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Equations of vibration of a two-dimensionally layered plate strictly based on the decision of various boundary-value problems

In this paper, the theory of oscillation of laminated plates of building structures is developed, which is rigorously grounded in the formulation of various boundary-value oscillation problems. When studying the oscillation of plates, the exact three-dimensional problem is replaced by a simpler, two-dimensional problem for the points of the middle plane of the plate, which imposes restrictions on the external conditions. These limitations boil down to the fact that external forces can not be high-frequency. Since the general equations of plate oscillation, the resulting wound contain derivatives of any order in terms of coordinates x, y and time t, are structured and therefore not suitable for solving applied problems and performing engineering calculations. For this, it is necessary to formulate approximate boundary-value oscillation problems.

Keywords: vibrations, a plate, a deformable medium, an elastic and viscoelastic medium.

Introduction. Materials used in building structures have elastic and viscoelastic properties, are anisotropic, multilayered and with other mechanical characteristics. Flat elements are components of many designs. The construction of general and approximate equations of oscillation of various types of flat elements presents an actual problem in the development of the theoretical foundations for calculating building structures and construction in general. Such problems include the problems of improving the models of the nonstationary nature of structures and their elements, the materials of which exhibit complex mechanical, rheological properties inherent in various building structures under the influence of various external factors.

In this paper, the theory of oscillation of laminated plates of building structures is developed, which is rigorously grounded in the formulation of various boundary-value oscillation problems.

Main part. Suppose that an infinite plate in thickness $2h_1$ is under the surface of a semi-infinite medium at depth $(h_0 - h_1)$. Plane XY will be placed in the middle plane of the plate at $z = 0$. The axis OZ is directed toward the outer surface of the outer layer. Denote the parameters of the layer by the index «1» , the upper layer $[-\infty < (x, y) < \infty; h_1 \leq z \leq (h_0 - h_1)]$ will be denoted by the index «2», and the lower half-space $[-\infty < (x, y) < \infty; -h_1 \leq z \leq 0]$ by the index «3».

We assume that the materials of the upper layer, plates and bases are homogeneous, isotropic, exhibit viscous properties.

We introduce the potentials $\Phi^{(l)}$ and $\Psi^{(l)}$ of longitudinal transverse waves in accordance with the well-known formulas

$$\vec{u}^{(l)} = grad\Phi^{(l)} + rot\vec{\Psi}^{(l)}, \tag{1}$$

where $\vec{u}^{(l)}$ — vectors of displacement of points in a layer, plates and bases.

In the potentials $\Phi^{(l)}$ and $\Psi^{(l)}$, the equations of motion of the layer, the plate and the base take the form:

$$N_l(\triangle\Phi^{(l)}) = \rho_l \frac{\partial^2\Phi^{(l)}}{\partial t^2}, \quad M_l(\triangle\vec{\Psi}^{(l)}) = \rho_l \frac{\partial^2\Psi^{(l)}}{\partial t^2}, \tag{2}$$

where the operator N_l is:

$$N_l = L_l + 2M_l.$$

$\triangle$-three-dimensional Laplace operator

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

L_l, M_l — viscoelastic operators.

By the Helmholtz theorem, in the absence of internal sources, the vector potential $\vec{\Psi}$ of transverse waves must satisfy the condition:

$$div\vec{\Psi}^{(l)} = 0, \tag{3}$$

which is the closing equation for finding four unknown potentials $\Phi^{(l)}, \Psi_1^{(l)}, \Psi_2^{(l)}, \Psi_3^{(l)}$.

The displacements $u,\ v,\ w$, deformations ε_{ij} and stresses in Cartesian coordinates through the potentials Φ and $\vec{\Psi}$ of longitudinal and transverse waves are determined from known formulas.

In [1] it is shown that the boundary problem oscillation plate located beneath the surface, reduces to a solution of integro-differential equations (2) at the boundary and initial conditions: the outer surface ($z = h_0$)

$$\sigma_{zz}^{(2)} = f_z^{(2)}(x,y,t); \quad \sigma_{jz}^{(2)} = f_{zj}^{(2)}(x,y,t); \tag{4}$$

at the contact boundary, the top layer-plate ($z = h_1$)

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}; \quad \sigma_{jz}^{(1)} = 0; \quad \sigma_{jz}^{(2)} = 0; \quad w^{(1)} = w^{(2)}; \tag{5}$$

at the plate boundary, the base ($z = -h_1$)

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(3)} + f_{3z}^{(3)}(x,y,t);$$

$$\sigma_{jz}^{(1)} = 0; \quad \sigma_{ij}^{(3)} + f_{zj}^{(3)}(x,y,t) = 0;$$

$$w^{(1)} = w^{(3)} + f_0^{(3)}(x,y,t) \quad (j = x,y). \tag{6}$$

In addition, the damping conditions at infinity must be satisfied, i.e. $z \to -\infty \quad \Phi^{(3)} = 0$;

$$\Psi_1^{(3)} = \Psi_2^{(3)} = \Psi_3^{(3)} = 0. \tag{7}$$

The initial conditions are zero, i.e.

$$\Phi^{(l)} = \frac{\partial \Phi^{(l)}}{\partial t} = \frac{\partial \vec{\Psi}_j^{(l)}}{\partial t} = \vec{\Psi} = 0, \quad (l = \overline{1,3}), \quad t = 0 \quad (j = 1,2,3). \tag{8}$$

The problem of oscillation of a plate in a differentiable medium is reduced to the study of equation (2), which satisfies the boundary conditions (4), (5), (6) and the initial conditions (8).

When studying the oscillation of plates, the exact three-dimensional problem is replaced by a simpler, two-dimensional problem for the points of the middle plane of the plate, which imposes restrictions on the external conditions. These limitations boil down to the fact that external forces can not be high-frequency.

The problem formulated above is solved by applying Fourier transforms in X and Y and Laplace transforms in t.

The general solution formulated by the three-dimensional problem with zero initial conditions was found in [1], and general expressions for displacements and stresses were obtained.

$$u^{(l)} = \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} + C_1 Q_{1n} \frac{\partial^2}{\partial x^2} \right) U^{(l)} + C_1 Q_{1n} \frac{\partial}{\partial x} \left(\frac{\partial V^{(l)}}{\partial y} + W^{(l)} \right) \right] \frac{z^{2n}}{(2n)!} \right\} +$$

$$+ \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} - D_1 Q_{1n} \frac{\partial^2}{\partial x^2} \right) U_1^{(l)} - D_1 Q_{1n} \frac{\partial}{\partial x} \left(\frac{\partial V_1^{(l)}}{\partial y} + \lambda_2^{(l)} W_1^{(l)} \right) \right] \frac{z^{2n+1}}{(2n+1)!} \right\};$$

$$v^{(l)} = \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} + C_1 Q_{1n} \frac{\partial^2}{\partial y^2} \right) V^{(l)} + C_1 Q_{1n} \frac{\partial}{\partial y} \left(\frac{\partial U^{(l)}}{\partial x} + W^{(l)} \right) \right] \frac{z^{2n}}{(2n)!} \right\} +$$

$$+ \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} - D_1 Q_{1n} \frac{\partial^2}{\partial y^2} \right) V_1^{(l)} - D_1 Q_{1n} \frac{\partial}{\partial y} \left(\frac{\partial U_1^{(l)}}{\partial x} + \lambda_2^{(l)} W_1^{(l)} \right) \right] \frac{z^{2n+1}}{(2n+1)!} \right\};$$

$$w^{(l)} = \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} + C_1 Q_{1n} \lambda_1^{(1)} \right) W^{(l)} + C_1 Q_{1n} \lambda_1^{(l)} \left(\frac{\partial U^{(l)}}{\partial x} + \frac{\partial V^{(l)}}{\partial y} \right) \right] \frac{z^{2n+1}}{(2n+1)!} \right\} +$$

$$+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(n)}-D_1Q_{1n}\lambda_2^{(1)}\right)W_1^{(l)}-D_1Q_{1n}\left(\frac{\partial U_1^{(l)}}{\partial x}+\frac{\partial V_1^{(l)}}{\partial y}\right)\right]\frac{z^{2n}}{(2n)!}\right\}; \tag{9}$$

$$C_1=1-\frac{N_1}{M_1};$$

$$Q_{1n}=\sum_{m=0}^{n-1}\lambda_1^{(n-m-1)}\cdot\lambda_2^{(m)};$$

$$D_1=1-\frac{M_1}{N_1},$$

where the operators $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ are equal

$$\lambda_1^{(1)}=\left[\rho_1N_1^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\triangle\right];$$

$$\lambda_2^{(1)}=\left[\rho_1M_1^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\triangle\right];$$

$$\triangle=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}. \tag{10}$$

$$\sigma_{xx}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[(1-C_1)\lambda_2^{(n)}+C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}+\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\right]\frac{\partial U^{(l)}}{\partial x}+\right.\right.$$

$$\left.+\left[C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}+\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)-(1+C_1)\lambda_2^{(n)}\right]\left(\frac{\partial V^{(l)}}{\partial y}+W^{(l)}\right)\right\}\frac{z^{2n}}{(2n)!}+$$

$$+\sum_{n=0}^{\infty}\left\{\left[2D_1Q_{1n}\left(\lambda_2^{(1)}+\frac{\partial^2}{\partial y^2}\right)+(1+\alpha D_1)\lambda_1^{(n)}\right]\frac{\partial U^{(l)}}{\partial x}+\right.$$

$$\left.\left.+\left[(1+\alpha D_1)\lambda_1^{(n)}-2D_1Q_{1n}\frac{\partial^2}{\partial x^2}\right]\left(\frac{\partial V_1^{(l)}}{\partial y}+\lambda_2^{(1)}W_1^{(l)}\right)\right\}\frac{z^{2n+1}}{(2n+1)!}\right];$$

$$\sigma_{yy}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[(1+C_1)\lambda_2^{(n)}+C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\frac{\partial V^{(l)}}{\partial y}+\right.\right.$$

$$\left.+\left[C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_1^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-(1+C_1)\lambda_2^{(n)}\right]\left(\frac{\partial V^{(l)}}{\partial x}+W^{(l)}\right)\right\}\frac{z^{2n}}{(2n)!}+$$

$$+\sum_{n=0}^{\infty}\left\{\left[1+2D_1\lambda_1^{(n)}+2D_1Q_{1n}\left(\lambda_2^{(n)}+\frac{\partial^2}{\partial x^2}\right)\right]\frac{\partial V^{(l)}}{\partial y}+\right.$$

$$\left.\left.+\left[(1+D_1)\lambda_1^{(n)}-2D_1Q_{1n}\frac{\partial^2}{\partial y^2}\right]\left(\frac{\partial U_1^{(l)}}{\partial x}+\lambda_2^{(1)}W_1^{(l)}\right)\right\}\frac{z^{2n+1}}{(2n+1)!}\right];$$

$$\sigma_{zz}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1Q_{1n}\left(\lambda_2^{(1)}-\Delta\right)-(1+C_1)\lambda_2^{(n)}\right]\left(\frac{\partial U^{(l)}}{\partial x}+\frac{\partial V^{(l)}}{\partial y}\right)+\right.\right.$$

$$\left.+[(1+C_1)\lambda_2^{(n)}+C_1Q_{1n}(\lambda_2^{(1)}-\Delta)W^{(1)}\right\}\frac{z^{2n}}{(2n)!}+\sum_{n=0}^{\infty}\left\{-\left[2D_1Q_{1n}\lambda_2^{(1)}+\lambda_1^{(n)}\right]\times\right.$$

$$\left.\left.\times\left(\frac{\partial U_1^{(l)}}{\partial x}+\frac{\partial V_1^{(l)}}{\partial y}\right)+\lambda_2^{(1)}\left[\lambda_1^{(n)}+2D_1Q_{1n}\Delta\right]W_1^{(1)}\right\}\frac{z^{2n+1}}{(2n+1)!}\right];$$

$$\sigma_{xy}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[2C_1Q_{1n}\frac{\partial^2}{\partial x^2}+\lambda_2^{(n)}\right]\frac{\partial V_1^{(l)}}{\partial y}+\left[\lambda_2^{(n)}+2C_1Q_{1n}\frac{\partial^2W^{(1)}}{\partial x\partial y}\right]\right\}\times\right.$$

$$\times\frac{z^{(2n)}}{(2n)!}+\sum_{n=0}^{\infty}\left\{\left[\lambda_1^{(n)}+D_1Q_{1n}\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\frac{\partial U_1^{(l)}}{\partial y}+\right.$$

$$\left.+\left[D_1Q_{1n}\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)+\lambda_1^{(n)}\right]\frac{\partial V_1^{(l)}}{\partial x}-2D_1Q_{1n}\lambda_2^{(1)}\frac{\partial^2 W_1^{(1)}}{\partial x\partial y}\right\}\frac{z^{2n+1}}{(2n+1)!}\Bigg];$$

$$\sigma_{xz}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1\left[2\lambda_1^{(1)}Q_{1n}+\lambda_2^{(n)}\right]\frac{\partial^2 V^{(1)}}{\partial x\partial y}+\left[2C_1Q_{1n}\lambda_1^{(1)}\frac{\partial^2}{\partial x^2}+\right.\right.\right.\right.$$

$$\left.+\lambda_2^{(n)}\left[(1-C_1)\lambda_1^{(1)}-C_1\frac{\partial^2}{\partial x^2}\right]V^{(1)}+\left[(1+C_2)\lambda_2^{(n)}+2C_1Q_{1n}\lambda_1^{(1)}\right]\frac{\partial W^{(1)}}{\partial x}\right\}\times$$

$$\times\frac{z^{2n+1}}{(2n+1)!}\Bigg]+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)D_1Q_{1n}+\lambda_1^{(n)}\right]V_1^{(1)}-\right.$$

$$\left.-2D_1Q_{1n}\frac{\partial^2 V_1^{(1)}}{\partial x\partial y}-\left[D_1Q_{1n}\left(\lambda_2^{(1)}-\Delta\right)-\lambda_1^{(n)}\right]\frac{\partial W_1^{(1)}}{\partial x}\right\}\frac{z^{2n}}{(2n)!}\Bigg];$$

$$\sigma_{yz}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1\left[2\lambda_1^{(1)}Q_{1n}+\lambda_2^{(n)}\right]\frac{\partial^2 U^{(1)}}{\partial x\partial y}+\left[2C_1Q_{1n}\lambda_1^{(1)}\frac{\partial^2}{\partial y^2}+\right.\right.\right.\right.$$

$$\left.+\lambda_2^{(n)}\left[(1-C_1)\lambda_1^{(1)}-C_1\frac{\partial^2}{\partial x^2}\right]V^{(1)}+\left[2C_1Q_{1n}\lambda_1^{(1)}+(1+C_2)\lambda_2^{(n)}\right]\frac{\partial W^{(1)}}{\partial x}\right\}\times$$

$$\times\frac{z^{2n+1}}{(2n+1)!}\Bigg]+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)D_1Q_{1n}+\lambda_1^{(n)}\right]V_1^{(1)}-\right.$$

$$\left.-2D_1Q_{1n}\frac{\partial^2 V_1^{(l)}}{\partial x\partial y}-\left[D_1Q_{1n}\left(\lambda_2^{(1)}-\Delta\right)-\lambda_1^{(n)}\right]\frac{\partial W_1^{(1)}}{\partial x}\right\}\frac{z^{2n}}{(2n)!}\Bigg],\qquad(11)$$

where the unknowns $U^{(1)}, V^{(1)}, W_1^{(1)}$ are the tangents and normal displacements of the points of the plane $z=0$ and the points of the middle plane of the plate, $U_1^{(1)}, V_1^{(1)}, W^{(1)}$ — the values of the derivatives along Z the transverse displacement or the values of the strain type (the deformation at $Z=0$).

In this case, the operators $\lambda_1^{(2)}$, $\lambda_2^{(1)}$ are two-dimensional integro-differential, describing the propagation of longitudinal and transverse waves in the plane $z=0$.

To find the unknowns $U^{(1)}$, $V^{(1)}$, $W_1^{(1)}$, $U_1^{(1)}$, $V_1^{(1)}$, $W^{(1)}$ we have the boundary conditions (4)–(6).

Using expressions (9) and (11) for stresses and displacements, substituting these expressions into the boundary conditions (4)–(6), equations are obtained for determining the unknown functions that are general solutions of the formulated problem and describing the vibrations of the three-dimensional medium.

To study the oscillations of rectangular plates in the plan, it is necessary to formulate boundary value problems.

Under the boundary-value problems of oscillation of a bounded plate in a plane located below the surface, we mean the derivation of the oscillation equation for the plates; the formulation of the boundary conditions along the edges of the plate and the initial conditions for the functions.

Since the general equations of plate oscillations obtained by the author [2] contain derivatives of any order in coordinates x, y and time t, are structured and therefore not suitable for solving applied problems and performing engineering calculations. For this, it is necessary to formulate approximate boundary-value oscillation problems.

In [3] an approximate equation of the transverse vibration of the plate was obtained for the transverse displacement $W_1^{(1)}$ of the middle plane of the plate in the form

$$A_1\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)+A_2\left(\frac{\partial^4 W_1^{(1)}}{\partial t^4}\right)+A_3\left(\triangle\,\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)+A_4(\triangle^3\,W_1^{(1)})+P(W_1^{(1)})=\Phi(x,y,t),\qquad(12)$$

where $A_j, P, \Phi(x,y,t)$ are equal

$$A_1=\rho_1M^{-1}h_1+\rho_2N_2^{-1}(h_0-h_1);$$

$$A_2=\rho_1^2M_1^{-1}(N_1^{-1}+3M_1^{-1})\frac{h_1^3}{6}+\rho_2N_2^{-1}\left[\rho_2N_2^{-1}\frac{(h_0-h_1)^3}{6}-\rho_1N_1^{-1}\frac{h_1^2(h_0-h_1)}{2}\right];$$

$$A_3=\left[(3-4M_2N_2^{-1})\frac{(h_0-h_1)^3}{6}-(2M_1N_1^{-1})\frac{h_1^2(h_0-h_1)}{2}\right]\rho_2N_2^{-1}-2\rho_1(3M_1^{-1}-2N_1^{-1})\frac{h_1^3}{3};$$

$$A_4=4(1-M_1N_1^{-1})\frac{h_1^3}{3}-4(1-M_2N_2^{-1})(M_2N_2^{-1})\frac{(h_0-h_1)^3}{6};$$

$$P=\frac{S}{2}\rho_1M_1\left\{\frac{\partial}{\partial t}+\frac{h_1^2}{2}\left[\rho_1(M_1^{-1}+3N_1^{-1})\left(\frac{\partial^3}{\partial t^3}\right)-4\left(\frac{\partial}{\partial t}\right)\triangle\right]+2(M_1N_1^{-1})(\rho_2N_2^{-1})\left(\frac{\partial^3}{\partial t^3}\right)h_1(h_0-h_1)\right\};$$

$$\Phi(x,y,t)=\left[1-(3-2M_1N_1^{-1})\frac{h_1^2}{2}\triangle+(\rho_1M_1^{-1})\left(\frac{\partial^2}{\partial t^2}\right)\frac{h_1^2}{2}\right]\times$$

$$\times\left\{F_3+M_2^{-1}f_z^{(3)}\left[(M_1N_1^{-1})(\rho_2N_2^{-1})\left(\frac{\partial^3}{\partial t^3}\right)(h_0-h_1)h_1\right]\right\}. \tag{13}$$

The reaction of the base P, is determined by the formula (13), contains both the velocity of the transverse displacements of the plane $z=0$ and the odd time derivatives.

Thus, the law of resistance $P(W_1^{(1)})$ (13) explicitly contains the parameters of the plate, the base and the upper layer.

Despite the fact that equation (12) is approximate, it is rather complicated. The operators (13) contain all the parameters and operators that characterize both the mechanical and rheological properties of the materials of the plates, the layers and the base and their thickness.

We derive the boundary conditions along the edges of the rectangular plate. For simplicity, let us consider a plane boundary $x=const$, for boundary conditions $y=const$, it is easy to write from the conditions for $x=const$, and for an arbitrary curvilinear boundary, using known formulas, through the boundary conditions at $x=const$, $y=const$.

The boundary conditions will be derived from the theory of thick plates or plates. Based on their boundary conditions on the surface of the plate $z=h$ or $z=-h$ obtain the dependence of the quantities $u_1^{(1)}, V_1^{(1)}$ on the transverse displacement $W_1^{(1)}$.

$$u_1^{(1)}=-\frac{\partial W_1^{(1)}}{\partial x};\qquad V_1^{(1)}=-\frac{\partial W_1^{(1)}}{\partial y}. \tag{14}$$

Hard edge fixing $x=const$. As is known from the theory of thick plates, there are two possible types of such fixation

$$u_1^{(1)}=v_1^{(1)}=w_1^{(1)}=0 \tag{15}$$

or

$$u_1^{(1)}=w_1^{(1)}=\sigma_{xy}^{(1)}=0. \tag{16}$$

Hingelessly supported edge $x=const$.

There are also two types of fastening for this fastening.

$$u_1^{(1)}=w_1^{(1)}=\sigma_{xx}^{(1)}=0 \tag{17}$$

or

$$w_1^{(1)}=\sigma_{xx}^{(1)}=\sigma_{xy}^{(1)}=0. \tag{18}$$

A stress-free edge.

For a free edge, the strict conditions have the form

$$\sigma_{xx}^{(1)}=\sigma_{xz}^{(1)}=\sigma_{xy}^{(1)}=0. \tag{19}$$

The rigid and hinged fastening is fairly simple and, using approximate expressions (13) and (15), for transverse displacement we obtain the boundary conditions:
for rigid fixing

$$W_1^{(1)}=\frac{\partial W_1^{(1)}}{\partial x}=0, \tag{20}$$

for articulation

$$W_1^{(1)} = \frac{\partial^2 W_1^{(1)}}{\partial x^2} = 0. \tag{21}$$

Consider a free edge. Using the first two conditions (19) and approximate expressions for the displacements u, v, w and the stresses σ_{ij} obtained in [6], we obtain

$$(2+3D_1)\frac{\partial^2 W_1^{(1)}}{\partial x^2} + (1+D_1)\left[2\frac{\partial^2 W_1^{(1)}}{\partial y^2} - \rho M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)\right] = 0,$$

$$\frac{\partial}{\partial x}\left[2\triangle W_1^{(1)} - \rho M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)\right] = 0. \tag{22}$$

Substituting the second derivative of $W_1^{(1)}$ with respect to time from the first expression (22) to the second, we obtain

$$(2+3D_1)\frac{\partial^2 W_1^{(1)}}{\partial x^2} + (1+D_1)\frac{\partial^2 W_1^{(1)}}{\partial y^2} - \rho(1+D_1)M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right) = 0;$$

$$\frac{\partial^3 W_1^{(1)}}{\partial x^3} = 0. \tag{23}$$

The third of the conditions (19) gives $\frac{\partial^3 W_1^{(1)}}{\partial x^3} = F(t)$, that is, in the first approximation $\frac{\partial W_1^{(1)}}{\partial x}$ does not depend on y and is determined after solving a particular problem.

The first term in (23) differs from the classical one, and the second term coincides. The first condition (23) takes into account the deformability of the edge over time and is analogous to the d'Alembert principle for the dynamics of a material point.

The general initial conditions for a plate as a three-dimensional body have the form:

$$u^{(1)} = v^{(1)} = w^{(1)} = 0; \qquad \frac{\partial u^{(1)}}{\partial t} = \frac{\partial v^{(1)}}{\partial t} = \frac{\partial w^{(1)}}{\partial t} = 0; \quad (t=0). \tag{24}$$

Using the relations (14) for displacements, we have

$$u^{(1)} = -\frac{\partial W_1^{(1)}}{\partial x} z + D_1 \frac{\partial}{\partial x}\triangle W_1^{(1)} \frac{z^3}{6};$$

$$v^{(1)} = -\frac{\partial W_1^{(1)}}{\partial y} z + D_1 \frac{\partial}{\partial y}\triangle W_1^{(1)} \frac{z^3}{6}; \tag{25}$$

$$w^{(1)} = W_1^{(1)} + \left[(2D_1 - 1)\triangle W_1^{(1)} + (1-D_1)\rho M^{-1}\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right]\frac{z^2}{2}.$$

In the beginning, we consider the initial conditions from (24) for the displacements themselves. Then from expressions (25) we obtain

$$\frac{\partial W_1^{(1)}}{\partial x} = 0; \qquad \frac{\partial}{\partial x}\triangle W_1^{(1)} = 0;$$

$$\frac{\partial W_1^{(1)}}{\partial y} = 0; \qquad \frac{\partial}{\partial y}\triangle W_1^{(1)} = 0; \tag{26}$$

$$W_1^{(1)} = 0; \qquad \frac{\partial^2 W_1^{(1)}}{\partial t^2} = 0. \tag{27}$$

Differentiating (25) with respect to u, using the second triple of initial conditions (24), we similarly obtain

$$\frac{\partial W_1^{(1)}}{\partial t} = \frac{\partial^3 W_1^{(1)}}{\partial t^3} = 0. \tag{28}$$

The initial conditions (26) and (27) give the necessary number of initial conditions for the transverse displacement $W_1^{(1)}$ of the fourth-order coordinate and time satisfying the hyperbolic equation.

Conclusions. The derivation of the boundary and initial conditions for the plate under the surface completely coincides with the analogous boundary initial conditions for the free plate, obtained in [2].

Thus, in formulating boundary value problems, the boundary conditions do not depend on the presence of the upper layer and the lower base.

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Әртүрлі шеттік есептердің қойылымымен қатаң негізделген екі өлшемді қатпарлы пластинканың тербеліс теңдеулері

Мақалада тербелістің әртүрлі шеттік есептерінің қойылымымен қатаң негізделген құрылыс конструкцияларындағы қатпарлы пластинкалардың тербеліс теориясы қарастырылды. Пластина тербелісін зерттегенде дәл үш өлшемді есеп пластинканың орта жазықтығының нүктелері үшін сыртқы әсерлерге шектеулер қоятын аса қарапайым, екі өлшемді есепке алмастырылды. Бұл шектеулер сыртқы әсерлердің жоғары жиілікте бола алмайтындығына келтіріледі. Алдында алынған пластина тербелісінің жалпы теңдеулерінің x, y координаталары және t уақыт бойынша кез келген ретті туындылары бар болғандықтан, құрылымы бойынша күрделі болып табылады. Сондықтан қолданбалы есептерді шығару және инженерлік есептеулер жүргізу үшін олар қажет емес. Ол үшін тербелістің жуық шеттік есептерін тұжырымдау керек.

Кілт сөздер: жоғары жиілік, қолданбалы есеп, өзгеріске ұшырайтын, серпімді орта, тербеліс теориясы.

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Уравнения колебания двумерной слоистой пластинки, строго обоснованные постановкой различных краевых задач

В статье развита теория колебания слоистых пластинок строительных конструкций, строго обоснованая постановкой различных краевых задач колебания. При исследовании колебания пластин точная трехмерная задача заменяется более простой, двумерной для точек срединной плоскости пластинки, что накладывает ограничения на внешние условия. Эти ограничения сводятся к тому, что внешние усилия не могут быть высокочастотными. Так как общие уравнения колебания пластин, полученные ранее, содержат производные любого порядка по координатам x, y и времени t , сложны по структуре и потому не пригодны для решения прикладных задач и проведения инженерных расчетов. Для этого необходимо сформулировать приближенные краевые задачи колебания.

Ключевые слова: вибрации, пластины, деформируемая среда, теория колебания слоистых пластинок.

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