in C_{01}^{\alpha,\alpha}(E), 0 < \alpha < 1$. For the solution $\{u(t), p\}$ of the source identification problem (1) the following stability inequality

$$\|u\|_{C(E)} + \|A^{-1}p\|_E \le M \left[\|\xi\|_E + \|f\|_{C(E)} \right] \tag{10}$$

and coercive inequality

$$\|u''\|_{C_{01}^{\alpha,\alpha}(E)} + \|Au\|_{C_{01}^{\alpha,\alpha}(E)} + \|p\|_E \le M[\|A\xi\|_E + \tfrac{1}{\alpha(1-\alpha)} \|f\|_{C_{01}^{\alpha,\alpha}(E)}], \tag{11}$$

hold, where M is independent of α, ξ and $f(t)$.

The proof of Theorem 1 is based on the formula (3) and estimates (5) and (7) on the Theorem on well-posedness of the nonlocal boundary value problem (2) [1].

Note that same results can be established for the solutions of the general source identification problems

$$\begin{cases} -u''(t) + Au(t) = f(t) + p, \quad 0 < t < 1, \\ u(0) = \sum\limits_{j=1}^N a_j u(t_j) + \varphi, \ u'(0) = u'(1) + \psi, u(\lambda) = \xi, \lambda \in (0,1), \end{cases}$$

where $0 < t_1 < ... < t_N \le 1$, if the operator

$$I - e^{-2B} - \sum\limits_{j=1}^N a_j \left(e^{-t_j B} - e^{-(2-t_j)B} - e^{-(1-t_j)B} + e^{-(1+t_j)B} \right)$$

has a bounded inverse in E and

$$\begin{cases} -u''(t) + Au(t) = f(t) + p, \quad 0 < t < 1, \\ u(0) = u(1) + \varphi, \ u'(0) = \sum\limits_{j=1}^N a_j u'(s_j) + \psi, u(\lambda) = \xi, \lambda \in (0,1), \end{cases}$$

where $0 < s_1 < ... < s_N \le 1$, if the operator

$$\left(I - e^{-B} \right)^2 - \sum\limits_{j=1}^N a_j \left(e^{-s_j B} + e^{-(2-s_j)B} - e^{-(1-s_j)B} - e^{-(1+s_j)B} \right)$$

has a bounded inverse in E .

Applications

In this section, we consider the applications of Theorem 1. First, we study the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions

$$
\begin{cases}
-\frac{\partial^2 u(t,x)}{\partial t^2} - a(x)\frac{\partial^2 u(t,x)}{\partial x^2} + \delta u(t,x) = f(t,x) + p(x), \\
0 < t < 1, 0 < x < l, \\
u(0,x) = u(1,x), u_t(0,x) = u_t(1,x), \ u(\lambda, x) = \xi(x), 0 \le x \le l, \\
u(t,0) = u(t,l), \ u_x(t,0) = u_x(t,l), \ 0 \le t \le 1,
\end{cases}
\tag{12}
$$

where $a(x)$, $\xi(x)$ and $f(t,x)$ are given sufficiently smooth functions and $a(x) > 0$, $0 < \lambda < 1, \delta > 0$ is a sufficiently large number. Assume that all compatibility conditions are satisfied.

We introduce the Banach spaces $C^\beta[0,l]$ $(0 < \beta < 1)$ of all continuous functions $\varphi(x)$ satisfying a Hölder condition for which the following norms are finite

$$
\| \varphi \|_{C^\beta[0,l]} = \| \varphi \|_{C[0,l]} + \sup_{0 \le x < x+\tau \le l} \frac{|\varphi(x+\tau) - \varphi(x)|}{\tau^\beta},
$$

where $C[0,l]$ is the space of the all continuous functions $\varphi(x)$ defined on $[0,l]$ with the usual norm

$$
\| \varphi \|_{C[0,l]} = \max_{0 \le x \le l} |\varphi(x)|.
$$

Theorem 2. For the solution of the source identification problem (12) the following stability and coercive stability estimates hold:

$$
\|u\|_{C(C^\beta[0,l])} \le M(\beta) \left[\|\xi\|_{C^\beta[0,l]} + \|f\|_{C(C^\beta[0,l])} \right],
$$

$$
\|u\|_{C_{01}^{2+\alpha,\alpha}(C^\beta[0,l])} + \|u\|_{C_{01}^{\alpha,\alpha}(C^{\beta+2}[0,l])} + \|p\|_{C^\beta[0,l]}
$$

$$
\le \frac{M(\beta)}{\alpha(1-\alpha)} \|f\|_{C_{01}^{\alpha,\alpha}(C^\beta[0,l])} + M(\beta) \|\xi\|_{C^{\beta+2}[0,l]},
$$

where $M(\beta)$ is independent of $\alpha, \xi(x)$ and $f(t,x), 0 < \alpha < 1, \ 0 < \beta < 1$.

The proof of Theorem 2 is based on the Theorem 1 and the positivity of the elliptic operator A in $C^\beta[0,l]$ [7].

Second, we investigate the source identification problem on the range $\{0 \le t \le 1, x \in R^n\}$

$$
\begin{cases}
-u_{tt}(t,x) + \sum_{|l|=2m} a_l(x) \frac{\partial^{|l|}}{\partial x_1^{l_1} \ldots \partial x_n^{l_n}} u(t,x) + \delta u(t,x) = f(t,x) + p(x), \\
0 < t < 1, x \in R^n, \\
u(0,x) = u(1,x), u_t(0,x) = u_t(1,x), \ u(\lambda, x) = \xi(x), x \in R^n
\end{cases}
\tag{13}
$$

for the $2m-$order multidimensional elliptic equation, where $a_l(x)$ $(l = (l_1, ..., l_n), |l| = 0, ..., 2m)$ and $\xi(x)$ are known sufficiently smooth functions, $a_l(x) > 0$, and $0 < \lambda < 1, \delta > 0$ are given real numbers. Assume that all compatibility conditions are satisfied and the symbol

$$
F^x(\zeta) = \sum_{|l|=2m} a_l(\zeta) (i\zeta_1)^{l_1} \ldots (i\zeta_n)^{l_n}, \ \zeta = (\zeta_1, \ldots, \zeta_n) \in R^n
$$

of the differential operator

$$F^x = \sum_{|l|=2m} a_l(\zeta) \frac{\partial^{|l|}}{\partial x_1^{l_1}...\partial x_n^{l_n}} \tag{14}$$

acting on functions in the space R^n, satisfies the inequalities

$$0 \le M_1 |\zeta|^{2m} \le (-1)^m F^x(\zeta) \le M_2 |\zeta|^{2m} < \infty,$$

for $\zeta \ne 0$.

Theorem 3. For the solution of the source identification problem (13) the following stability and coercive stability estimates are satisfied:

$$\|u\|_{C(C^\mu(R^n))} \le M(\mu) \left[\|\xi\|_{C^\mu(R^n)} + \|f\|_{C(C^\mu(R^n))} \right],$$

$$\|u\|_{C_{01}^{2+\alpha,\alpha}(C^\mu(R^n))} + \sum_{|l|=2m} \left\| \frac{\partial^{|l|}u}{\partial x_1^{l_1}...\partial x_n^{l_n}} \right\|_{C_{01}^{\alpha,\alpha}(C^\mu(R^n))} + \|p\|_{C^\mu(R^n)}$$

$$\le \frac{M(\mu)}{\alpha(1-\alpha)} \|f\|_{C_{01}^{\alpha,\alpha}(C^\mu(R^n))} + M(\mu) \sum_{|l|=2m} \left\| \frac{\partial^{|l|}\xi}{\partial x_1^{l_1}...\partial x_n^{l_n}} \right\|_{C^\mu(R^n)}$$

where $M(\mu)$ is independent of α, $\xi(x)$ and $f(t,x), 0 < \alpha < 1, \ 0 < \mu < 1$.

The proof of Theorem 3 is based on the Theorem 1 and the positivity of the elliptic operator A^x in $C^\mu(R^n)$ [7] and the coercivity estimate for an operator A^x in $C^\mu(R^n)$ [8].

Third, let $\Omega = (0,1)^n$ be the open cube in R^n with suitable boundary S, $\overline{\Omega} = \Omega \cup S$. In $[0,1] \times \Omega$, we study the source identification problem

$$\begin{cases} -u_{tt}(t,x) - \sum_{k=1}^n a_k(x) u_{x_k x_k}(t,x) + \delta u(t,x) = f(t,x) + p(x), \\ \\ x = (x_1,...,x_n) \in \Omega, 0 < t < 1, \\ \\ u(0,x) = u(1,x), u_t(0,x) = u_t(1,x), \ u(\lambda,x) = \xi(x), x \in \overline{\Omega}, \\ \\ u(t,x) = 0, \ 0 \le t \le 1, \ x \in S \end{cases} \tag{15}$$

for the multidimensional elliptic equation. Here $a_r(x)$ $(x \in \Omega)$ and $\varphi(x), \psi(x), \xi(x)$ $(x \in \overline{\Omega})$ are given sufficiently smooth functions, and $0 < \lambda < T, \delta > 0$ are known numbers. Assume that all compatibility conditions are satisfied.

Denote by $C_{01}^\beta(\overline{\Omega})(\beta = (\beta_1,...,\beta_n), \beta_i, 1 \le i \le n)$, the Banach spaces of continuous functions satisfying a Hölder condition with weight $x_k^{\beta_k}(1 - x_k - h_k)^{\beta_i}, 0 \le x_k < x_k + h_k \le 1, 1 \le k \le n$ and the indicator β which equipped with the corresponding norm

$$\|f\|_{C_{01}^\beta(\overline{\Omega})} = \|f\|_{C(\overline{\Omega})}$$
$$+ \sup_{\substack{0 \le x_k < x_k+h_k \le 1, \\ 1 \le k \le n}} |f(x+h) - f(x)| \prod_{k=1}^n \left(\frac{x_k}{h_k} \right)^{\beta_{ki}} (1 - x_k - h_k)^{\beta_k}.$$

It is well known that the differential expression

$$A^x u = -\sum_{k=1}^n a_k \ u_{x_k x_k} + \delta u \tag{16}$$

defines a positive operator A^x acting on $C_{01}^{\beta}(\overline{\Omega})$ with domain $D(A^x) \subset C_{01}^{2+\beta}(\overline{\Omega})$ and satisfying the boundary condition $u = 0$ on S.

Theorem 4. For the solution of the source identification problem (15) the following stability and coercive stability estimates hold

$$\|u\|_{C(C_{01}^{\mu}(\overline{\Omega}))} \leq M(\mu) \left[\|f\|_{C(C_{01}^{\mu}(\overline{\Omega}))} + \|\xi\|_{C_{01}^{\mu}(\overline{\Omega})} \right], \tag{17}$$

$$\|u\|_{C_{01}^{2+\alpha,\alpha}(C_{01}^{\mu}(\overline{\Omega}))} + \sum_{k=1}^{n} \|u_{x_k x_k}\|_{C_{0T}^{\alpha,\alpha}(C_{01}^{\mu}(\overline{\Omega}))} + \|p\|_{C_{01}^{\mu}(\overline{\Omega})}$$

$$\leq \frac{M(\mu)}{\alpha(1-\alpha)} \|f\|_{C_{0T}^{\alpha,\alpha}(C_{01}^{\mu}(\overline{\Omega}))} + M(\mu) \|\xi\|_{C_{01}^{\mu+2}(\overline{\Omega})}, \tag{18}$$

$$0 < \alpha < 1, \mu = (\mu_1, ..., \mu_n), \ 0 < \mu_i < 1, \ 1 \leq i \leq n,$$

where $M(\mu)$ is independent of α, $\xi(x)$ and $f(t,x)$.

Fourth, in $[0,1] \times \Omega$, we consider the source identification problem

$$\begin{cases} -u_{tt}(t,x) - \sum_{k=1}^{n} a_k(x)u_{x_k x_k}(t,x) + \delta u(t,x) = f(t,x) + p(x), \\ \\ x \in \Omega, 0 < t < 1, \\ \\ u(0,x) = u(1,x), u_t(0,x) = u_t(1,x), \ u(\lambda,x) = \xi(x), x \in \overline{\Omega}, \\ \\ \frac{\partial}{\partial \overrightarrow{n}} u(t,x) = 0, \ 0 \leq t \leq 1, \ x \in S \end{cases} \tag{19}$$

for the multidimensional elliptic equation. Assume that all compatibility conditions are satisfied. The differential expression (16) defines a positive operator A^x acting on $C_{01}^{\beta}(\overline{\Omega})$ with domain $D(A^x) \subset C_{01}^{2+\beta}(\overline{\Omega})$ and satisfying the boundary condition $\frac{\partial u}{\partial \overrightarrow{n}} = 0$ on S. Therefore, by using Theorem 1, we can get the following result.

Theorem 5. For the solution of the source identification problem (19) the stability and coercive stability estimates (17) and (18) respectively are valid.

Conclusion

In the present paper, the well-posedness of the source identification problem for the abstract elliptic equation in Banach spaces is investigated. The exact estimates for the solution of this problem in Hölder norms are established. In future investigation, absolute stable difference schemes for approximately solution of the source identification problem for elliptic differential equations will be constructed and investigated.

Acknowledgement

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Банах кеңістігінде дереккөзді сәйкестендіруде эллипстік есептің корректілігі туралы ескерту

Банах кеңістігінде эллипстік дифференциалдық теңдеу үшін дереккөзді сәйкестендіру мәселесі қарастырылған. Хелдер нормасында дереккөздерді сәйкестендіру есебін шешу үшін дәл бағамы алынды. Қосымшаларда дереккөзді сәйкестендірудің төрт эллипстік есебі зерттелген. Эллипстік теңдеу үшін дереккөздерді сәйкестендіру есебін шешу үшін мәжбүрлі орнықтылық және орнықтылық бағамы алынған.

Кілт сөздер: корректілігі, эллипстік теңдеу, позитивті, мәжбүрлі орнықтылық, дереккөзді сәйкестендіру, дәл бағамы, шеттік есеп.

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Замечание о корректности эллиптической задачи идентификации источника в банаховом пространстве

Исследована проблема идентификации источника для эллиптического дифференциального уравнения в банаховом пространстве. Получены точные оценки для решения задачи идентификации источника в нормах Хелдера. В приложениях исследованы четыре эллиптических задачи идентификации источника. Получены оценки устойчивости и коэрцитивной устойчивости для решения задач идентификации источника для эллиптических уравнений.

Ключевые слова: корректность, эллиптические уравнения, позитивность, коэрцитивная устойчивость, идентификация источника, точные оценки, краевая задача.

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