UDC 517.98

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On multipliers in weighted Sobolev spaces. Part II

Let X, Y be Banach spaces whose elements are functions $y\colon \Omega \to \mathbb{R}$. We say that a function $z\colon \Omega \to \mathbb{R}$ is a pointwise multiplier on the pair (X, Y), if $Tx = zx \in Y$ and the operator $T\colon X \to Y$ is bounded. $M(X \to Y)$ denotes the multiplier space on the pair (X, Y). We introduce the norm $\|z;\, M(X \to Y)\| = \|T;\, X \to Y\|$ in $M(X \to Y)$. Let $1 \le p < \infty$. Let m be an integer. $W^m_{p,\omega_0,\omega_1}$ denotes the weighted Sobolev space with the finite norm $\|u\|_{W^m_{p,\omega_0,\omega_1}} = \|u; W^m_{p,\omega_0,\omega_1}\| = \|\omega_0^{1/p}|\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p,v}}$. The aim of this work is to obtain descriptions of multiplier spaces for the pair of weighted Sobolev spaces $(W^l_{p,\rho,v}, W^m_{q,\omega_0,\omega_1})$ in the case $1 \le q < p < \infty$.

Key words: weighted Sobolev space, pointwise multiplier.

Let Ω be a domain (an open connected set) in the n-dimensional Euclidian space $\mathbb{R}^n$ with the norm $|x| = \left(\sum\limits_{i=1}^{n} x_i^2\right)^{1/2}$. We denote by $L_p(\Omega)$, $1 \le p < \infty$, the space of all real valued measurable functions $f\colon \Omega \to \mathbb{R}$ with the finite norm $\|f\|_{L_p(\Omega)} = \|f;\, L_p(\Omega)\| = \left(\int_\Omega |f(x)|^p\, dx\right)^{\frac{1}{p}}$. We denote by $L_{p,loc}(\Omega)$ the space of functions f defined a.e. in Ω such that $f \in L_p(F)$ for any compact $F \subset \Omega$. Here $L^+_{p,loc}(\Omega)$ is the space of all a.e. positive functions of $L_{p,loc}(\Omega)$, $L_{loc}(\Omega) = L_{1,loc}(\Omega)$, $L^+_{loc}(\Omega) = L^+_{1,loc}(\Omega)$. A function v of $L^+_{loc}(\Omega)$ is called weight in Ω. Let α be a measure on Ω. Below $L_{p,\alpha}(\Omega)$ is the space of all real valued functions equipped with the finite weighted Lebesgue norm $\|u\|_{L_{p,\alpha}(\Omega)} = \left(\int\limits_\Omega |u|^p\, d\alpha(x)\right)^{\frac{1}{p}}$ $(1 \le p < \infty)$. If $d\alpha(x) = v(x)\, dx$, $v \in L^+_{loc}(\Omega)$, we write $L_{p,v}(\Omega)$. Note that $L_p(\Omega) = L_{p,v}(\Omega)$, if $v \equiv 1$. By C^∞, C^∞_0 we denote the space of all infinitely differentiable functions in $\mathbb{R}^n$ and the space of functions of C^∞ with compact support $\operatorname{supp} f$ in $\mathbb{R}^n$, respectively. When the domain is not indicated in the notation of a space or a norm then it is assumed to be $\mathbb{R}^n$. Throughout the paper we assume that $0 < m < l$ are integers.

Let $1 \le p < \infty$. Let m be an integer, $\omega_0,\ \omega_1 \in L^+_{loc}$. We denote by $W^m_{p,\omega_0,\omega_1}$ the completion of the set of $u \in C^\infty_0$ in the finite norm

$$\|u\|_{W^m_{p,\omega_0,\omega_1}} = \|u; W^m_{p,\omega_0,\omega_1}\| = \||\nabla_m u|\|_{L_{p,\omega_0}} + \|u\|_{L_{p,\omega_1}},$$

where $|\nabla_m u| = \left(\sum\limits_{|\alpha|=m} |D^\alpha u|^2\right)^{1/2}$. Here $W^m_{p,\omega} = W^m_{p,\omega_0,\omega_1}$ with $\omega_0 = 1,\ \omega_1 = \omega$, $W^m_p = W^m_{p,\omega_0,\omega_1}$ with $\omega_0 = 1,\ \omega_1 = 1$. By $W^m_{p,loc}$ we denote the space [1] $\{u\colon \eta u \in W^m_p \quad \text{for all} \quad \eta \in C^\infty_0\}$. Here I^n is the family of all cubes Q in the form

$$Q = Q_h = Q_h(x) = \{y \in \mathbb{R}^n\colon |y_i - x_i| < \frac{h}{2}, i = 1, \ldots, n\},\ \lambda Q = Q_{\lambda h}(x).$$

By c we denote constants depending only on the assigned numerical parameters, for example, $c = c(l, p, n)$, etc.

Let $h(\cdot)$ be a positive locally bounded function in $\mathbb{R}^n$. $\mathfrak{B}$ denotes the family (basis) of cubes $Q(x) = Q_{h(x)}(x)$, $x \in \mathbb{R}^n \setminus e$, where e is a set with measure 0. We use the following notation

$$\mathfrak{B} = \{Q(x)\} \quad \text{or} \quad \mathfrak{B} = \{Q(x) = Q_h(x)\}.$$

Definition. Let $\rho \in L^+_{loc.}$ We say that a weight ρ satisfies the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x)\}$, if there exist $b > 1$ such that for a.a. x

$$b^{-1}\rho(x) < \rho(y) < b\,\rho(x), \quad \text{for a.a. } y \in Q(x).$$

Let ρ satisfy the slow variation condition. We denote by $W^m_p(\rho^\mu, \rho^\nu)$ the space $W^m_{p,\omega_0,\omega_1}$ with $\omega_0 = (\rho^\mu)^p$, $\omega_1 = (\rho^\nu)^p$.

Theorem A [1]. Let $1 < p, q < \infty$, $pl > n$. Then

$$\|\gamma; M(W^l_p \to W^m_q)\| \le c \sup_{\{Q_1\}} \|\gamma; W^m_q(Q_1)\|.$$

Let us denote by $\Sigma\mathfrak{B}_{(\tau)}$ $(0 < \tau \le 1)$ the set of all finite or countable subfamilies $\{\tau Q^j\} \subset \mathfrak{B}$, in which the cubes $\tau Q^j = \tau Q(x^j)$ are pairwise disjoint. Further, we take

$$T_{(\tau)} = \sup_{\{Q^j\} \subset \Sigma\mathfrak{B}_{(\tau)}} \left\{ \sum_{\{Q^j\}} \left[\int_{Q^j} \rho^{s(l-n/p)q}(x) \left(|\nabla_m \gamma|^q + \rho^{-smq}|\gamma|^q \right) dx \right]^{p/(p-q)} \right\}^{(p-q)/pq}.$$

Theorem 1. Let $1 \le q < p < \infty$, $pl > n$, $-\infty < \mu, s < \infty$. Let $\gamma \in W^m_{q,loc}$. Assume that ρ satisfies the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x) = Q_{h(x)}(x)\}$, $h(x) = \rho(x)^s$. Then the following statemens are true:

(a) If $T = T_{(1)} < \infty$,
then $\gamma \in M(W^l_p(\rho^\mu, \rho^{\mu-sl}) \to W^m_q(\rho^\mu, \rho^{\mu-sm}))$ and the norm satisfies the following inequality

$$\|\gamma; M(W^l_p(\rho^\mu, \rho^{\mu-sl}) \to W^m_q(\rho^\mu, \rho^{\mu-sm}))\| \le c\,T.$$

(b) If $\gamma \in M(W^l_p(\rho^\mu, \rho^{\mu-sl}) \to W^m_q(\rho^\mu, \rho^{\mu-sm}))$,
then $\infty > \|\gamma; M(W^l_p(\rho^\mu, \rho^{\mu-sl}) \to W^m_q(\rho^\mu, \rho^{\mu-sm}))\| \ge c\,T_{(1/2)}$.

Proof. (a) Let $u \in C^\infty_0$, $F = \operatorname{supp} u$. Let $\{(Q^j, \frac{2}{3}Q^j), j \in J\}$ be a Besicovitch double covering extracted from the family $\{Q(x), x \in F\}$ $(Q^j = Q(x^j)$. Let $\{\psi_j\}_{j\in J}$ be a partition of unity corresponding to the double covering, namely, $\psi_j \in C^\infty_0$, $0 \le \psi_j \le 1$, $\sum\limits_{j\in J} \psi_j = 1$ and $\sup|D^\alpha \psi_j| \le c\,h_j^{-|\alpha|}$ for all multiindexes $\alpha = (\alpha_1, \dots, \alpha_n)$ of order $|\alpha| = \sum_i |\alpha_i|$ ([2], Chapter 2). We have $(\gamma u)(x) = u(x) \sum\limits_{j\in J} \psi_j(x)\gamma(x) = u(x) \sum\limits_{j\in J} \gamma_j(x)$, $\gamma_j = \gamma\psi_j$, $j \in J$,

$$D^\alpha \gamma(x) = D^\alpha \left(\sum_{j\in J} \gamma\psi_j \right)(x) = \sum_{j\in J} D^\alpha (\gamma\psi_j)(x) = \sum_{j\in J} D^\alpha \gamma_j.$$

Moreover, there exist finite constants $\widetilde{\varkappa}_1$, $\widetilde{\varkappa}_2 > 0$, such that

$$\int_F \left(\rho^{\mu q}(x)|\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x)|\gamma u|^q\right) dx \leq$$
$$\leq \widetilde{\varkappa}_1^q \sum_{k\in J}\int_{Q^k}\sum_{j\in J}\rho^{\mu q}(x)\left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x)|(\gamma_j u)(x)|^q\right) dx \leq$$
$$\leq c\,\widetilde{\varkappa}_1^q \sum_{j\in J}\sum_{k\in J}\int_{Q^k\cap Q^j}\rho^{\mu q}(x)\left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x)|(\gamma_j u)(x)|^q\right) dx \leq$$
$$\leq c\,\widetilde{\varkappa}_1^q \sum_{j\in J}\int_{Q^j}\rho^{\mu q}(x)\left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x)|(\gamma_j u)(x)|^q\right) dx \leq$$
$$\leq c\,\widetilde{\varkappa}_1^q\widetilde{\varkappa}_2 \max_{1\leq i\leq\widetilde{\varkappa}_2}\sum_{j\in J_i}\int_{Q^j}\rho^{\mu q}(x)\left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x)|(\gamma_j u)(x)|^q\right) dx \leq$$
$$\leq c\,\widetilde{\varkappa}_1^q\widetilde{\varkappa}_2 \sum_{j\in J_{i_0}}\int_{Q^j}\rho^{\mu q}(x)\left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x)|(\gamma_j u)(x)|^q\right) dx,$$

where the maximum coincides with the sum by the subfamily J_{i_0}.

Let $U \in C_0^\infty$ be the continuation of $\tilde{u}(\xi) = u(x^j + h_j\xi)$ from $Q_1 = Q_1(0)$ to the cube $\frac{3}{2}Q_1$ such that $\operatorname{supp} U \subset \frac{3}{2}Q_1$ and

$$\|U; W_p^l\| \leq c_5\,\|\tilde{u}; W_p^l(Q_1)\| \tag{1}$$

(see [3]). Let $\tilde{\gamma}_j(\xi) = \gamma(x^j + h_j\xi)\psi_j(x^j + h_j\xi)$. Note that $\operatorname{supp}\tilde{\gamma}_j(\xi) \subset \bar{Q}_1$. Thus, by using (1), Theorem A, we obtain

$$\int_{Q^j}\left(|\nabla_m(u\gamma_j)|^q\rho^{\mu q}(x) + |u\gamma_j|^q\rho^{(\mu-sm)q}(x)\right) dx \leq \rho_j^{\mu q}h_j^{n-mq}\int_{Q_1}\left(|\nabla_m(\tilde{u}\tilde{\gamma}_j)|^q + |\tilde{u}\tilde{\gamma}_j|^q\right) d\xi \leq$$
$$\leq \rho_j^{\mu q}h_j^{n-mq}\|\tilde{\gamma}_j; M(W_p^l \to W_q^m)\|^q\|\tilde{u}; W_p^l(Q_1)\|^{q/p} \leq$$
$$\leq c\,\rho_j^{\mu q}h_j^{n-mq}\sup_x\int_{Q_1(y)\cap Q_1(0)}\left(|\nabla_m\tilde{\gamma}_j|^q + |\tilde{\gamma}_j|^q\right) d\xi \times \left(h_j^{lp-n}\int_{Q^j}|\nabla_l u|^p + h_j^{-n}\int_{Q^j}|u|^p\right)^{q/p} \leq$$
$$\leq c\int_{Q_1(0)}\left(|\nabla_m\tilde{\gamma}_j|^q + |\tilde{\gamma}_j|^q\right) d\xi \times h_j^{n-mq}\left(h_j^{lp-n}\int_{Q^j}\rho^{\mu p}|\nabla_l u|^p + h_j^{-n}\rho^{\mu p}\int_{Q^j}|u|^p\right)^{q/p} =$$
$$= c\,h_j^{(l-n/p)q}\left(\int_{Q^j}\left(|\nabla_m\gamma_j|^q + h^{-mq}|\gamma_j|^q\right) dx\right) \times \left(\int_{Q^j}\left(\rho^{\mu p}(x)|\nabla_l u|^p + \rho^{(\mu-sl)p}(x)|u|^p\right) dx\right)^{q/p}.$$

For each $x \in Q^j$, we have

$$|\nabla_m\gamma_j(x)| \leq c\sum_{|\alpha|=m}|D^\alpha(\gamma\psi_j)(x)| \leq \sum_{|\alpha|=m}\sum_{0\leq\beta\leq\alpha}|D^\beta\gamma(x)||D^{\alpha-\beta}\psi_j(x)| \leq$$
$$\leq c\sum_{|\alpha|=m}\sum_{0\leq\beta\leq\alpha}|D^\beta\gamma(x)|h_j^{-|\alpha-\beta|} \leq c\sum_{k=0}^m|\nabla_k\gamma(x)|h_j^{k-m}. \tag{2}$$

By (2) and embedding theorems of Sobolev spaces $W_p^l(Q_1)$ [4], we have

$$\int_{Q^j}|\nabla_m\gamma_j|^q \leq c\left(\sum_{k=0}^m h_j^{k-m}\|\nabla_m\gamma; L_q(Q^j)\|\right)^q \leq c\int_{Q^j}\left(|\nabla_m\gamma|^q + h^{-mq}|\gamma|^q\right) dx. \tag{3}$$

Inequality (3) implies that

$$\int_{Q^j}\left(|\nabla_m(u\gamma_j)|^q\rho^{\mu q}(x)+|u\gamma_j|^q\rho^{(\mu-sm)q}(x)\right)dx\le$$

$$\le c\,h_j^{(l-n/p)q}\left(\int_{Q^j}\left(|\nabla_m\gamma|^q+h^{-mq}|\gamma|^q\right)dx\right)\times\left(\int_{Q^j}\left(\rho^{\mu p}(x)|\nabla_l u|^p+\rho^{(\mu-sl)p}(x)|u|^p\right)dx\right)^{q/p}. \quad (4)$$

By (4) and the Holder inequality, we have

$$\int_F\left(\rho^{\mu q}(x)|\nabla_m(\gamma u)|^q+\rho^{(\mu-sm)q}(x)|\gamma u|^q\right)dx\le$$

$$\le c\,\widetilde{\varkappa}_1^q\widetilde{\varkappa}_2\sum_{j\in J_{i_0}}\left(\int_{Q^j}\rho^{s(l-n/p)q}(x)\left(|\nabla_m\gamma|^q+\rho^{-smq}|\gamma|^q\right)dx\right)\times$$

$$\times\left(\int_{Q^j}\left(|\rho^{\mu}\nabla_l u|^p+|\rho^{(\mu-sl)}u|^p\right)dx\right)^{q/p}\le \quad (5)$$

$$\le c\left\{\sum_{j\in J_{i_0}}\left[\int_{Q^j}\rho^{s(l-n/p)q}(x)\left(|\nabla_m\gamma|^q+\rho^{-smq}|\gamma|^q\right)dx\right]^{p/(p-q)}\right\}^{(p-q)/p}\times$$

$$\times\|u;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^q\le c\,T^q\|u;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^q.$$

Hence it follows the upper estimate of $\|\gamma;M(W_p^l(\rho^\mu,\rho^{\mu-sl})\to W_q^m(\rho^\mu,\rho^{\mu-sm}))\|$.

(b) We take $\eta\in C_0^\infty(Q_1)$, $0\le\eta\le1$, $\eta=1$ in $\frac12Q_1$. Assume that there exist $\varphi_j\in C_0^\infty(Q^j)$, $\varphi_j(x)=\eta\left(\frac{x-x^j}{h_j}\right)$, such that $\varphi_j=1$ in $\frac12Q^j$. Here $\{Q^j,j\in\Lambda\}\subset\Sigma\mathfrak{B}$, $Q^j=Q(x^j)$. Then

$$\frac{\|\gamma\varphi_j;W_q^m(\rho^\mu,\rho^{\mu-sm})\|^q}{\|\varphi_j;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^q}\ge c\,\frac{\rho_j^{\mu q}\int_{\frac12Q^j}\left(|\nabla_m\gamma|^q+\rho_j^{-smq}|\gamma|^q\right)dx}{\rho_j^{\mu q-s(l-n/p)q}}=$$

$$=c\left(\rho_j^{s(l-n/p)q}\int_{\frac12Q^j}|\nabla_m\gamma|^q\,dx+\rho_j^{s(l-m-n/p)q}\int_{\frac12Q^j}|\gamma|^q\,dx\right)=c\,a_j^q,$$

where $a_j^q=\rho_j^{s(l-n/p)q}\int_{\frac12Q^j}|\nabla_m\gamma|^q\,dx+\rho_j^{s(l-m-n/p)q}\int_{\frac12Q^j}|\gamma|^q\,dx$.

Let $u_j=a_j^{q/(p-q)}\frac{\varphi_j}{\|\varphi_j;W_p^l(\rho^\mu,\rho^{\mu-sl})\|}$. We take $u=\sum\limits_{j\in\Lambda}u_j$. Then

$$\|\gamma u_j;W_q^m(\rho^\mu,\rho^{\mu-sm})\|^q\ge a_j^{q^2/(p-q)}\left(\frac{\|\gamma\varphi_j;W_q^m(\rho^\mu,\rho^{\mu-sm})\|}{\|\varphi_j;W_p^l(\rho^\mu,\rho^{\mu-sl})\|}\right)^q\ge c\,a_j^{pq/(p-q)}.$$

In addition, $\|u_j;W_p^l(\rho^\mu,\rho^{\mu-sl})\|=a_j^{q/(p-q)}$. So that,

$$\|u;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^p=\sum_{j\in\Lambda}\|u_j;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^p=\sum_{j\in\Lambda}a_j^{pq/(p-q)}\le$$

$$\le c\sum_{j\in\Lambda}\|\gamma u_j;W_q^m(\rho^\mu,\rho^{\mu-sm})\|^q=c\,\|\gamma u;W_q^m(\rho^\mu,\rho^{\mu-sm})\|^q\le$$

$$\le c\,\|\gamma;M(W_p^l(\rho^\mu,\rho^{\mu-sl})\to W_q^m(\rho^\mu,\rho^{\mu-sm}))\|^q\|u;W_p^l(\rho^\mu,\rho^{\mu-sl})\|^q$$

for $u = \sum\limits_{j\in\Lambda} u_j$, which implies

$$\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^{p-q} \le c\,\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \to W_q^m(\rho^\mu, \rho^{\mu-sm}))\|^q < \infty$$

for $u \in C^\infty \cap W_p^l(\rho^\mu, \rho^{\mu-sl})$. Next, we show

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \to W_q^m(\rho^\mu, \rho^{\mu-sm}))\|^{pq/(p-q)} \ge c\,\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p =$$
$$= c\sum_{j\in\Lambda} \|u_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p = c\sum_{j\in\Lambda} a_j^{pq/(p-q)} =$$
$$= c\sum_{j\in\Lambda}\left[\rho_j^{s(l-n/p)q}\int_{\frac{1}{2}Q^j} |\nabla_m\gamma|^q\,dx + \rho_j^{s(l-m-n/p)q}\int_{\frac{1}{2}Q^j}|\gamma|^q\,dx\right]^{p/(p-q)}.$$

Thus, we have the following final estimate

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \to W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \ge c\,T_{(1/2)}.$$

The proof of Theorem 1 is complete.

Remark. Since a lattice of unit cubes is contained in $\{Q_1(x), x \in \mathbb{R}^n\}$, Theorem 1 implies the two-sided estimate of $\|\gamma; M(W_p^l \to W_q^m)\|$ $(1 \le q < p < \infty)$, obtained in [1].

Theorem 2. Let $1 \le q < p < \infty$, $pl > n$, $-\infty < \mu, s < \infty$. Let $\gamma \in W^m_{q,loc}$. Let ρ satisfy the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x) = Q_{h(x)}(x)\}$, where $h(x) = \rho(x)^s$. Then

$$W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)}) \cap W^m_{q,loc} \subset M(W_p^l(\rho^\mu, \rho^{\mu-sl}), W_q^m(\rho^\mu, \rho^{\mu-sm})).$$

And the following inequality holds:

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}), W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \le c\,\|\gamma; W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)})\|.$$

Proof. Let $u \in C_0^\infty$, $F = \operatorname{supp} u$. By using (), we have the estimate

$$\int_F \left(\rho^{\mu q}(x)|\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x)|\gamma u|^q\right)dx \le$$
$$\le c\left\{\sum_{j\in J_{i_0}}\left[\int_{Q^j}\rho^{s(l-n/p)q}(x)\left(|\nabla_m\gamma|^q + \rho^{-smq}|\gamma|^q\right)dx\right]^{p/(p-q)}\right\}^{(p-q)/p}\times$$
$$\times\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \le c\,T^q\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q.$$

Let us continue to estimate ()

$$\int_F \left(\rho^{\mu q}(x)|\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x)|\gamma u|^q\right)dx \le$$
$$\le c\left\{\int_{Q^j}\rho^{s(l-n/p)q}(x)\left(|\nabla_m\gamma|^q + \rho^{-smq}|\gamma|^q\right)dx\right\}\times$$
$$\times\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \le c\,\|\gamma; W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)})\|^q\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q,$$

which implies the statement of Theorem 2. The proof of the theorem is complete.

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А.Мырзағалиева

Салмақты Соболев кеңістіктеріндегі мультипликаторлар жайлы. II-бөлім

X, Y — $y\colon \Omega \to \mathbb{R}$ функцияларынан тұратын банах кеңістіктері болсын. Егер $Tx = zx \in Y$ және $T\colon X \to Y$ операторы шенелген болса, онда $z\colon \Omega \to \mathbb{R}$ функциясы (X, Y) жұбындағы нүктелік мультипликатор деп аталады. $M(X \to Y)$ арқылы (X, Y) жұбындағы мультипликаторлар кеңістігін белгілейміз. $M(X \to Y)$ мультипликаторлар кеңістігінде норманы келесідей анықтаймыз: $\|z;\, M(X \to Y)\| = \|T;\, X \to Y\|$. $1 \le p < \infty$, m – бүтін сан болсын. $W^m_{p,\omega_0,\omega_1}$ арқылы салмақты Соболев кеңістігін белгілеп, норманы келесідей анықтаймыз: $\|u\|_{W^m_{p,\omega_0,\omega_1}} = \|u; W^m_{p,\omega_0,\omega_1}\| = \|\omega_0^{1/p}|\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p,v}}$. Аталмыш жұмыстың мақсаты — салмақты Соболев кеңістіктерінің $(W^l_{p,\rho,v}, W^m_{q,\omega_0,\omega_1})$ жұбы үшін мультипликаторлар кеңістіктерін сипаттау.

А.Мырзагалиева

О мультипликаторах в весовых пространствах Соболева. Часть II

Пусть X, Y — банаховы пространства функций $y\colon \Omega \to \mathbb{R}$. Функция $z\colon \Omega \to \mathbb{R}$ называется точечным мультипликатором в паре (X, Y), если $Tx = zx \in Y$ и оператор $T\colon X \to Y$ ограничен. Через $M(X \to Y)$ обозначается пространство мультипликаторов в паре (X, Y). В $M(X \to Y)$ вводится норма $\|z;\, M(X \to Y)\| = \|T;\, X \to Y\|$. Пусть $1 \le p < \infty$, m – целое. Через $W^m_{p,\omega_0,\omega_1}$ обозначается весовое пространство Соболева с конечной нормой вида $\|u\|_{W^m_{p,\omega_0,\omega_1}} = \|u; W^m_{p,\omega_0,\omega_1}\| = \|\omega_0^{1/p}|\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p,v}}$. Цель данной работы заключается в описании пространств мультипликаторов для пары весовых пространств Соболева $(W^l_{p,\rho,v}, W^m_{q,\omega_0,\omega_1})$.

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