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Characterizing the Ordered AG-Groupoids Through the Properties of Their Different Classes of Ideals

In this article, we have presented some important charcterizations of the ordered non-associative semigroups in relation to their ideals. We have initially characterized the ordered AG-groupoid through the properties of the their ideals, then we characterized the two important classes of these AG-groupoids, namely the regular and intragregular non-associative AG-groupoids. Our aim is also to encourage the research and the maturity of the associative algebraic structures by studying a class of non-associative and non-commutative algebraic structures called the ordered AG-groupoid.

Keywords: Ordered AG-groupoids, left (right, interior, quasi-, bi-, generalized bi-) ideals, regular (intra-regular) ordered AG-groupoids.

Introduction

In 1972, a generalization of commutative semigroups has been established by Kazim et. al [1]. In ternary commutative law: $abc = cba$, they introduced the braces on the left side of this law and explored a new pseudo associative law, that is $(ab)c = (cb)a$. They have called the left invertive law of this law. A groupoid S is said to be a left almost semigroup (abbreviated as LA-semigroup) if it satisfies the left invertive law : $(ab)c = (cb)a$. This structure is also known as Abel-Grassmann's groupoid (abbreviated as AG-groupoid) in [2]. An AG-groupoid is a midway structure between an abelian semigroup and a groupoid. Mushtaq et. al [3], investigated the concept of ideals in AG-groupoids.

In [4] (resp. [5]), a groupoid S is said to be medial (resp. paramedial) if $(ab)(cd) = (ac)(bd)$ (resp. $(ab)(cd) = (db)(ca)$). In [1], an AG-groupoid is medial, but in general an AG-groupoid needs not to be paramedial. Every AG-groupoid with left identity is paramedial by Protic et. al [2] and also satisfies $a(bc) = b(ac)$, $(ab)(cd) = (dc)(ba)$.

In [6, 7], if $(S, \cdot, \leq)$ is an ordered semigroup and $\emptyset \neq A \subseteq S$, we define a subset of S as follows : $(A] = \{s \in S : s \leq a$ for some $a \in A\}$. A non-empty subset A of S is called a subsemigroup of S if $A^2 \subseteq A$.

A non-empty subset A of S is called a left (resp. right) ideal of S if following hold (1) $SA \subseteq A$ (resp. $AS \subseteq A$). (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$. Equivalent definition: A is called a left(resp. right) ideal of S if $(A] \subseteq A$ and $SA \subseteq A$ (resp. $AS \subseteq A$).

A non-empty subset A of S is called an interior (resp. quasi-) ideal of S if (1) $SAS \subseteq A$ (resp. $(AS] \cap (SA] \subseteq A$). (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$.

A subsemigroup (A non-empty subset) A of S is called a bi- (generalized bi-) ideal of S if (1) $ASA \subseteq A$. (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$. Every bi-ideal of S is a generalized bi-ideal of S.

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In [7, 8], an ordered semigroup is said to be regular if for every $a \in S$,there exists an element $x \in S$ such that $a \leq axa$. Equivalent definitions are as follows: (1) $A \subseteq (ASA]$ for every $A \subseteq S$. (2) $a \in (aSa]$ for every $a \in S$.

In [9,10], an ordered semigroup S is intra-regular if for every $a \in S$ there exist elements $x, y \in S$ such that $a \leq xa^2y$. Equivalent definitions are as follows: (1) $A \subseteq (SA^2S]$ for every $A \subseteq S$. (2) $a \in (Sa^2S]$ for every $a \in S$.

We will define left (right, interior, quasi-, bi-, generalized bi-) ideals in ordered AG-groupoids. We will establish a study by discussing the different properties of such ideals. We will also characterize regular (resp. intra-regular, both regular and intra-regular) ordered AG-groupoids by the properties of left (right, quasi-, bi-, generalized bi-) ideals.

Ideals in Ordered AG-groupoids

An ordered AG-groupoid S, is a partially ordered set, at the same time an AG-groupoid such that $a \leq b$, implies $ac \leq bc$ and $ca \leq cb$ for all $a, b, c \in S$. Two conditions are equivalent to the one condition $(ca)d \leq (cb)d$ for all $a, b, c, d \in S$.

Example 1. Consider a set $S = \{e, f, a, b, c\}$ with the following multiplication "$\cdot$" and order relation "$\leq$"

$\cdot$	e	f	a	b	c
e	e	f	a	b	c
f	f	f	f	b	c
a	a	f	c	b	c
b	c	c	c	f	b
c	b	b	b	c	f

$\leq = \{(e, e), (e, a), (e, b), (e, c), (f, f), (f, b), (f, c), (a, a), (a, c), (b, b), (b, c), (c, c)\}$.

Then $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity e.

For $\emptyset \neq A \subseteq S$, we define a subset $(A] = \{s \in S : s \leq a$ for some $a \in A\}$ of S and obviously $A \subseteq (A]$. For $\emptyset \neq A, B \subseteq S$, then $((A]] = (A], (A](B] \subseteq (AB]$, $((A](B]] = (AB]$, if $A \subseteq B$, then $(A] \subseteq (B]$, $(A \cap B] \neq (A] \cap (B]$, in general.

For $\emptyset \neq A \subseteq S$. Then A is called an ordered AG-subgroupoid of S if $A^2 \subseteq A$. A is called a left (resp. right) ideal of S if the following hold (1) $SA \subseteq A$ (resp. $AS \subseteq A$). (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$. A is called an ideal of S if A is both a left and a right ideal of S.

We denote by $L(a), R(a), I(a)$ the left ideal, the right ideal and the ideal of S, respectively, generated by a. we have $L(a) = \{s \in S : s \leq a$ or $s \leq xa$ for some $x \in S\} = (a \cup Sa]$, $R(a) = (a \cup aS]$, $I(a) = (a \cup Sa \cup aS \cup (Sa)S]$.

A non-empty subset A of an ordered AG-groupoid S is called an interior (resp. quasi-) ideal of S if (1) $(SA)S \subseteq A$ (resp. $(AS] \cap (SA] \subseteq A$). (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$.

An AG-subgroupoid A of S is called a bi-ideal of S if (1) $(AS)A \subseteq A$. (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$. A non-empty subset A of S is called generalized bi-ideal of S if (1) $(AS)A \subseteq A$. (2) If $a \in A$ and $b \in S$ such that $b \leq a$ implies $b \in A$.

Now we give the imperative properties of such ideals of an ordered AG-groupoid S, which will be play a vital rule in the later sections. Specifically we show:

(1) Let S be an ordered AG-groupoid with left identity e. Then every right ideal of S is a ideal of S.

(2) Let S be an ordered AG-groupoid with left identity e, such that $(xe)S = xS$ for all $x \in S$. Then every quasi-ideal of S is a bi-ideal of S.

Lemma 1. Let S be an ordered AG-groupoid with left identity e. Then $SS = S$ and $eS = S = Se$.

Proof: Since $SS \subseteq S$ and $x = ex \in SS$, i.e., $S \subseteq SS$, thus $SS = S$. Obviously, $eS = S$ and $Se = (SS)\,e = (eS)\,S = SS = S$.

Lemma 2. Let S be an odered AG-groupoid with left identity e and $a \in S$. Then Sa is a smallest left ideal of S containing a.

Proof: Let $x \in Sa$ and $s \in S$, this implies that $x = s_1 a$, where $s_1 \in S$. Now

$$\begin{aligned} sx &= s(s_1a) = (es)(s_1a) = ((s_1a)s)e = ((s_1a)(es))e \\ &= ((s_1e)(as))e = (e(as))(s_1e) = (as)(s_1e) = ((s_1e)s)a \in Sa. \end{aligned}$$

Thus $sx \in Sa$ and $(Sa] \subseteq Sa$. Since $a = ea \in Sa$, hence Sa is a left ideal of S containing a. Let I be another left ideal of S containing a. Since $sa \in I$, because I is a left ideal of S. But $sa \in Sa$, this means that $Sa \subseteq I$. Therefore Sa is a smallest left ideal of S containing a.

Lemma 3. Let S be an odered AG-groupoid with left identity e and $a \in S$. Then aS is a left ideal of S.

Proof: Straight forward.

Proposition 1. Let S be an ordered AG-groupoid with left identity e and $a \in S$. Then $aS \cup Sa$ is a smallest right ideal of S containing a.

Proof: Let $x \in aS \cup Sa$. We have to show that $(aS \cup Sa)S \subseteq aS \cup Sa$. Now

$$\begin{aligned} (aS \cup Sa)S &= (aS)S \cup (Sa)S = (SS)a \cup (Sa)(eS) \\ &\subseteq Sa \cup (Se)(aS) = Sa \cup S(aS) \\ &= Sa \cup a(SS) \subseteq Sa \cup aS = aS \cup Sa. \end{aligned}$$

Thus $(aS \cup Sa)S \subseteq aS \cup Sa$ and $(aS \cup Sa] \subseteq aS \cup Sa$. Therefore $aS \cup Sa$ is a right ideal of S. Since $a \in Sa$, i.e., $a \in aS \cup Sa$. Let I be another right ideal of S containing a. Now $aS \in IS \subseteq I$ and $Sa = (SS)a = (aS)S \in (IS)S \subseteq IS \subseteq I$, i.e., $aS \cup Sa \subseteq I$. Hence $aS \cup Sa$ is a smallest right ideal of S containing a.

Lemma 4. Let S be an ordered AG-groupoid with left identity e. Then every right ideal of S is an ideal of S.

Proof: Let R be a right ideal of S and $r \in R, s \in S$. Now $sr = (es)r = (rs)e \in (RS)S \subseteq RS \subseteq R$. Thus $SR \subseteq R$ and $(R] \subseteq R$. Hence R is an ideal of S.

Lemma 5. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$. Then $(AS)S \subseteq AS$ and $(AS]S \subseteq (AS]$.

Proof: Since

$$\begin{aligned} (AS)S &= (AS)(eS) = (Ae)(SS) \subseteq (Ae)S = AS. \\ \text{and } (AS]S &= (AS](S] \subseteq ((AS)S] \subseteq (AS]. \end{aligned}$$

Remark 1. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$, then $(AS]$ is an ideal of S.

Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$ and $A, B \subseteq S$. Then $(AS)(BS) \subseteq (AB)S$ and $(AS](BS] \subseteq ((AB)S]$. Similarly $(SA)(SB) \subseteq S(AB)$ and $(SA](SB] \subseteq (S(AB)]$.

In general for $A_1, A_2, ..., A_n \subseteq S$, then $(A_1S)(A_2S)...(A_nS) \subseteq (A_1A_2, ..., A_n)S$ and $(A_1S](A_2S]...(A_nS] \subseteq ((A_1A_2, ..., A_n)S]$.

Similarly, $(SA_1)(SA_2)...(SA_n) \subseteq S(A_1A_2, ..., A_n)$ and $(SA_1](SA_2]....(SA_n] \subseteq (S(A_1A_2, ..., A_n)]$.

Lemma 6. Let S be an ordered AG-groupoid. A is a right ideal of S and B is a right ideal of A, then $(B] = B$.

Proof: Since $(B] = \{s \in S \mid s \leq b \text{ for some } b \in B\}$ and $s \in (B]$, this implies that there exists an element $s \in S$ such that $s \leq b$ for some $b \in B \subseteq A$. Thus $S \ni s \leq b \in A$. Now $A \ni s \leq b \in B$ and B is a right ideal of A, i.e., $s \in B$, so $(B] \subseteq B$. Since $B \subseteq (B]$, thus $(B] = B$.

Proposition 2. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. A is a right ideal of S and B is a right ideal of A such that $(B^2] = B$. Then B is an ideal of S.

Proof: We have to show that B is a right ideal of S. Now

$$\begin{aligned} BS &= (B^2]S = (B^2](S] \subseteq (B^2S] = ((BB)S] \\ &= ((SB)B] \subseteq ((SB)A] = ((SB)(eA)] \\ &= ((Se)(BA)] = (B((Se)A)] = (B((Ae)S)] \\ &= (B(AS)] \subseteq (BA] \subseteq (B] = B \text{ by the Lemma 6.} \end{aligned}$$

Thus $BS \subseteq B$ and $(B] \subseteq B$, i.e., B is a right ideal of S. Hence B is an ideal of S by the Lemma 4.

Lemma 7. Let S be an ordered AG-groupoid. A is a left ideal of S and B is a left ideal of A, then $(B] = B$.

Proof: Same as Lemma 6.

Proposition 3. Let S be an ordered AG-groupoid with left identity e. A is a left ideal of S and B is a left ideal of A such that $(B^2] = B$. Then B is left ideal of S.

Proof: We have to show that B is a left ideal of S. Now

$$\begin{aligned} SB &= S(B^2] = (S](B^2] \subseteq (SB^2] = (S(BB)] \\ &= ((Se)(BB)] = ((SB)(eB)] \\ &\subseteq ((SA)(eB)] \subseteq (AB] \subseteq (B] = B, \text{ by the Lemma 7.} \end{aligned}$$

Thus $SB \subseteq B$ and $(B] \subseteq B$. Hence B is a left ideal of S.

Lemma 8. Every two-sided ideal of S is an interior ideal of S.

Proof: Straight forward.

Proposition 4. Let S be an ordered AG-groupoid with left identity e. Then any non-empty subset I of S is an ideal of S if and only if I is an interior ideal of S.

Proof: Suppose that I is an interior ideal of S. Let $i \in I$ and $s \in S$. Now $is = (ei)s \in (SI)S \subseteq I$, this implies that $IS \subseteq I$ and $(I] \subseteq I$, i.e., I is a right ideal of S. Hence I is an ideal of S by the Lemma 4. Converse is true by the Lemma 8.

Lemma 9. Every right (two-sided) ideal of S is a bi-ideal of S.

Proof: Straight forward.

Lemma 10. Every bi-ideal of S is a generalized bi-ideal of S.

Proof: Obvious.

Lemma 11. Every left (right, two-sided) ideal of S is a quasi-ideal of S.

Proof: Let I be a right ideal of S. Now $(IS] \cap (SI] \subseteq (IS] \subseteq (I] \subseteq I$ and $(I] \subseteq I$. Thus I is a quasi-ideal of S.

Proposition 5. Every quasi-ideal of S is an ordered AG-subgroupoid of S.

Proof: Suppose that I is a quasi-ideal of S. Now $II \subseteq IS \subseteq (I](S] \subseteq (IS]$ and $II \subseteq SI \subseteq (S](I] \subseteq \subseteq (SI]$, i.e., $I^2 = II \subseteq (IS] \cap (SI] \subseteq I$. Hence I is an AG-subgroupoid of S.

Proposition 6. Let R be a right ideal and L be a left ideal of an ordered AG-groupoid S, respectively. Then $R \cap L$ is a quasi-ideal of S.

Proof: Since $((R \cap L)S] \cap (S(R \cap L)] \subseteq (RS] \cap (SL] \subseteq (R] \cap (L] \subseteq R \cap L$ and $(R \cap L] = R \cap L$. Thus $R \cap L$ is a quasi-ideal of S.

Lemma 12. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$. Then every quasi-ideal of S is a bi-ideal of S.

Proof: Let Q be a quasi-ideal of S. Now $(QS)Q \subseteq (SS)Q \subseteq SQ \subseteq (SQ]$ and $(QS)Q \subseteq (QS)S = = (QS)(eS) = (Qe)(SS) = (Qe)S = QS \subseteq (QS]$, thus $(QS)Q \subseteq (QS] \cap (SQ] \subseteq Q$. Therefore $(QS)Q \subseteq Q$ and $(Q] \subseteq Q$. Hence Q is a bi-ideal of S.

Regular Ordered AG-groupoids

An ordered AG-groupoid S is called regular if for every $a \in S$, there exists an element $x \in S$ such that $a \leq (ax)a$. Equivalent definitions are as follows:

(1) $A \subseteq ((AS)A]$ for every $A \subseteq S$.

(2) $a \in ((aS)a]$ for every $a \in S$.

An ideal I of an ordered AG-groupoid S is called idempotent if $(I^2] = I$.

In this section, we characterize regular ordered AG-groupoids by the properties of (left, right, quasi-, bi-, generalized bi-) ideals.

Lemma 13. Every right ideal of a regular ordered AG-groupoid S

Proof: Let R be a right ideal of S. Let $r \in R$ and $a \in S$, this implies that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $ar \leq ((ax)a)r = (ra)(ax) \in RS \subseteq R$, thus $SR \subseteq R$ and $(R] = R$. Hence R is an ideal of S.

Lemma 14. Every ideal of a regular ordered AG-groupoid S is an idempotent.

Proof: Suppose that I is an ideal of S and $(I^2] = (II] \subseteq (I] = I$. Let $a \in I$, this mean that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $a \leq (ax)a \in (IS)I \subseteq II = I^2$, i.e., $I \subseteq (I^2]$. Therefore $(I^2] = I$.

Remark 2. Every right ideal of a regular ordered AG-groupoid S is an idempotent.

Proposition 7. Let S be a regular ordered AG-groupoid. Then any non-empty subset I of S is an ideal of S if and only if I is an interior ideal of S.

Proof: Assume that I is an interior ideal of S. Let $a \in I$ and $s \in S$, then there exists an element $x \in S$, such that $a \leq (ax)a$. Now $as \leq ((ax)a)s = (sa)(ax) \in (SI)S \subseteq I$. Thus $IS \subseteq I$ and $(I] = I$, i.e., I is a right ideal of S. Hence I is an ideal of S by the Lemma 4. Converse is true by the Lemma 13.

Proposition 8. Let S be a regular ordered AG-groupoid with left identity e. Then $(IS] \cap (SI] = I$, for every right ideal I of S.

Proof: Let I be an ideal of S. This implies that $(IS] \cap (SI] \subseteq I$, because every ideal of S is a quasi-ideal of S. Let $a \in I$, this means that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $a \leq (ax)a \in (IS)I \subseteq II \subseteq IS$, i.e., $I \subseteq (IS]$. Now $a \leq (ax)a = (ax)(ea) = (ae)(xa) \in II \subseteq SI$, i.e., $I \subseteq (SI]$. Thus $I \subseteq (IS] \cap (SI]$. Hence $(IS] \cap (SI] = I$.

Lemma 15. Let S be a regular ordered AG-groupoid. Then $(RL] = R \cap L$, for every right ideal R and every left ideal L of S.

Proof: Since $(RL] \subseteq (RS] \subseteq (R] = R$ and $(RL] \subseteq (SL] \subseteq (L] = L$, i.e., $(RL] \subseteq R \cap L$. Let $a \in R \cap L$, this implies that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $a \leq (ax)a \in (RS)L \subseteq RL$, i.e., $R \cap L \subseteq (RL]$. Therefore $(RL] = R \cap L$.

Theorem 1. Let S be an ordered AG-groupoid with left identity e such that $(xe)\, S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is a regular.

(2) $R \cap L = (RL]$ for every right ideal R and every left ideal L of S.

(3) $Q = ((QS)Q]$ for every quasi-ideal Q of S.

Proof: Suppose that (1) holds. Let Q be a quasi-ideal of S and $a \in Q$, this implies that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $a \leq (ax)a \in (QS)Q$, i.e., $Q \subseteq ((QS)Q] \subseteq (Q] = Q$, because every quasi-ideal of S is a bi-ideal of S. Hence $Q = ((QS)Q]$, i.e., $(1) \Rightarrow (3)$. Assume that (3) holds, let R be a right ideal and L be a left ideal of S. Then R and L be quasi-ideals of S by the Lemma 11, so $R \cap L$ be a quasi-ideal of S. Now $R \cap L = (((R \cap L)S)(R \cap L)] \subseteq ((RS)L] \subseteq (RL]$. Since $(RL] \subseteq R \cap L$, so $(RL] = R \cap L$, i.e., $(3) \Rightarrow (2)$. Suppose that (2) is true, let $a \in S$, then Sa is a left ideal of S containing a by the Lemma 2 and $aS \cup Sa$ is a right ideal of S containing a by the

Proposition 1. By (2),

$$
\begin{aligned}
(aS \cup Sa) \cap Sa &= ((aS \cup Sa)(Sa)] = ((aS)(Sa) \cup (Sa)(Sa)]. \\
(Sa)(Sa) &= ((Se)a)(Sa) = ((ae)S)(Sa) = (aS)(Sa).
\end{aligned}
$$

Thus

$$
\begin{aligned}
(aS \cup Sa) \cap Sa &= ((aS)(Sa) \cup (Sa)(Sa)] \\
&= ((aS)(Sa) \cup (aS)(Sa)] = ((aS)(Sa)].
\end{aligned}
$$

Since $a \in (aS \cup Sa) \cap Sa$, Implies $a \in ((aS)(Sa)]$. Then $a \leq (ax)(ya) = ((ya)x)a = (((ey)a)x)a = = (((ay)e)x)a = ((xe)(ay))a = (a((xe)y))a \in (aS)a$ for any $x, y \in S$, i.e., $a \in ((aS)a]$. Hence a is regular, so S is a regular, i.e., $(2) \Rightarrow (1)$.

Theorem 2. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is a regular.

(2) $Q = ((QS)Q]$ for every quasi-ideal Q of S.

(3) $B = ((BS)B]$ for every bi-ideal B of S.

(4) $G = ((GS)G]$ for every generalized bi-ideal G of S.

Proof: $(1) \Rightarrow (4)$, is obvious. $(4) \Rightarrow (3)$, since every bi-ideal of S is a generalized bi-ideal of S by the Lemma 10. $(3) \Rightarrow (2)$, since every quasi-ideal of S is bi-ideal of S by the Lemma 12. $(2) \Rightarrow (1)$, by the Theorem 1.

Theorem 3. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is a regular.

(2) $Q \cap I = ((QI)Q]$ for every quasi-ideal Q and every ideal I of S.

(3) $B \cap I = ((BI)B]$ for every bi-ideal B and every ideal I of S.

(4) $G \cap I = ((GI)G]$ for every generalized bi-ideal G and every ideal I of S.

Proof: Suppose that (1) is true. Let G be a generalized bi-ideal and I be an ideal of S. Now $((GI)G] \subseteq ((SI)S] \subseteq (I] = I$ and $((GI)G] \subseteq ((GS)G] \subseteq (G] = G$, thus $((GI)G] \subseteq G \cap I$. Let $a \in G \cap I$, this means that there exists an element $x \in S$ such that $a \leq (ax)a$. Now $a \leq (ax)a = = (((ax)a)x)a = ((xa)(ax))a = (a((xa)x))a \in (GI)G$, thus $G \cap I \subseteq ((GI)G]$. Hence $G \cap I = ((GI)G]$, i.e., $(1) \Rightarrow (4)$. $(4) \Rightarrow (3)$, since every bi-ideal of S is a generalized bi-ideal of S by the Lemma 10. $(3) \Rightarrow (2)$, since every quasi-ideal of S is a bi-ideal of S by the Lemma 12. Assume that (2) is true. Now $Q \cap S = ((QS)Q]$, i.e., $Q = ((QS)Q]$, where Q is a quasi-ideal of S. Hence S is a regular by the Theorem 1.

Theorem 4. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is a regular.

(2) $R \cap Q \subseteq (RQ]$ for every quasi-ideal Q and every right ideal R of S.

(3) $R \cap B \subseteq (RB]$ for every bi-ideal B and every right ideal R of S.

(4) $R \cap G \subseteq (RG]$ for every generalized bi-ideal G and every right ideal R of S.

Proof: $(1) \Rightarrow (4)$, is obvious. $(4) \Rightarrow (3)$, since every bi-ideal of S is a generalized bi-ideal of S. $(3) \Rightarrow (2)$, since every quasi-ideal of S is a bi-ideal of S by the Lemma 12. . Suppose that (2) is true. Now $R \cap Q = Q \cap R \subseteq (RQ]$, where Q is a left ideal and R is right ideal of S, because every left ideal of S is a quasi-ideal of S. Since $(RQ] \subseteq R \cap Q$, thus $R \cap Q = (RQ]$. Hence S is a regular, by the Theorem 1.

Intra-regular Ordered AG-groupoids

An ordered AG-groupoid S is called intra-regular if for every $a \in S$, there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Equivalent definitions are as follows:

(1) $A \subseteq ((SA^2)S]$ for every $A \subseteq S$.

(2) $a \in ((Sa^2)S]$ for every $a \in S$.

In this section, we characterize intra-regular ordered AG-groupoids by the properties of (left, right, quasi-, bi-, generalized bi-) ideals.

Lemma 16. Every left (right) ideal of an intra-regular ordered AG-groupoid S is an ideal of S.

Proof: Let R be a right ideal of S. Let $r \in R$ and $a \in S$, this implies that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now $ar \leq ((xa^2)y)r = (ry)(xa^2) \in RS \subseteq R$. Thus $SR \subseteq R$ and $(R] \subseteq R$. Hence R is an ideal of S.

Lemma 17. Every ideal of an intra-regular ordered AG-groupoid S with left identity e, is an idempotent.

Proof: Suppose that I is an ideal of S and $(I^2] = (II] \subseteq (I] = I$. Let $a \in I$, this means that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now

$$\begin{aligned} a &\leq (xa^2)y = (x(aa))y = (a(xa))y \\ &= (a(xa))(ey) = (ae)((xa)y) = (xa)((ae)y) \in II. \end{aligned}$$

Thus $a \in (II] = (I^2]$. Therefore $(I^2] = I$.

Proposition 9. Let S be an intra-regular ordered AG-groupoid with left identity e. Then any non-empty subset I of S is an ideal of S if and only if I is an interior ideal of S.

Proof: Assume that I is an interior ideal of S. Let $i \in I$ and $a \in S$, then there exist elements $x, y \in S$ such that $x \leq (yx^2)z$. Now

$$\begin{aligned} ia &\leq i((xa^2)y) = i((x(aa))y) \\ &= i((a(xa))y) = i((a(xa))(ey)) \\ &= i((ae)((xa)y)) = i((xa)((ae)y)) \\ &= (xa)(i((ae)y)) = (xi)(a((ae)y)) \in (SI)S \subseteq I. \end{aligned}$$

Thus $IS \subseteq I$ and $(I] \subseteq I$, i.e., I is a right ideal of S. So I is an ideal of S by the Lemma 16. Converse is obvious.

Lemma 18. Let S be an intra-regular ordered AG-groupoid with left identity e. Then $L \cap R \subseteq (LR]$ for every left ideal L and every right ideal R of S.

Proof: Let $a \in L \cap R$, where L is a left ideal and R is a right ideal of S, respectively, this implies that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now

$$\begin{aligned} a &\leq (xa^2)y = (x(aa))y = (a(xa))y = (a(xa))(ey) \\ &= (ae)((xa)y) = (xa)((ae)y) \in LR. \\ &\Rightarrow L \cap R \subseteq (LR]. \end{aligned}$$

Theorem 5. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is an intra-regular.

(2) $L \cap R \subseteq (LR]$ for every left ideal L and every right ideal R of S.

Proof: Since (1) $\Rightarrow$ (2) holds by the Lemma 18. Suppose that (2) holds and $a \in S$, then Sa is a left ideal of S containing a and $aS \cup Sa$ is a right ideal of S containing a. By our supposition

$$\begin{aligned} Sa \cap (aS \cup Sa) &\subseteq ((Sa)(aS \cup Sa)] = ((Sa)(aS) \cup (Sa)(Sa)]. \\ (Sa)(aS) &= (Sa)((ea)S) = (Sa)((Sa)e) = (Sa)((Sa)(ee)) \\ &= (Sa)((Se)(ae)) = (Sa)(S(ae)) = (Sa)(Sa). \end{aligned}$$

Thus

$$\begin{aligned}(aS \cup Sa) \cap Sa &\subseteq ((Sa)(aS) \cup (Sa)(Sa)] \\ &= ((Sa)(Sa) \cup (Sa)(Sa)] \\ &= ((Sa)(Sa)] = (S^2a^2] = (Sa^2] \\ &= (S(a^2e)] = ((SS)(a^2e)] = ((eS)(a^2S)] = (S(a^2S)] \\ &= (a^2(SS)] = ((ea^2)(SS)] = ((Sa^2)(Se)] = ((Sa^2)S].\end{aligned}$$

Since $a \in (aS \cup Sa) \cap Sa$, implies $a \in ((Sa^2)S]$, thus a is an intra regula. Hence S is an intra-regular, i.e., $(2) \Rightarrow (1)$.

Theorem 6. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is an intra-regular.

(2) $Q \cap I = ((QI)Q]$ for every quasi-ideal Q and every ideal I of S.

(3) $B \cap I = ((BI)B]$ for every bi-ideal B and every ideal I of S.

(4) $G \cap I = ((GI)G]$ for every generalized bi-ideal G and every ideal I of S.

Proof: Suppose that (1) holds. Let $a \in G \cap I$, where G is a generalized bi-ideal and I is an ideal of S, this implies that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now

$$\begin{aligned}a &\leq (xa^2)y = (x(aa))y = (a(xa))y = (y(xa))a. \\ y(xa) &\leq y(x((xa^2)y)) = y((xa^2)(xy)) = (xa^2)(y(xy)) \\ &= (xa^2)(xy^2) = (x(aa))m, \text{ say } xy^2 = m \\ &= (a(xa))m = (m(xa))a. \\ m(xa) &\leq m(x((xa^2)y)) = m((xa^2)(xy)) = (xa^2)(m(xy)) \\ &= (x(aa))n, \text{ say } m(xy) = n \\ &= (a(xa))n = (n(xa))a \\ &= va, \text{ say } n(xa) = v. \\ &\Rightarrow y(xa) = (m(xa))a = (va)a = (va)(ea) = (ve)(aa) = a((ve)a).\end{aligned}$$

Thus $a \leq (xa^2)y = (y(xa))a = (a((ve)a))a \in (GI)G$. This means that $a \in ((GI)G]$, i.e., $G \cap I \subseteq ((GI)G]$. Now $((GI)G] \subseteq ((SI)S] \subseteq (I] = I$ and $((GI)G] \subseteq ((GS)G] \subseteq (G] = G$, thus $((GI)G] \subseteq G \cap I$. Hence $G \cap I = ((GI)G]$, i.e., $(1) \Rightarrow (4)$. $(4) \Rightarrow (3)$, every bi-ideal of S is a generalized bi-ideal of S by the Lemma 10. $(3) \Rightarrow (2)$, every quasi-ideal of S is a bi-ideal of S by the Lemma 12. Assume that (2) is true and let R be a right ideal and I be a two-sided ideal of S. Now $I \cap R = ((RI)R] \subseteq ((SI)\,R] \subseteq (IR]$, since every right ideal of S is a quasi-ideal of S. Therefore S is an intra-regular by the Theorem 5, i.e., $(2) \Rightarrow (1)$.

Theorem 7. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is an intra-regular.

(2) $L \cap Q \subseteq (LQ]$ for every quasi-ideal Q and every left ideal L of S.

(3) $L \cap B \subseteq (LB]$ for every bi-ideal B and every left ideal L of S.

(4) $L \cap G \subseteq (LG]$ for every generalized bi-ideal G and every left ideal L of S.

Proof: Suppose that (1) holds. Let $a \in L \cap G$, where L is a left ideal and G is a generalized bi-ideal of S, this means that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now $a \leq (xa^2)y = (x(aa))y = = (a(xa))y = (y(xa))a \in LG$, i.e., $a \in (LG]$. Thus $L \cap G \subseteq (LG]$, i.e., $(1) \Rightarrow (4)$. $(4) \Rightarrow (3)$, every bi-ideal of S is a generalized bi-ideal of S. $(3) \Rightarrow (2)$, every quasi-ideal of S is a bi-ideal of S. Assume that (2) is true and let R be a right ideal of S and L be a left ideal of S. Now $L \cap R \subseteq (LR]$, where R is a quasi-ideal of S. Hence S is an intra-regular by the Theorem 5, i.e., $(2) \Rightarrow (1)$.

Theorem 8. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is an intra-regular.

(2) $L \cap Q \cap R \subseteq ((LQ)R]$ for every quasi-ideal Q, every right ideal R and every left ideal L of S.

(3) $L \cap B \cap R \subseteq ((LB)R]$ for every bi-ideal B, every right ideal R and every left ideal L of S.

(4) $L \cap G \cap R \subseteq ((LG)R]$ for every generalized bi-ideal G, every right ideal R and every left ideal L of S.

Proof: Suppose that (1) holds. Let $a \in L \cap G \cap R$, where L is a left ideal, G is a generalized bi-ideal and R is a right ideal of S, this implies that there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Now

$$\begin{aligned} a &\leq (xa^2)y = (x(aa))y = (a(xa))y = (y(xa))a. \\ y(xa) &\leq y(x((xa^2)y)) = y((xa^2)(xy)) = (xa^2)(y(xy)) \\ &= (xa^2)(xy^2) = (x(aa))m, \text{ say } xy^2 = m \\ &= (a(xa))m = (m(xa))a. \end{aligned}$$

Thus $a \leq (xa^2)y = (y(xa))a = ((m(xa))a)a \in (LG)R$, i.e., $a \in ((LG)R]$. Hence $L \cap G \cap R \subseteq$ $\subseteq ((LG)R]$, i.e., $(1) \Rightarrow (4)$. $(4) \Rightarrow (3)$, every bi-ideal of S is a generalized bi-ideal of S. $(3) \Rightarrow (2)$, every quasi-ideal of S is a bi-ideal of S. Assume that (2) is true. Now

$$\begin{aligned} L \cap S \cap R &\subseteq ((LS)R] = (((eL)S)R] = (((SL)e)R] = (((SL)(ee))R] \\ &= (((Se)(Le))R] \subseteq ((S(Le))R] \subseteq ((SL)R] \subseteq (LR]. \\ &\Rightarrow L \cap R \subseteq (LR]. \end{aligned}$$

Hence S is an intra-regular by the Theorem 5, i.e., $(2) \Rightarrow (1)$.

Regular and Intra-regular Ordered AG-groupoids

In this section, we characterize both regular and intra-regular ordered AG-groupoids by the properties of (left, right, quasi-, bi-, generalized bi-) ideals.

Theorem 9. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is a regular and an intra-regular.

(2) $(B^2] = B$ for every bi-ideal B of S.

(3) $B_1 \cap B_2 = (B_1B_2] \cap (B_2B_1]$ for all bi-ideals B_1, B_2 of S.

Proof: Suppose that (1) holds and B be a bi-ideal of S. Since $(B^2] = (BB] \subseteq (B] = B$. Let $a \in B$, this implies that there exists an element $x \in S$ such that $a \leq (ax)a$, also there exist elements $y, z \in S$ such that $a \leq (ya^2)z$. Now

$$\begin{aligned} a &\leq (ax)a \leq (ax)((ya^2)z) = (((ya^2)z)x)a. \\ ((ya^2)z)x &= (xz)(ya^2) = m(ya^2), \text{ say } m = xz \\ &= m(y(aa)) = m(a(ya)) = a(m(ya)) \\ &\leq ((ax)a)(m(ya)) = ((ax)m)(a(ya)) \\ &= ((mx)a)(a(ya)) = (na)(a(ya)), \text{ say } n = mx \\ &= ((en)a)(a(ya)) = ((an)e)(a(ya)) \\ &= ((an)a)(e(ya)) = ((an)a)(ya) = (sa)(ya), \text{ say } s = an \\ &= (aa)(ys) = (aa)t, \text{ say } t = ys \\ &\leq (((ax)a)a)t = ((aa)(ax))t = (t(ax))(aa) \\ &= (a(tx))(aa) = (aw)(aa), \text{ say } w = tx. \end{aligned}$$

Thus $a \leq (((ya^2)z)x)a \leq ((aw)(aa))a \in ((BS)B)B \subseteq B^2$, i.e., $a \in (B^2]$. So $B \subseteq (B^2]$, i.e., $(B^2] = B$. Hence (1) $\Rightarrow$ (3) . Assume that (2) is true. Let B_1, B_2 be bi-ideals of S, then $B_1 \cap B_2$ be also a bi-ideal of S. Now $B_1 \cap B_2 = ((B_1 \cap B_2)(B_1 \cap B_2)] \subseteq (B_1B_2]$ and $B_1 \cap B_2 = ((B_1 \cap B_2)(B_1 \cap B_2)] \subseteq$ $\subseteq (B_2B_1]$, thus $B_1 \cap B_2 \subseteq (B_1B_2] \cap (B_2B_1]$. First of all we have to show that $(B_1B_2]$ is a bi-ideal of S. It is enough to show that $((B_1B_2]S)(B_1B_2] \subseteq (B_1B_2]$. Now

$$\begin{aligned}
((B_1B_2]S)(B_1B_2] &= ((B_1B_2](S])(B_1B_2] \\
&\subseteq ((B_1B_2)S](B_1B_2] \\
&\subseteq (((B_1B_2)S)(B_1B_2)] \\
&= (((B_1B_2)(SS))(B_1B_2)] \\
&= (((B_1S)(B_2S))(B_1B_2)] \\
&= (((B_1S)B_1)((B_2S)B_2)] \subseteq (B_1B_2] \\
&\Rightarrow (((B_1B_2)S)(B_1B_2))] \subseteq (B_1B_2].
\end{aligned}$$

Thus $(B_1B_2]$ is a bi-ideal of S, similarly $(B_2B_1]$ is also a bi-ideal of S. Then $(B_1B_2] \cap (B_2B_1]$ is also a bi-ideal of S. Now

$$\begin{aligned}
(B_1B_2] \cap (B_2B_1] &= (((B_1B_2] \cap (B_2B_1])((B_1B_2] \cap (B_2B_1])] \\
&\subseteq ((B_1B_2](B_2B_1]] \subseteq (((B_1B_2)(B_2B_1)]] \\
&= ((B_1B_2)(B_2B_1)] \subseteq ((B_1S)(SB_1)] \\
&= (((SB_1)S)B_1] = ((((Se)B_1)S)B_1] \\
&= ((((B_1e)S)S)B_1] = (((B_1S)S)B_1] \\
&= (((SS)B_1)B_1] = ((SB_1)B_1] = (((Se)B_1)B_1] \\
&= (((B_1e)S)B_1] = ((B_1S)B_1] \subseteq (B_1] \\
&\Rightarrow (B_1B_2] \cap (B_2B_1] \subseteq (B_1] = B_1.
\end{aligned}$$

Similarly, we have $(B_1B_2] \cap (B_2B_1] \subseteq (B_2] = B_2$, thus $(B_1B_2] \cap (B_2B_1] \subseteq B_1 \cap B_2$. Therefore $B_1 \cap B_2 = (B_1B_2] \cap (B_2B_1]$, i.e., (2) $\Rightarrow$ (3) . Suppose that (3) holds, let R be right ideal of S and I be an ideal of S. Then R and I be bi-ideals of S, because every right ideal and two sided ideal of S is bi-ideal of S by the Lemma 9. Now $R \cap I = (RI] \cap (IR]$, this implies that $R \cap I \subseteq (RI] \cap (IR]$. Thus $R \cap I \subseteq (RI]$ and $R \cap I \subseteq (IR]$, where I is also a left ideal of S. Since $(RI] \subseteq R \cap I$, i.e., $(RI] = R \cap I$, thus S is a regular by the Theorem 1. Also, $R \cap I \subseteq (IR]$, thus S is an intra-regular by the Theorem 5. Hence (3) $\Rightarrow$ (1) .

Theorem 10. Let S be an ordered AG-groupoid with left identity e such that $(xe)\,S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is regular and intra-regular.

(2) Every quasi-ideal of S is an idempotent.

Proof: Suppose that (1) holds. Let Q be a quasi-ideal of S and $(Q^2] = (QQ] \subseteq (Q] = Q$, i.e., $(Q^2] \subseteq Q$. Let $a \in Q$, this implies that there exists an element $x \in S$ such that $a \leq (ax)a$, also there exist elements $y, z \in S$ such that $a \leq (ya^2)z$. Now

$$\begin{aligned}
a &\leq (ax)a \leq (ax)((ya^2)z) = (((ya^2)z)x)a. \\
((ya^2)z)x &= (xz)(ya^2) = m(ya^2), \text{ say } m = xz \\
&= m(y(aa)) = m(a(ya)) = a(m(ya)) \\
&\leq ((ax)a)(m(ya)) = ((ax)m)(a(ya)) \\
&= ((mx)a)(a(ya)) = (qa)(a(ya)), \text{ say } q = mx \\
&= ((eq)a)(a(ya)) = ((aq)e)(a(ya)) \\
&= ((aq)a)(e(ya)) = ((aq)a)(ya) = (sa)(ya), \text{ say } s = aq \\
&= (aa)(ys) = (aa)t, \text{ say } t = ys \\
&\leq (((ax)a)a)t = ((aa)(ax))t = (t(ax))(aa) \\
&= (a(tx))(aa) = (aw)(aa), \text{say } w = tx
\end{aligned}$$

Thus $a \leq (((ya^2)z)x)a \leq ((aw)(aa))a \in ((QS)Q)Q \subseteq QQ \subseteq Q^2$, i.e., $a \in (Q^2]$, because every quasi-ideal of S is a bi-ideal of S by the Lemma 12. Thus $Q \subseteq (Q^2]$, i.e., $(Q^2] = Q$. Hence $(1) \Rightarrow (2)$. Assume that (2) is true. Let $a \in S$, then Sa is a left ideal of S containing a, so Sa is a quasi-ideal of S, because every left ideal of S is a quasi-ideal of S. Now $Sa = ((Sa)^2] = ((Sa)(Sa)]$, i.e., $a \in ((Sa)(Sa)]$. Thus S is an intra-regular by the Theorem 5. Now $Sa = ((Sa)(Sa)] = (((Se)a)(Sa)] = (((ae)S)(Sa)] = ((aS)(Sa)]$, i.e., $a \in ((aS)(Sa)]$. Thus S is a regular by the Theorem 1. Therefore $(2) \Rightarrow (1)$.

Theorem 11. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is regular and intra-regular.

(2) Every quasi-ideal of S is an idempotent.

(3) Every bi-ideal of S is an idempotent.

Proof: $(1) \Rightarrow (3)$, by the Theorem 9. $(3) \Rightarrow (2)$, every quasi-ideal of S is a bi-ideal of S, by the Lemma 12. $(2) \Rightarrow (1)$, by the Theorem 10.

Theorem 12. Let S be an ordered AG-groupoid with left identity e such that $(xe)S = xS$ for all $x \in S$. Then the following conditions are equivalent.

(1) S is regular and intra-regular.

(2) $Q_1 \cap Q_2 \subseteq (Q_1Q_2]$ for all quasi-ideals Q_1, Q_2 of S.

(3) $Q \cap B \subseteq (QB]$ for every quasi-ideal Q and every bi-ideal B of S.

(4) $B \cap Q \subseteq (BQ]$ for every bi-ideal B and every quasi-ideal Q of S.

(5) $B_1 \cap B_2 \subseteq (B_1B_2]$ for all bi-ideals B_1, B_2 of S.

Proof: Suppose that (1) holds. Let B_1, B_2 be bi-ideals of S, then $B_1 \cap B_2$ be also a bi-ideal of S. Since every bi-ideal of S is an idempotent by the Theorem 9, then $B_1 \cap B_2 = ((B_1 \cap B_2)^2] = ((B_1 \cap B_2)(B_1 \cap B_2)] \subseteq (B_1B_2]$. Hence $(1) \Rightarrow (5)$. Since $(5) \Rightarrow (4) \Rightarrow (2)$ and $(5) \Rightarrow (3) \Rightarrow (2)$, because every quasi-ideal of S is a bi-ideal of S by the Lemma 12. Assume that (2) is true. Now $R \cap L \subseteq (RL]$, where R is a right ideal and L is a left ideal of S. Since $(RL] \subseteq R \cap L$, i.e., $R \cap L = (RL]$, thus S is regular. Again by (2) $L \cap R \subseteq (LR]$, thus S is an intra-regular. Therefore $(2) \Rightarrow (1)$.

Conclusion

In this article, we have characterized the non-associative ordered semigroups in terms of their one-sided ideals, ideals, interior ideals, bi-ideals and quais ideals. We have also characterized the intraregular and regular orderded AG-groupoids through the properties of their ideals.

References

1 Kazim, M. A. & Naseerudin, M. (1972). On almost semigroups. *Alig. Bull. Math., 2,* 1-7.

2 PV, P. (1995). AG-test and some general properties of Abel-Grassmann's groupoids. *Pure Mathematics and Applications, 6, 4,* 371-383.

3 Mushtaq, Q. & Yusuf, S. M., (1978). On LA-semigroups. *Alig. Bull. Math., 8,* 65-70.

4 Jezek, J. & Kepka, T. (1983). Medial groupoids. *Rozpravy CSAV Rada Mat. a Prir. Ved, 93, 2,* 93.

5 Cho, J. R., Jezek, J. & Kepka, T. (1999). Paramedial groupoids. *Czechoslovak Mathematical Journal, 49, 2,* 277-290.

6 Kehayopulu, N. (1990). Remark on ordered semigroups. *Math. Japon., 35, 6,* 1061-1063.

7 Kehayopulu, N. (1990). On left regular ordered semigroups. *Math. Japon., 35, 6,* 1051-1060.

8 Kehayopulu, N. (1993, December). On intra-regular ordered semigroups. *In Semigroup Forum, Springer-Verlag, 46, 1,* 271-278.

9 Kehayopulu, N. (1997). On regular ordered semigroups. *Mathematica japonicae, 45, 3,* 549-553.

10 Kehayopulu, N. (1998). On completely regular ordered semigroups. *Sci. Math, 1, 1,* 27-32.

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Реттелген AG-группоидтардың әртүрлі идеалды кластарының қасиеттері бойынша сипаттамасы

Мақалада ассоциативті емес жартылай группалардың идеалдарына қатысты кейбір маңызды сипаттамалар ұсынылған. Біріншіден, біз реттелген AG-группоидты оның идеалының қасиеттері тұрғысынан сипаттадық, содан кейін осы AG-группоидтардың екі маңызды класына, яғни регулярлық және ішкі регулярлық емес ассоциативті емес AG-группоидтарға сипаттама бердік. Біздің мақсатымыз – реттелген AG-группоид деп аталатын ассоциативті емес және коммутативті емес алгебралық құрылымдар класын зерттеу арқылы ассоциативті алгебралық құрылымдарды зерттеу мен дамытуды ынталандыру.

Кілт сөздер: реттелген AG-группоидтар, солға (оң, ішкі, квази-, би-, жалпыланған би-) идеалдар, регулярлық (ішкі регулярлық) реттелген AG-группоидтар.

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Характеризация упорядоченных AG-группоидов через свойства их различных классов идеалов

В статье представлены некоторые важные характеристики упорядоченных неассоциативных полугрупп относительно их идеалов. Сначала были охарактеризован упорядоченный AG-группоид через свойства его идеалов, затем два важных класса этих AG-группоидов, а именно, регулярные и внутрирегулярные неассоциативные AG-группоиды. Цель настоящей работы — стимулирование исследования и развитие ассоциативных алгебраических структур путем изучения класса неассоциативных и некоммутативных алгебраических структур, называемых упорядоченным AG-группоидом.

Ключевые слова: упорядоченные AG-группоиды, левые (правые, внутренние, квази-, би-, обобщенные би-) идеалы, регулярные (внутрирегулярные) упорядоченные AG-группоиды.