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## On closed mappings uniform spaces

Uniform spaces are an important class of spaces in general topology. The purpose of the study is to prove new theorems concerning the properties of uniform spaces. The  $u$  — continuous,  $u$  — closed,  $z_u$  — closed,  $u$  — perfect mappings have been determined and their some properties have been established. The importance of these mappings classes is caused by that  $u$  — closed mappings are a subclass of the closed mappings class, and the closed mappings class is a subclass of the  $z_u$  — closed mappings.

*Key words:* uniformly continuous mapping, perfect mapping, bicomact.

### 1. Introduction

Z. Frolik [1] introduced  $z$  — closed mappings, which are a natural generalization of the closed mappings ([2])

*Definition 1.1* [1]. A continuous mapping  $f: X \rightarrow Y$  a topological space  $X$  into a topological space  $Y$  is called  $z$  — closed, if the image  $f(F)$  of any functionally closed ( $\equiv$  zero set)  $F$  in  $X$  is a closed set in  $Y$ .

Below the uniform analogues of closed and  $z$  — closed mappings have been determined. Everywhere necessary information and denotations are taken from books [3–6].

Every uniform space be  $uX$ , where  $u$  be a uniformity in a uniform coverings terms,  $f: uX \rightarrow vY$  be a mapping of uniform space  $uX$  into uniform space  $vY$  and if  $f(F) = Y$ , then the mapping  $f$  is surjective. We denote  $C^*(uX)$  to be a ring of all bounded uniformly continuous functions on  $uX$ ,  $\mathfrak{Z}(uX) = \{f^{-1}(0) : f \in C^*(uX)\}$  be a set of all uniformly zero-sets ( $\equiv$  uniformly closed sets [5]), of the uniform space  $uX$ .

Let  $u_R R$  be a set of real numbers  $R$  with natural uniformity  $u_R$ , generated by the metrics  $\rho(x, y) = |x - y|$  for any  $x, y \in R$ , and  $u_I I$  be a segment  $I = [0, 1]$  with uniformity  $u_I$ , induced by the uniformity

*Definition 1.2* [5]. A mapping  $f: uX \rightarrow vY$  is called  $u$  — continuous, if the inverse image  $f^{-1}(F) \in \mathfrak{Z}(uX)$  ( $f^{-1}(U) \in \mathcal{L}(uX)$ ) for any  $F \in \mathfrak{Z}(vY)$  ( $U \in \mathcal{L}(vY)$ ).

*Remark 1.1.* Every uniformly continuous mapping  $f: uX \rightarrow vY$  is  $u$  — continuous. If  $\mathcal{U}_f$  and  $\mathcal{V}_f$  — are fine uniformities of Tychonoff spaces  $X$  and  $Y$  respectively, then for mapping  $f: u_f X \rightarrow v_f Y$ .  $u_f$  — continuity is equivalent to the continuity of mapping  $f: X \rightarrow Y$ . There are  $u$  — continuous mappings  $f: uX \rightarrow vY$ , which are not uniformly continuous.  $\mathcal{U}_{\mathbb{R}}$

*Theorem 1.1* [5]. Let  $g_1^{-1}(0) = F_1 \in \mathfrak{Z}(uX)$  and  $g_2^{-1}(0) = F_2 \in \mathfrak{Z}(uX)$ , where  $g_1, g_2 \in C^*(uX)$  and  $F_1 \cap F_2 = \emptyset$ . Then the function  $f: uX \rightarrow u_I I$ , determined as  $f(x) = |g_1(x)| / (|g_1(x)| + |g_2(x)|)$  for any  $x \in X$ , is a  $u$  — function.

*Example 1.1.* Let  $X = [-1; 0) \cup (0; 1]$  and uniformity  $\mathcal{U}$  on  $X$  is induced by the uniformity  $\mathcal{U}_{\mathbb{R}}$  of  $\mathbb{R}$ . The sets  $[-1; 0)$  and  $(0; 1]$  are not uniformly separated, hence, there is no uniformly continuous function on the uniform space  $uX$ , which separates these sets. Functions  $g_i: uX \rightarrow u_{\mathbb{R}} \mathbb{R}$ ,  $i = 1, 2$ , determined as  $g_1(x) = \rho(x, [-1; 0))$  and  $g_2(x) = \rho(x, (0; 1])$  are uniformly continuous. Then the function  $f(x) = g_1(x) / (g_1(x) + g_2(x))$  is an example of the  $u$  — continuous function, which is not uniformly continuous.

### 2. Main results

*Example 2.1.* Let  $\varepsilon > 0$  and  $\mathbb{R}^+ = (0; +\infty)$ . A uniformity  $\mathcal{U}_{\mathbb{R}}$  of real numbers  $\mathbb{R}$  is generated by the basis  $\mathcal{B}$ , consisting of uniform coverings  $\alpha_{\varepsilon} = \{O_{\varepsilon}(x) : x \in \mathbb{R}\}$ , where  $O_{\varepsilon}(x) = (x - \varepsilon, x + \varepsilon)$  is open interval with center at point  $x$  of length  $2\varepsilon$ . Let  $\mathcal{P}(\mathbb{R})$  be a set of all finite subsets of  $\mathbb{R}$  and  $\mathbb{R}^+$ . For any  $\varepsilon \in \mathbb{R}^+$  any  $A \in \mathcal{P}(\mathbb{R})$  suppose  $\alpha_{\varepsilon, A} = \{O_{\varepsilon}(x) \setminus A : x \in \mathbb{R} \setminus A\} \cup \{a : a \in A\}$ . A family  $\mathcal{B}' = \{\alpha_{\varepsilon, A} : \varepsilon \in \mathbb{R}^+, A \in \mathcal{P}(\mathbb{R})\}$  is a basis of

some uniformity  $\mathcal{U}'$  on  $\mathbb{R}$ , more strong, than uniformity  $\mathcal{U}_{\mathbb{R}}$ . Really,  $\alpha_{\varepsilon_1, A_1} \cap \alpha_{\varepsilon_2, A_2} = \alpha_{\varepsilon, A_1 \cup A_2}$  where  $\varepsilon = \min \{\varepsilon_1, \varepsilon_2\}$  and the covering  $\alpha_{\delta, A}$  is starry inscribed to the covering,  $\alpha_{\varepsilon, A}$  where  $\delta = \frac{\varepsilon}{3}$ . We note, that  $\mathcal{U}_{\mathbb{R}} \subset \mathcal{U}'$  and  $\mathcal{U}'$  generates discrete topology on the  $\mathbb{R}$ .

*Proposition 2.1.* A set of rational numbers  $\mathbb{Q}$  is not uniformly zero-set in the uniform space  $u'\mathbb{R}$ , i.e.  $\mathbb{Q} \notin \mathfrak{Z}(u'\mathbb{R})$ .

*Proof.* We suppose, that  $\mathbb{Q} \in \mathfrak{Z}(u'\mathbb{R})$ , i.e. there is such uniformly continuous function  $f \in C^*(u'\mathbb{R})$ , that  $\mathbb{Q} = f^{-1}(0)$ . Then for any  $n \in \mathbb{N}$  there exist  $\varepsilon_n > 0$  and  $A_n \in \mathcal{P}(\mathbb{R})$  such that the family  $f(\alpha_{\varepsilon_n, A_n})$  is inscribed to the covering  $\alpha_{\frac{1}{n}}$ , i.e. for any  $y \in O_{\varepsilon_n}(x)$  the formula  $|f(x) - f(y)| < \frac{1}{n}$  is provided for all  $x \in \mathbb{R}$ . Let  $x \notin A = \bigcup_{n=1}^{\infty} A_n$ , in force of everywhere density of  $\mathbb{Q}$  in  $\mathbb{R}$ , there is such  $y \in \mathbb{Q} \setminus A$  that  $|x - y| < \varepsilon_n$ , hence for all  $x \in \mathbb{R}$  for all such  $x \notin A$  and  $|x - y| < \varepsilon_n$  we have  $|f(x)| < \frac{1}{n}$  for any  $n \in \mathbb{N}$ , i.e.  $f(x) = 0$  for any  $x \in \mathbb{R} \setminus A$ . Thus,  $\mathbb{R} \setminus A = f^{-1}(0)$ , i.e.  $\mathbb{R} = \mathbb{Q} \cup A$  is contradiction, since  $\mathbb{Q}$  and  $A$  are countable sets, and  $\mathbb{R}$  is uncountable.

The proposition is proved.

We consider the function  $h: u'\mathbb{R} \rightarrow u_{\mathbb{R}}\mathbb{R}$ , determined as  $g(x) = 0$ , if  $x \in \mathbb{Q}$  and  $g(x) = 1$ , if  $x \in \mathbb{R} \setminus \mathbb{Q}$ . Then  $h$  is continuous function, which is not  $u'$ -continuous function, since  $g^{-1}(0) = \mathbb{Q} \notin \mathfrak{Z}(u'\mathbb{R})$ . By means of example 2.1, it is naturally to determine a special closed mapping of uniform spaces.

*Definition 2.1.* A mapping  $f: uX \rightarrow vY$  is called  $u$ -closed if  $f$  is a  $u$ -continuous and for any closed set  $F$  in  $X$  the image  $f(F)$  is closed in  $Y$ .

*Definition 2.2.* A mapping  $f: uX \rightarrow vY$  is called  $z_u$ -closed, if  $f$  is a  $u$ -continuous and for any uniformly closed set  $F \in \mathfrak{Z}(uX)$  the image  $f(F)$  is closed in  $Y$ .

Obviously, every  $u$ -closed mapping is  $z_u$ -closed. It takes place the next simple

*Proposition 2.2.* Every  $u$ -closed mapping  $f: uX \rightarrow vY$  is  $z_u$ -closed.

*Theorem 2.1.* A mapping  $f: uX \rightarrow vY$  is  $z_u$ -closed if and only if for every point  $y \in Y$  and every cozero-set  $U \in \mathcal{L}(uX)$ , containing  $f^{-1}(y)$ , i.e.  $f^{-1}(y) \subset U$ , there is such open neighborhood  $V$  of point  $y \in Y$ , that  $f^{-1}(V) \subset U$ .

*Proof.* Necessity. Let the mapping  $f: uX \rightarrow vY$  be a  $z_u$ -closed and  $y \in Y$  be an arbitrary point and uniformly cozero-set  $U \in \mathcal{L}(uX)$ , containing  $f^{-1}(y)$ , i.e.  $f^{-1}(y) \subset U$ . Then  $X \setminus U \in \mathfrak{Z}(uX)$  is uniformly zero-set and  $f(X \setminus U)$  is closed in  $Y$ . Set  $V = Y \setminus f(X \setminus U)$  is open in  $Y$  and  $y \in V$ , i.e.  $V$  is open neighborhood of point  $y$ . The next calculations:  $f^{-1}(V) = f^{-1}(Y \setminus f(X \setminus U)) = X \setminus f^{-1}(f(X \setminus U)) \subset X \setminus (X \setminus U) = U$  are provided, i.e.  $f^{-1}(V) \subset U$ .

*Sufficiency.* Conversely, let the condition of theorem is provided  $F \in \mathfrak{Z}(uX)$  be an arbitrary uniformly zero-set. The set  $U = X \setminus F \in \mathcal{L}(uX)$  is uniformly cozero-set and for any  $y \in Y \setminus f(F)$  we have  $f^{-1}(y) \subset X \setminus f(F) \subset X \setminus (f^{-1}(f(F))) \subset X \setminus F = U$ . Then there is an open neighborhood  $V_y$  of point  $y \in Y \setminus f(F)$  such, that  $f^{-1}(V_y) \subset U$ . Suppose  $V = \bigcup \{V_y: y \in Y \setminus f(F)\}$ . Then  $V$  is open  $Y$  and  $Y \setminus f(F) \subset V$  and  $f^{-1}(V) \subset U$ , i.e.  $f^{-1}(V) \cap F = \emptyset$ . Then  $V \cap f(F) = \emptyset$ , i.e.  $V \subset Y \setminus f(F)$ . Consequently,  $f(F) = Y \setminus V$ , i.e. the set  $f(F)$  is closed.

The theorem is proved completely.

The next theorem demonstrates, when  $z_u$ -closed of mappings implies  $u$ -closeness.

*Theorem 2.2.* If a mapping  $f: uX \rightarrow vY$  is closed and  $f^{-1}(y)$  is Lindelof for any point  $y \in Y$ , then the mapping  $f$  is  $u$ -closed.

*Proof.* Let  $y \in Y$  be an arbitrary point,  $f^{-1}(y)$  be a Lindelof and  $U$  be an arbitrary open set, containing  $f^{-1}(y)$ , i.e.  $f^{-1}(y) \subset U$ . Family  $\mathcal{L}(uX)$  is a basis of topology of the uniform space  $uX$  [5], hence for any point  $x \in f^{-1}(y) \subset U$  there exists such uniformly cozero-set  $V_x \in \mathcal{L}(uX)$ , which is the open neighborhood of  $x$ , then  $x \in V_x \subset U$ . Then the family  $\{V_x: x \in f^{-1}(y)\}$  is open covering of Lindelof space  $f^{-1}(y)$ . Let  $\{V_{x_n}: n \in \mathbb{N}\}$  be a countable sub covering. Since  $V_{x_n} \in \mathcal{L}(uX)$  for all  $n \in \mathbb{N}$ , then  $V' = \bigcup \{V_{x_n}: n \in \mathbb{N}\}$  is uniformly cozero-set [5] and  $f^{-1}(y) \subset V' \subset U$ . By  $z_u$ -closeness of mapping  $f: uX \rightarrow vY$ , there is such open neighborhood  $V$  of point  $y \in Y$ , that  $f^{-1}(V) \subset V' \subset U$ . Then, on one of the closed mappings criterion [3], it follows, that the mapping  $f: uX \rightarrow vY$  is  $u$ -closed.

The theorem is proved.

*Corollary 2.1.* Let  $f: uX \rightarrow vY$  be a bicomact  $u$ -continuous mapping, i.e.  $f^{-1}(y)$  is bicomact for any  $y \in Y$ . Then the next conditions are equivalent:

- 1)  $f: uX \rightarrow vY$  is  $z_u$ -closed.
- 2)  $f: uX \rightarrow vY$  is  $u$ -closed.

*Proof.* 1)  $\implies$  2). It follows immediately from the Theorem 2.2.

2) $\Rightarrow$ 1). It follows from the Proposition 2.2.

Corollary 2.1. allows to define a special perfect mapping.

*Definition 2.3.* A mapping  $f : uX \rightarrow vY$  is called  $u$ -perfect, if it is  $u$ -closed and bicomcompact.

*Remark 2.1.* Obviously, every uniformly perfect mapping ([1])  $f : uX \rightarrow vY$  is  $u$ -perfect, and every  $u$ -perfect mapping  $f : uX \rightarrow vY$  is perfect.

$$(s_{u_f}X, s\mathcal{U}_f) (s_{u_a}X, s\mathcal{U}_a),$$

*Example 2.2.* Let  $X$  be a locally bicomcompact Tychonoff space and  $aX$  its one-point Alexandroff bicomcompactification. Let  $\mathcal{U}_f$  be a fine uniformity on  $X$ , and  $\mathcal{U}_a$  be a minimal precompact uniformity on  $X$  (see [3, 7], Ex. 10) then  $\mathcal{U}_a \subset \mathcal{U}_f$  and  $\mathcal{U}_a \neq \mathcal{U}_f$ , as for the Samuel bicomcompactifications  $(s_{u_f}X, s\mathcal{U}_f)$  and  $(s_{u_a}X, s\mathcal{U}_a)$ , we have  $s_{u_f}X = \beta X$  is Stone-Cech bicomcompactification and  $s_{u_a}X = aX$  is the Aleksandroff bicomcompactification. Obviously,  $\beta X \neq aX$  (it is suppose that there is more than one uniformity on  $X$ ). A identical mapping  $1_x : \mathcal{U}_a X \rightarrow \mathcal{U}_f X$  is a topological homeomorphism, it is not  $u$ -continuous mapping. Thus, the class of perfect and closed mappings more wider than the class of  $u$ -perfect and  $u$ -closed mappings.

The next properties of  $u$ -continuous mapping of the uniform spaces are take please.

*Proposition 2.3.* A composition  $g \circ f : uX \rightarrow wZ$  of  $u$ -continuous mappings  $f : uX \rightarrow vY$  and  $g : vY \rightarrow wZ$  is  $u$ -continuous mapping.

*Proof.* Immediately follows from the definition of  $u$ -continuous mapping (Definition 1.2).

*Theorem 2.3.* If a composition  $g \circ f : uX \rightarrow wZ$  of  $u$ -continuous mappings  $f : uX \rightarrow vY$  and  $g : vY \rightarrow wZ$  is  $z_u$ -closed mapping, then restriction  $g|_{f(X)} : v'f(X) \rightarrow wZ$ , where  $\mathcal{V}' = \mathcal{V} \wedge f(X)$ , is  $z_u$ -closed mapping.

*Proof.* Let  $N \in \mathfrak{Z}(v'f(X))$ , i.e.  $N$  is a uniformly closed in  $f(X)$ . Then from the properties of the uniformly closed sets [3] it is follows there such  $N' \in \mathfrak{Z}(vY)$  exists, that  $N = N' \cap f(X)$ . Then  $f^{-1}(N') \in \mathfrak{Z}(uX)$  and  $g \circ f : uX \rightarrow wZ$  is  $z_u$ -closed mapping by the condition of the theorem. We have  $g|_{f(X)}(N) = g|_{f(X)}(N' \cap f(X)) = g(N' \cap f(X)) = (g \circ f)(f^{-1}(N'))$  and  $g|_{f(X)}(N)$  is closed in  $Z$ .

The theorem is proved.

*Corollary 2.2.* If a composition  $g \circ f : uX \rightarrow wZ$  of  $u$ -continuous mappings  $f : uX \rightarrow vY$  and  $g : vY \rightarrow wZ$  is  $u$ -closed mapping, then restriction  $g|_{f(X)} : v'f(X) \rightarrow wZ$ , where  $\mathcal{V}' = \mathcal{V} \wedge f(X)$ , is  $u$ -closed mapping.

*Proof.* Proof follows from the  $z_u$ -closeness of any  $u$ -closed mapping (Proposition 2.5.).

*Proposition 2.4.* Let  $f : uX \rightarrow v$  be  $u$ -continuous mapping and  $u'X'$  be a uniform subspace of  $uX$ . Then restriction  $f|_{X'} : u'X' \rightarrow v'f(X')$ , where  $\mathcal{V}' = \mathcal{V} \wedge f(X')$ , is  $u$ -continuous mapping too.

*Proof.* Let  $F$  be a uniformly closed in  $f(X')$ , i.e.  $F \in \mathfrak{Z}(v'f(X'))$ . Then there such function  $f \in C^*(v'f(X'))$  exists, that  $F = g^{-1}(0)$ . By the Katetov Theorem [7], there such function  $h \in C^*(vY)$  exists, that  $h|_{f(X')} = g$ . Then a function  $h \circ f : uX \rightarrow u_{\mathbb{R}}\mathbb{R}$  is  $u$ -continuous and  $(h \circ f)|_{X'} = g \circ f|_{X'}$ . Hence we have  $(g \circ f|_{X'})^{-1}(0) = (h \circ f)^{-1}|_{X'} = f^{-1}(h^{-1}(0)) \cap X' = f^{-1}(g^{-1}(0)) \in \mathfrak{Z}(u'X')$ , where and  $f^{-1}(g^{-1}(0)) \cap X' = f^{-1}(h^{-1}(0))$ .

The proposition is proved.

*Proposition 2.5.* Let  $f : uX \rightarrow vY$  be  $z_u$ -closed mapping and  $v'Y'$  be uniform subspace of  $vY$ , where  $\mathcal{V}' = \mathcal{V} \wedge Y'$  and  $Y' \subset Y$ . Then a mapping of restriction  $f|_{f^{-1}(Y')} : u'f^{-1}(Y) \rightarrow v'Y'$ , where  $\mathcal{U}' = \mathcal{U} \wedge f^{-1}(Y')$ , is  $z_u$ -closed mapping too.

*Proof.* It follows from the equality  $f|_{f^{-1}(Y')}(N \cap f^{-1}(Y')) = f(N) \cap Y'$  for any  $N \in \mathfrak{Z}(X)$ .

The proposition is proved.

*Proposition 2.6.* Let  $f : uX \rightarrow uY$  be  $u$ -closed mapping and  $u'X'$  be closed uniform subspace of  $uX$ . Then a constriction  $f|_{X'} : u'X' \rightarrow v'f(X')$ , where  $\mathcal{U}' = \mathcal{U} \wedge f(X)$ , is a  $u$ -closed mapping too.

*Proof.* It follows from Proposition 2.13 and definitions of  $u$ -closed mappings.

*Theorem 2.4.* Let  $f : uX \rightarrow vY$  and  $g : uX \rightarrow wZ$  be a subjective  $u$ -continuous mapping of the uniform spaces  $uX, vY, wZ$  and  $f$  is a  $u$ -closed mapping. Then diagonal product  $f \Delta g : uX \rightarrow v \times wY \times Z$ , where  $\mathcal{V} \times \mathcal{W}$  is the product of the uniformities  $\mathcal{V}$  and  $\mathcal{W}$ , is  $u$ -closed mapping.

*Proof.* For a diagonal mapping  $f \Delta g : uX \rightarrow v \times wY \times Z$ , by the definition, we have  $(f \Delta g)(x) = (f(x), g(x))$  Let  $i_X : uX \rightarrow uX$  and  $i_Z : wZ \rightarrow wZ$  be identical uniform homeomorphisms. Suppose  $f \times i_Z : u \times wX \times Z \rightarrow v \times wY \times Z$ ,  $i_X \Delta g : uX \rightarrow u \times wX \times Z$ , where  $(f \times i_Z) : (x, z) = (f(x), z)$  and  $(i_X \Delta g)(x) =$

$(x, g(x))$  for any  $x \in X$  and  $z \in Z$ . If  $M \subset Z$  and  $F \subset X$  are closed sets, then  $f(F) \times M$  is closed subset of  $Y \times Z$ , hence,  $(f \times i_Z)(F, M) = f(F) \times M$  and  $f \times i_Z$  is  $u$  — closed mapping. The mapping  $i_X \Delta g: X \rightarrow X \times Z$  is uniform homeomorphism of the space  $uX$  and  $\Gamma_g = \{(x, g(x)): x \in X\}$  is a graph of a mapping  $g$  it is a closed subspace of  $u \times wX \times Z$ . The closeness of the graph  $\Gamma_g$  in  $X \times Z$  follows from the  $uX$  and  $wZ$  are Hausdorff spaces. Then mapping  $f \Delta g$  is a composition of the mappings  $i_X \Delta g: uX \rightarrow u \times wX \times Z$  and  $f \times i_Z|_{\Gamma_g}: v'\Gamma_g \rightarrow v \times wY \times Z$ , where  $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$  and mapping  $f \times i_Z|_{\Gamma_g}$  is  $u$  — closed as a restriction of the closed mapping  $f \times i_Z$  onto the closed subspace  $\Gamma_g \subset X \times Z$ , and  $f \times i_Z|_{\Gamma_g}: v'\Gamma_g$  is a uniform homeomorphism. Thus,  $f \Delta g = (f \times i_Z)|_{\Gamma_g} \circ (i_X \Delta g)$  is a  $u$  — closed mapping. We have a diagram.

The theorem is proved.

$$\begin{array}{ccc} X & \xrightarrow{i_X \Delta g} & \Gamma_g \xrightarrow{(f \times i_Z)|_{\Gamma_g}} Y \times Z \\ & \searrow & \nearrow \\ & & f \Delta g \end{array}$$

*Theorem 2.5.* Let  $f: uX \rightarrow vY$  and  $g: uX \rightarrow wZ$  are a subjective  $u$  — continuous mappings of the uniform spaces  $uX, vY, wZ$  and a composition is  $g \circ f: uX \rightarrow wZ$  is  $u$  — closed mapping. Then the mapping  $f: X \rightarrow vY$  is  $u$  — closed too.

*Proof.* By the condition of theorem  $g \circ f: uX \rightarrow wZ$  is  $u$  — closed and  $f: uX \rightarrow vY$  is a  $u$  — continuous mapping, according to the Theorem 2.4.,  $f \Delta (g \circ f): uX \rightarrow u \times wX \times Z$  is a  $u$  — closed mapping. By the surjectivity of mappings  $f$  and  $g \circ f$  we have  $(f \Delta (g \circ f))(x) = (f(x), (g \circ f)(x))$  for any  $x \in X$ . Then  $\{(f(x), (g \circ f)(x)): x \in X\} = \{f(x), g(f(x)): x \in X\} = \{(y, g(y)): y \in Y\} = \Gamma_g$ .

Obviously, that  $(f \Delta (g \circ f))(x) = \{f(x), g(f(x)): x \in X\} = \{(y, g(y)): y \in Y\} = \Gamma_g$ . The graph  $\Gamma_g$  is closed subspace  $Y \times Z$  and the mapping  $\pi_Y|_{\Gamma_g}: v'\Gamma_g \rightarrow vY$ , where  $\pi_Y: v \times wY \times Z \rightarrow vY$  and  $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$ , is uniform homeomorphism mapping. Then  $f = \pi_Y|_{\Gamma_g} \circ (f \Delta (g \circ f)): uX \rightarrow vY$  is  $u$  — closed mapping as a composition of the uniform homeomorphism  $\pi_Y|_{\Gamma_g}: v'\Gamma_g \rightarrow vY$ , and  $u$  — closed mapping  $f \Delta (g \circ f): uX \rightarrow u \times wX \times Z$ . The next diagram takes place. We note, that the closeness of graph  $\Gamma_g$  in  $Y \times Z$  is essential, as soon as for any

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ & \searrow & & \nearrow & \\ & & Y & \times & Z \\ & \downarrow f & & \leftarrow \pi_Y|_{\Gamma_g} & \\ & & Y & & \end{array}$$

Closed  $F \subset X, (f \Delta (g \circ f))(F) = F'$  is closed in  $Y \times Z$  and its image  $\pi_Y|_{\Gamma_g}(F')$  is closed in  $Y$ . It means, that  $f(F) = \pi_Y|_{\Gamma_g}(F')$  and  $f(F)$  is closed in  $Y$ , i.e.  $f$  is  $u$  — closed.

The theorem is proved.

### References

- 1 Frolik Z. Applications of complete families of continuous functions to the theory of  $Q$ -spaces // Journal Czech.-Math. — 1961. — No. 11. — P. 115–133.
- 2 Энгелкинг Р. Общая топология. — М.: Мир, 1986.
- 3 Isbell J.R. Uniform spaces // Providence. — 1964.
- 4 Бурбаки Н. Общая топология. Основные структуры. — М.: Наука, 1968.
- 5 Charalambous M.G. Uniform Dimension Functions: PhD dis. — Univ. of London, 1971.
- 6 Gillman L., Jerison M. Rings of continuous functions. — New York, 1960.
- 7 Katetov M. On real-valued functions in topological spaces // Journal Fund.-Math. — 1951. — No. 38. — P. 85–91.

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**Бірқалыпты кеңістіктердің тұйық бейнелеулері туралы**

Бірқалыпты кеңістіктер жалпы топологияда кеңістіктердің маңызды класы болып табылады. Зерттеудің мақсаты — бірқалыпты кеңістіктердің қасиеттеріне қатысты жаңа теоремаларды дәлелдеу. Мақалада  $z_u$ -тұйық бейнелеулерінің сипаттамасы қойылған.  $z_u$ -тұйықталуы және биокомпакт бейнелеулері кластағы  $u$ -тұйықталуы тепе-теңдігі дәлелдеді,  $u$ -тұйық бейнелеулері үшін негізгі қасиеттері белгіленді.

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**О замкнутых отображениях равномерных пространств**

Равномерные пространства являются важным классом пространств в общей топологии. Цель исследования состоит в доказательстве новых теорем, касающихся свойств равномерных пространств. В статье установлена характеристика  $z_u$ - замкнутых отображений, доказана равносильность  $z_u$ - замкнутости и  $u$ - замкнутости в классе бикompактных отображений. Также для  $u$ -замкнутых отображений установлены их основные свойства.

## References

- 1 Frolik Z. *Journal Czech.-Math.*, 1961, 11, p. 115–133.
- 2 Angelking R. *General topology*, Moscow: Mir, 1986.
- 3 Isbell J.R. *Providence*, 1964.
- 4 Burbaki N. *General topology. Core Structures*, Moscow: Nauka, 1968.
- 5 Charalambous M.G. *Uniform Dimension Functions*: PhD dis., Univ. of London, 1971.
- 6 Gillman L., Jerison M. *Rings of continuous functions*, New York, 1960.
- 7 Katetov M. *Journal Fund.-Math.*, 1951, 38, p. 85–91.